Chapter 11

11.1-1 a)

$$F(z)=\sum_{k=1}^{\infty}\gamma^{k-1}z^{-k}=\frac{1}{\gamma}\sum_{k=1}^{\infty}\left(\frac{\gamma}{z}\right)^k$$

$$=\frac{1}{\gamma}\left[\frac{\gamma}{z}+\left(\frac{\gamma}{z}\right)^2+\left(\frac{\gamma}{z}\right)^3+\cdots\right]$$

$$=\frac{1}{\gamma}\left[-1+\left(1+\frac{\gamma}{z}+\left(\frac{\gamma}{z}\right)^2+\left(\frac{\gamma}{z}\right)^3+\cdots\right)\right]$$

$$=\frac{1}{\gamma}\left[-1+\frac{1}{1-\frac{\gamma}{z}}\right]=\frac{1}{z-\gamma}$$

b)

$$F(z)=\sum_{k=m}^{\infty}z^{-k}=z^{-m}+z^{-(m+1)}+z^{-(m+2)}+\cdots$$

$$=z^{-m}\left[1+\frac{1}{z}+\frac{1}{z^2}+\cdots\right]$$

$$=z^{-m}\left(\frac{1}{1-\frac{1}{z}}\right)=\frac{z}{z^m(z-1)}$$

c)

$$F(z)=\sum_{k=0}^{\infty}\frac{\gamma^k}{k!}z^{-k}=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\gamma}{z}\right)^k$$

note that $e^x=\sum_{k=0}^{\infty}\frac{1}{k!}x^k$ so : $F(z)=e^{\gamma/z}$

d)

$$F[z]=\sum_{k=0}^{\infty}\frac{1}{k!}(\ln\alpha)^k z^{-k}=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\ln\alpha}{z}\right)^k$$

using the result in part c : $F(z)=e^{\ln\alpha/z}=(e^{\ln\alpha})^{1/z}=\alpha^{1/z}$

11.1-2 a)

$$f[k]=(2)^{k+1}u[k-1]+(e)^{k-1}u[k]=4(2)^{k-1}u[k-1]+\frac{1}{e}(e)^k u[k]$$

therefore: $F[z]=\frac{4}{z-2}+\frac{1}{e}\frac{z}{z-e}$

11-1.2 b)

$$f[k] = k\gamma^k u[k-1] = k\gamma^k u[k] - 0 = k\gamma^k\ u[k]$$

hence: $$F[z] = \frac{\gamma z}{(z-\gamma)^2}$$

c) $$f[k] = [(2)^{-k} \cos\frac{\pi k}{3}]\,u[k-1] = (2)^{-k}\cos\frac{\pi k}{3}\,u[k] - \delta[k]$$

therefore: $$F(z) = \frac{z(z-0.25)}{z^2-0.5z+0.25} - 1 = \frac{0.25(z-1)}{z^2-0.5z+0.25}$$

d) Because $k(k-1)(k-2)=0$ for $k=0,1,2$

$$f[k] = k(k-1)(k-2)2^{k-3}u[k-m] = k(k-1)(k-2)(2)^{k-3}\ u[k]$$

$k=0,1$ or 2 therefor $f[k] = (2)^{-3}\{k(k-1)(k-2)2^k\ u[k]\}$

and $$F(z) = (2)^{-3}\left[\frac{3!\ (2)^3 z}{(z-2)^4}\right] = \frac{6z}{(z-2)^4}$$

11-1.3 a)

$$\frac{F(z)}{z} = \frac{z-4}{(z-2)(z-3)} = \frac{2}{z-2} + \frac{1}{z-3}$$

$$F(z) = 2\frac{z}{z-2} - \frac{z}{z-3}$$

$$f[k] = [2\ (2)^k - (3)^k]\,u[k]$$

b)

$$\frac{F(z)}{z} = \frac{z-4}{z(z-2)(z-3)} = \frac{-2/3}{z} + \frac{1}{z-2} + \frac{1/3}{z-3}$$

$$F(z) = -\frac{2}{3} + \frac{z}{z-2} - \frac{1}{3}\frac{z}{z-3}$$

$$f[k] = -\frac{2}{3}\delta[k] + [(2)^k - \frac{1}{3}(3)^k]\,u[k]$$

c) $$\frac{F(z)}{z} = \frac{e^{-2}-2}{(z-e^{-2})(z-2)} = \frac{1}{z-e^{-2}} - \frac{1}{z-2}$$

$$F(z) = \frac{z}{z-e^{-2}} - \frac{z}{z-2}$$

$$f[k] = [e^{-2k} - 2^k]\,u[k]$$

d) $$\frac{F(z)}{z} = \frac{2z+3}{(z-1)(z-2)(z-3)} = \frac{5/2}{z-1} - \frac{7}{z-2} + \frac{9/2}{z-3}$$

$$F(z) = \frac{5}{2}\frac{z}{z-1} - 7\left(\frac{z}{z-2}\right) + 9/2\,\frac{z}{z-3}$$

$$f[k] = \left[\tfrac{5}{2} - 7(2)^k + \frac{9}{2}(3)^k\right] u[k]$$

e) $$\frac{F(z)}{z} = \frac{-5z+22}{(z+1)(z-2)^2} = \frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}$$

multiply both sides by z and let $z \to \infty$. This yields

$$0 = 3 + k + 0 \Rightarrow k = -3$$

$$F(z) = 3\frac{z}{z+1} - 3\frac{z}{z-2} + 4\frac{z}{(z-2)^2}$$

$$f[k] = [3(-1)^k - 3(2)^k + 2k(2)^k]\,u[k]$$

f) $$\frac{F(z)}{z} = \frac{1.4z + 0.08}{(z-0.2)(z-0.8)^2} = \frac{1}{z-0.2} + \frac{k}{z-0.8} + \frac{2}{(z-0.8)^2}$$

Multiply both Sides by z and let $z \to \infty$, this yields:

$$0 = 1 + k \Rightarrow k = -1$$

$$F(z) = \frac{z}{z-0.2} - \frac{z}{z-0.8} + 2\,\frac{z}{(z-0.8)^2}$$

$$f[k] = \left[(0.2)^k - (0.8)^k + \frac{5}{2}k(0.8)^k\right] u[k]$$

g) We use pair 12C with $A=1$, $B=-2$, $a=-0.5$, $|\gamma|=1$. Therefore

$$r=\sqrt{4}=2 \qquad \beta=\cos^{-1}\left(\frac{0.5}{1}\right)=\frac{\pi}{3} \qquad \theta=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{3}$$

$$f[k]=2(1)^k\cos\left(\frac{\pi k}{3}+\frac{\pi}{3}\right)u[k]=2\cos\left(\frac{\pi k}{3}+\frac{\pi}{3}\right)u[k]$$

h)

$$\frac{F(z)}{z}=\frac{2z^2-0.3z+0.25}{z(z^2+0.6z+0.25)}=\frac{1}{z}+\frac{Az+B}{z^2+0.6z+25}$$

Multiply both sides by z and let $z\to\infty$. This yields

$$2=1+A \Rightarrow A=1$$

Setting $z=1$ on both sides yields

$$\frac{1.95}{1.85}=1+\frac{1+B}{1.85} \Rightarrow B=-0.9$$

$$F(z)=1+\frac{z(z-0.9)}{z^2+0.6z+0.25}$$

For the second fraction on right side, we use pair 12C with $A=1$

$B=-0.9$ $a=0.3$ and $|\gamma|=0.5$ This yields

$$r=\sqrt{10} \qquad \beta=\cos^{-1}\left(\frac{-0.3}{0.5}\right)=2.214 \qquad \theta=\tan^{-1}\left(\frac{1.2}{0.4}\right)=1.249$$

$$f[k]=\delta[k]+\sqrt{10}\,(0.5)^k\cos(2.214k+1.249)u[k]$$

i)

$$\frac{F(z)}{z}=\frac{2(3z-23)}{(z-1)(z^2-6z+25)}=\frac{-2}{z-1}+\frac{Az+B}{z^2-6z+25}$$

Multiply both sides by z and let $z\to\infty$. This yields

$$0=-2+A \Longrightarrow A=2$$

Set $Z = 0$ on both sides to obtain.

$$\frac{46}{25} = 2 + \frac{B}{25} \Rightarrow B = -A$$

$$F(z) = -2 + \frac{z}{z-1} + \frac{z(2z-4)}{z^2 - 6z + 25}$$

For the second fraction on the right-hand side, we use pair 12c with

$A = 2$, $B = -4$, $a = -3$ and $|\gamma| = 5$

$$r = \frac{\sqrt{17}}{2} \qquad \beta = \cos^{-1}\left(\frac{3}{5}\right) = 0.927 \qquad \theta = \tan^{-1}\left(\frac{-1}{4}\right) = -0.25$$

$$f[k] = \left[-2 + \frac{\sqrt{17}}{2}(5)^k \cos(0.927k - 0.25)\right] u[k]$$

j)

$$\frac{F(z)}{z} = \frac{3.83z + 11.34}{(z-2)(z^2 - 5z + 25)} = \frac{1}{z-2} + \frac{Az + B}{z^2 - 5z + 25}$$

Multiply both sides by z and let $z \to \infty$. This yields:

$$0 = 1 + A \Longrightarrow A = -1$$

Setting $z = 0$ on both sides yields

$$\frac{11.34}{-50} = -\frac{1}{2} + \frac{B}{25} \Longrightarrow B = 6.83$$

$$F(z) = \frac{z}{z-2} + \frac{z(-z + 6.83)}{z^2 - 5z + 25}$$

For the second fraction on right-hand side, use pair 12c with $A = -1$

$B = 6.83$, $a = -2.5$ and $|\gamma| = 5$.

$$r = \sqrt{2} \qquad \beta = \cos^{-1}(0.5) = \frac{\pi}{3} \qquad \theta = \tan^{-1}\left(\frac{-4.33}{-4.33}\right) = -\frac{3\pi}{4}$$

$$f[k] = [(2)^k + \sqrt{2}(5)^k \cos(\frac{\pi}{3}k - \frac{3\pi}{4})]\, u[k]$$

k)

$$\frac{F(z)}{z} = \frac{z(-2z^2+8z-7)}{(z-1)(z-2)^3} = \frac{1}{z-1} + \frac{k_1}{z-2} + \frac{k_2}{(z-2)^2} + \frac{2}{(z-3)^2}$$

Multiply both sides by z and let $z \to \infty$. This yields

$$-2 = 1 + k_1 \Longrightarrow k_1 = -3$$

Set $z = 0$ on both sides to obtain

$$0 = -1 + 3/2 + k_2/4 - 1/4 \Longrightarrow k_2 = -1$$

$$F(z) = \frac{z}{z-1} - 3\frac{z}{z-2} - \frac{z}{(z-2)^2} + 2\frac{z}{(z-2)^3}$$

$$f[k] = [1 - 3(2)^k - \frac{k}{2}(2)^k + \frac{1}{4}k(k-1)2^k]\, u[k]$$

11.1-4

Long division of $2z^3 + 13z^2 + z$ by $z^3 + 7z^2 + 2z + 1$ yields

$$F(z) = 2 - \frac{1}{z} + \frac{4}{z^2} + \cdots$$

Therefore $f[0] = 2$, $f[1] = -1$, $f[2] = 4$

11.1-5

$$F(z) = \frac{\gamma z}{z^2 - 2\gamma z + \gamma^2}$$

Long division yields:

$$\frac{\gamma z}{z^2 - 2\gamma z + \gamma^2} = \frac{\gamma}{z} + 2(\frac{\gamma}{z})^2 + 3(\frac{\gamma}{z})^3 + \cdots$$

Therefore $f[0] = 0$, $f[1] = \gamma$, $f[2] = 2\gamma^2$, $f[3] = 3\gamma^3$, ... and

$$f[k] = k\gamma^k u[k]$$

11.2-1

$$f[k] = u[k] - u[k-m]$$

$$F(z) = \frac{z}{z-1} - z^{-m}\frac{z}{z-1} = \frac{1-z^{-m}}{1-z^{-1}}$$

11.2-2

$$f[k] = \delta[k-1] + 2\delta[k-2] + 3\delta[k-3] + 4\delta[k-4] + 3\delta[k-5] + 2\delta[k-6]$$
$$+ \delta[k-7]$$

Therefore

$$F(z) = \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \frac{4}{z^4} + \frac{3}{z^5} + \frac{2}{z^6} + \frac{1}{z^7}$$
$$= \frac{z^6 + 2z^5 + 3z^4 + 4z^3 + 3z^2 + 2z + 1}{z^7}$$

Alternate method

$$f[k] = k\{u[k] - u[k-5]\} + (-k+8)\{u[k-5] - u[k-9]\}$$

$$= k\,u[k] - 2k\,u[k-5] + k\,u[k-9] + 8u[k-5] - 8u[k-9]$$

$$= k\,u[k] - 2\{(k-5)u[k-5] + 5u[k-5]\} + (k-9)u[k-9] + 9u[k-9]$$
$$+ 8u[k-5] - 8u[k-9] =$$

$$k\,u[k] - 2(k-5)u[k-5] + (k-9)u[k-9] - 2u[k-5] + u[k-9]$$

Therefore

$$F(z) = \frac{z}{(z-1)^2} - \frac{2z}{z^5(z-1)^2} + \frac{z}{z^9(z-1)^2} - \frac{2z}{z^5(z-1)} + \frac{z}{z^9(z-1)}$$

$$= \frac{z}{z^9(z-1)^2}\left[z^9 - 2z^4 + 1 - 2z^4(z-1) + (z-1)\right]$$

$$= \frac{1}{z^7(z-1)^2}\left[z^8 - 2z^4 + 1\right]$$

Note the two answers are identical

11.2-3 a) $f[k] = k^2 \gamma^k u[k]$

Repeated application of Eq (11.22) To $\gamma^k u[k] \Longleftrightarrow \frac{z}{z-\gamma}$ yields

$$k\gamma^k u[k] \Longleftrightarrow \frac{\gamma z}{(z-\gamma)^2}$$

$$k^2\gamma^k u[k] \Longleftrightarrow \frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$$

b) Considering above result, let $\gamma = 1$ Then we have $k^2 u[k] \Longleftrightarrow \frac{z(z+1)}{(z-1)^3}$

Application of Eq (11.22) to this result yields

$$k^3 u[k] = -z\frac{d}{dz}\left[\frac{z(z+1)}{(z-1)^3}\right] = \frac{z(z^2+4z+1)}{(z-1)^4}$$

c) $$f[k] = a^k \{u[k] - u[k-m]\}$$

$$= a^k u[k] - a^m a^{k-m} u[k-m]$$

$$F(z) = \frac{z}{z-a} - \frac{a^m z}{z-a} z^{-m} = \frac{z}{z-a}\left[1-\left(\frac{a}{z}\right)^m\right]$$

d) $$f[k] = ke^{-2k}u[k-m] = (k-m+m)e^{-2(k-m+m)}u[k-m]$$

$$= e^{-2m}(k-m)e^{-2(k-m)}u[k-m] + me^{-2m}e^{-2(k-m)}u[k-m]$$

$$F(z) = e^{-2m}\frac{e^{-2}z}{(z-e^{-2})^2} z^{-2} + me^{-2m}\left(\frac{z}{z-e^{-2}}\right)z^{-2}$$

$$= \frac{e^{-2m}}{z(z-e^{-2})^2}\left[\frac{1}{e^2}(1-m) + mz\right]$$

11.2-4 pair 2

$$u[k] = \delta[k] + \delta[k-1] + \delta[k-2] + \delta[k-3] + \dots$$

$$u[k] \Longleftrightarrow 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$$

Repeated application of Eq (11.22) to pair 2 yields pair 3, 4 and 5.

Application of Eq (11.21) to pair 2 yields 7, and application of time-delay (11.16a) to pair 7 yields pair 6. Repeated application of Eq (11.22) to pair 7 yields pair 8 and 9.

11.3-1

$$y[k+1] - \gamma y[k] = f[k+1]$$

with $y[0] = -M$, $f[k] = P u[k-1]$

$$F(z) = \frac{P}{z-1}$$

$$y[k] \Longleftrightarrow Y(z) \qquad y[k+1] \Longleftrightarrow zY(z) + Mz$$

The z transform of the system equation is

$$zY(z) + Mz - \gamma Y(z) = \frac{Pz}{z-1}$$

$$(z-\gamma)Y(z) = -M + \frac{Pz}{z-1}$$

and

$$Y(z) = \frac{-Mz}{z-\gamma} + \frac{Pz}{(z-\gamma)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{-M}{z-\gamma} + \frac{P}{(z-\gamma)(z-1)} = \frac{-M}{z-\gamma} + \frac{P}{\gamma-1}\left[\frac{1}{z-\gamma} - \frac{1}{z-1}\right]$$

$$Y(z) = -M\frac{z}{z-\gamma} + \frac{P}{\gamma-1}\left[\frac{z}{z-\gamma} - \frac{z}{z-1}\right]$$

$$y[k] = \left[-M\gamma^k + \frac{P(\gamma^k - 1)}{r}\right] u[k] \qquad r = \gamma - 1$$

The loan balance is zero for $k = N$, that is $y[N]=0$, setting $k = N$ we have:

$$y[N] = \left[-M\gamma^N + \frac{P(\gamma^N - 1)}{r}\right] = 0$$

This yields:

$$P = \frac{r\gamma^N}{\gamma^N - 1} M$$

11.3-2 The Z-transform of the equation yields

$$zY(z) - z + 2Y(z) = 2F(z) - zf(0)$$

$$f(k) = ee^{-k}u[k] \quad \text{and} \quad F(z) = \frac{ez}{z - e^{-1}}, \quad f[0] = e$$

therefore:

$$(z+2)Y(z) = z - ez + \frac{ez^2}{z - e^{-1}} = \frac{z(1-e)(z-e^{-1}) + ez^2}{z - e^{-1}}$$

$$\frac{Y(z)}{z} = \frac{z + 1 - e^{-1}}{(z+2)(z-e^{-1})} = \frac{1}{2e+1}\left[\frac{e+1}{z+2} + \frac{e}{z - e^{-1}}\right]$$

$$Y(z) = \frac{1}{2e+1}\left[(e+1)\frac{z}{z+2} + e\frac{z}{z - e^{-1}}\right]$$

$$y[k] = \frac{1}{2e+1}\left[(e+1)(-2)^k + e^{-(k-1)}\right]u[k]$$

11.3-3 The system equation in delay form is

$$2y[k] - 3y[k-1] + y[k-2] = 4f[k] - 3f[k-1]$$

Also

$$y[k] \Longleftrightarrow Y(z) \quad y[k-1] \Longleftrightarrow \frac{1}{z}Y(z) \quad y[k-2] \Longleftrightarrow \frac{1}{z^2}Y(z) + 1$$

$$f[k] \Longleftrightarrow F(z) = \frac{z}{z - 0.25} \quad f[k-1] \Longleftrightarrow \frac{1}{z - 0.25}$$

The Z transform of the equation is is

$$2Y(z) - \frac{3}{z}Y(z) + \frac{1}{z^2}Y(z) + 1 = \frac{4z}{z - 0.25} - \frac{3}{z - 0.25} = \frac{4z - 3}{z - 0.25}$$

11.3-5 System equation in delay form is

$$4y[k]+4y[k-1]+y[k-2]=f[k-1]$$

Also, $y[k] \Leftrightarrow Y(z)$ $y[k-1] \Leftrightarrow \frac{1}{z}Y(z)$ $y[k-2] \Leftrightarrow \frac{1}{z^2}Y(z)+1$

$f[k] \Leftrightarrow \frac{z}{z-1}$ $f[k-1] \Leftrightarrow \frac{1}{z-1}$ $(f[-1]=0)$

The z transform of the system equation is

$$4Y(z)+\frac{4}{z}Y(z)+\frac{1}{z^2}Y[z]+1=\frac{1}{z-1}$$

Also

$$\frac{4z^2+4z+1}{z^2}Y(z)=\frac{2-z}{z-1}$$

and $\frac{Y(z)}{z}=\frac{z(2-z)}{4(z-1)(z^2+z+0.25)}=\frac{z(2-z)}{4(z-1)(z+0.5)^2}=\frac{1}{4}\left[\frac{4/9}{z-1}-\frac{13/9}{z+0.5}+\frac{5/6}{(z+0.5)^2}\right]$

$$Y(z)=\frac{1}{4}\left[\frac{4}{9}\frac{z}{z-1}-\frac{13}{9}\frac{z}{z+0.5}+\frac{5}{6}\frac{z}{(z+0.5)^2}\right]$$

$$y[k]=\left[\frac{1}{9}-\frac{13}{36}(-0.5)^k-\frac{5}{12}k(-0.5)^k\right]u[k]$$

11.3-6 The system in delay form is

$$y[k]-3y[k-1]+2y[k-2]=f[k-1]$$

Also $y[k] \Leftrightarrow Y(z)$ $y[k-1] \Leftrightarrow \frac{1}{z}Y(z)+2$ $y[k-2] \Leftrightarrow \frac{1}{z^2}Y(z)+\frac{2}{z}+3$

$f[k] \Leftrightarrow F(z)$ $f[k-1] \Leftrightarrow \frac{1}{z}F(z)$

$$F(z)=\frac{z}{z-3}$$

The Z transform of the system equation is

$$Y(z)-3\left[\frac{1}{z}Y(z)+2\right]+2\left[\frac{1}{z^2}Y(z)+\frac{2}{z}+3\right]=\frac{1}{z-3}$$

$$\left(1-\frac{3}{z}+\frac{2}{z^2}\right)Y(z)=-\frac{4}{z}+\frac{1}{z-3}=\frac{-3z+12}{z(z-3)}$$

or

$$\left(2 - \frac{3}{z} + \frac{1}{z^2}\right) Y(z) = -1 + \frac{4z-3}{z-0.25} = \frac{3z-2.75}{z-0.25}$$

and

$$\frac{Y(z)}{z} = \frac{z(3z-2.75)}{(2z^2-3z+1)(z-0.25)} = \frac{z(3z-2.75)}{2(z-0.5)(z-1)(z-0.25)} = \frac{5/2}{z-1/2} +$$

$$\frac{1/3}{z-1} - \frac{4/3}{z-0.25}$$

11.3-4 For initial conditions $y[0], y[1]$, we require equation in advance form:

$$2y[k+2] - 3y[k+1] + y[k] = 4f[k+2] - 3f[k+1]$$

Also, $y[k] \Longleftrightarrow Y(z)$ $\quad y[k+1] \Longleftrightarrow zY(z) - \frac{3}{2}z$ $\quad y[k+2] \Longleftrightarrow z^2Y(z) - \frac{3}{2}z^2 - \frac{35}{4}z$

$f[k] \Longleftrightarrow F(z) = \frac{z}{z-0.25}$ $\quad f[k+1] \Longleftrightarrow zF(z) - z = \frac{0.25z}{z-0.25}$

and

$$f[k+2] \Longleftrightarrow z^2F(z) - z^2 - \frac{1}{4}z = \frac{z}{16(z-0.25)}$$

The z transform of the equation is

$$2\left[z^2Y(z) - \frac{3}{2}z^2 - \frac{35}{4}z\right] - 3\left[zY(z) - \frac{3}{2}z\right] + Y(z) = \frac{-z/2}{z-0.25}$$

or $$(2z^2 - 3z + 1)Y(z) = \frac{z(3z^2 + 12.25z - 3.75)}{(z-0.25)}$$

and $$\frac{Y(z)}{z} = \frac{3z^2 + 12.25z - 3.75}{2(z-0.25)(z-1)(z-0.5)} = \frac{46/3}{z-1} - \frac{4/3}{z-0.25} - \frac{25/2}{z-0.5}$$

$$Y(z) = \frac{46}{3}\frac{z}{z-1} - \frac{4}{3}\frac{z}{z-0.25} - \frac{25}{2}\frac{z}{z-0.5}$$

$$y[k] = \left[\frac{46}{3} - \frac{4}{3}(0.25)^k - \frac{25}{2}(0.5)^k\right]u[k]$$

$$\frac{Y(z)}{z} = \frac{-3z+12}{(z^2-3z+2)(z-3)} = \frac{-3z+12}{(z-1)(z-2)(z-3)} = \frac{9/2}{(z-1)} - \frac{6}{z-2} + \frac{3/2}{z-3}$$

$$Y(z) = \frac{9}{2}\frac{z}{z-1} - 6\frac{z}{z-2} + \frac{3}{2}\frac{z}{z-3}$$

$$y[k] = [\tfrac{9}{2} - 6(2)^k + \tfrac{3}{2}(3)^k]u[k]$$

11.3-7 The system equation in delay form is:

$$y[k] - 2y[k-1] + 2y[k-2] = f[k-2]$$

$$y[k] \Longleftrightarrow Y(z) \quad y[k-1] \Longleftrightarrow \frac{1}{z}Y(z)+1 \quad y[k-2] \Longleftrightarrow \frac{1}{z^2}Y(z)+\frac{1}{z}$$

$$f[k-2] \Longleftrightarrow \frac{1}{z^2}F(z) \quad \text{and} \quad F(z) = \frac{z}{z-1}$$

The z-transform of the difference equation is

$$Y(z) - 2\left[\frac{1}{z}Y(z)+1\right] + 2\left[\frac{1}{z^2}Y(z)+\frac{1}{z}\right] = \frac{1}{z(z-1)}$$

$$\frac{z^2-2z+2}{z^2}Y(z) = \frac{2z^2-4z+3}{z(z-1)}$$

$$\frac{Y(z)}{z} = \frac{2z^2-4z+3}{(z-1)(z^2-2z+2)} = \frac{1}{z-1} + \frac{z-1}{z^2-2z+2}$$

$$Y(z) = \frac{z}{z-1} + \frac{z(z-1)}{z^2-2z+2}$$

For the second fraction of the right-hand side, we use pair 12c with

$A=1$, $B=-1$, $a=-1$, $|\gamma|^2=2$. This yields $r=1$, $\beta=\pi/4$ and $\theta=0$, Therefore:

$$y[k] = [1+(\sqrt{2})^k\cos(\tfrac{\pi}{4}k)]u[k]$$

11.3-8

The equation in advance form is

$$y[k+2]+2y[k+1]+2y[k]=f[k+1]+2f[k]$$

$$y[k] \Leftrightarrow Y(z) \qquad y[k+1] \Leftrightarrow zY(z) \qquad y[k+2] \Rightarrow z^2Y(z)-z$$

$$f[k] \Leftrightarrow F(z) \qquad f[k+1] \Leftrightarrow zF(z)-z \qquad \text{and } F(z)=\frac{z}{z-e}$$

the z-transform of the difference equation is

$$z^2Y(z)-z+2zY[z]+2Y[z]=\frac{z^2}{z-e}-z+\frac{2z}{z-e}=\frac{z(e+2)}{z-e}$$

$$(z^2+2z+2)Y[z]=z+\frac{z(e+2)}{z-e}=\frac{z(z+2)}{z-e}$$

Therefore:

$$\frac{Y[z]}{z}=\frac{z+2}{(z-e)(z^2+2z+2)}=\frac{0.318}{z-e}+\frac{-0.318z-0.502}{z^2+2z+2}$$

$$Y[z]=0.318\frac{z}{z-e}-\frac{z(0.318z+0.502)}{z^2+2z+2}$$

for the second fraction on the right-hand side, we use pair 12c with

$A=0.318$, $B=0.502$, $a=1$, $|\gamma|^2=2$ and $r=0.367$, $\beta=\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=\frac{3\pi}{4}$

$$\theta=\tan^{-1}\left(\frac{-0.184}{0.318}\right)=-0.525$$

$$y[k]=[0.318(e)^k-0.367(\sqrt{2})^k\cos(\tfrac{3\pi}{4}k-0.525)]u[k]$$

11.3-9

$$f[k]=e e^k u[k] \qquad F(z)=\frac{ez}{z-e}$$

$$Y(z)=F(z)H(z)=\frac{ez^2}{(z-e)(z+0.2)(z-0.8)}$$

There fore:

$$\frac{Y[z]}{z} = \frac{2z+3}{(z-1)(z-2)(z-3)} = \frac{5/2}{z-1} - \frac{7}{z-2} + \frac{9/2}{z-3}$$

$$Y[z] = \frac{5}{2}\frac{z}{z-1} - 7\frac{z}{z-2} + \frac{9}{2}\frac{z}{z-3}$$

$$y[k] = \left[\frac{5}{2} - 7(2)^k + 9/2\,(3)^k\right]u[k]$$

11.3-11

a) $f[k] = 4^{-k}u[k] = (\frac{1}{4})^k u[k]$ so that $F[z] = \frac{z}{z-1/4}$, and

$$Y[z] = F[z]H[z] = \frac{6z(5z-1)}{(z-1/4)(6z^2-5z+1)} = \frac{z(5z-1)}{(z-\frac{1}{4})(z-\frac{1}{3})(z-\frac{1}{2})}$$

There fore

$$\frac{Y[z]}{z} = \frac{5z-1}{(z-1/4)(z-1/3)(z-1/2)} = \frac{12}{z-1/4} - \frac{48}{z-1/3} + \frac{36}{z-1/2}$$

$$Y[z] = 12\frac{z}{z-1/4} - 48\frac{z}{z-1/3} + 36\frac{z}{z-1/2}$$

$$y[k] = [12(1/4)^k - 48(1/3)^k + 36(1/2)^k]\,u[k]$$

$$= 12[4^{-k} - 4(3)^{-k} + 3(2)^{-k}]\,u[k]$$

b) Here the input is $4^{-(k-2)}u[k-2]$ which is identical to the input in part (a) delayed by 2 units Therefore the response will be the output in part (a) delayed by 2 units (time-invariance property). Therefore.

$$y[k] = 12\,[4^{-(k-2)} - 4(3)^{-(k-2)} + 3(2)^{-(k-2)}]\,u[k-2]$$

c) Here the input can be expressed as: $f[k] = 4^{-(k-2)}u[k] = 16(4)^{-k}u[k]$

This input is 16 times the input in part (a). There fore the response will be 16 times the output in part (a) (linearity property). Therefore

$$y[k] = 192\,[4^{-k} - 4(3)^{-k} + 3(2)^{-k}]\,u[k]$$

d) Here the input can be expressed as

$$f[k] = 4^{-k}u[k-2] = \frac{1}{16}\,4^{-(k-2)}\,u[k-2]$$

This input is $\frac{1}{16}$ times the input in part (b). Therefore the response will be $\frac{1}{16}$ times the output in part (b), so

$$y[k] = \frac{3}{4}\,[4^{-(k-2)} - 4(3)^{-(k-2)} + 3(2)^{-(k-2)}]\,u[k-2]$$

11.3-12

$$Y[z] = F[z]\,H[z] = \frac{z(2z-1)}{(z-1)(z^2-1.6z+0.8)}$$

$$\frac{Y[z]}{z} = \frac{2z-1}{(z-1)(z^2-1.6z+0.8)} = \frac{5}{z-1} - \frac{5(z-1)}{z^2-1.6z+0.8}$$

$$Y[z] = 5\frac{z}{z-1} - 5\,\frac{z(z-1)}{z^2-1.6z+0.8}$$

For the second fraction on the right hand side, we use pair 12c with $A=1$

$B=-1$, $a=-0.8$, $\gamma = \frac{2}{\sqrt{5}}$, $|\gamma|^2 = 0.8$. Therefore

$r = 1.118$, $\beta = \cos^{-1}\left(\frac{0.8\sqrt{5}}{2}\right) = 0.464$ $\quad\theta = \tan^{-1}\left(\frac{0.2}{0.4}\right) = 0.464$

$$y[k] = \left[5 - 5(1.118)\left(\frac{2}{\sqrt{5}}\right)^k \cos(0.464k + 0.464)\right]u[k]$$

$$= \left[5 - 5.59\left(\frac{2}{\sqrt{5}}\right)^k \cos(0.464k + 0.464)\right]u[k]$$

11.3-13 a) $H[z] = \frac{z}{z+2}$ b) $H[z] = \frac{4z^2-3z}{2z^2-3z+1}$ c) $H[z] = \frac{z}{4z^2+4z+1}$

d) We convert the equation to advanced operator form. This yields

$(E^2+2E+2)y[k] = (E+2)f[k]$. Therefore $H[z] = \frac{z+2}{z^2+2z+2}$

11.3-14 a)

$$H[z] = \frac{z^2+3z+3}{z^2+3z+2} = \frac{z^2+3z+3}{(z+1)(z+2)}$$

$$\frac{H[z]}{z} = \frac{z^2+3z+3}{z(z+1)(z+2)} = \frac{3/2}{z} - \frac{1}{1+z} + \frac{1/2}{z+2}$$

$$H[z] = \frac{3}{2} - \frac{z}{z+1} + \frac{1}{2}\frac{z}{z+2}$$

$$h[k] = [\tfrac{3}{2}\delta[k] - (-1)^k + \tfrac{1}{2}(-2)^k]u[k]$$

b) $$H[z] = \frac{2z^2-z}{z^2+2z+1} = \frac{z(2z-1)}{(z+1)^2}$$

Therefore

$$\frac{H[z]}{z} = \frac{2z-1}{(z+1)^2} = \frac{2}{z+1} - \frac{3}{(z+1)^2}$$

$$H[z] = 2\left(\frac{z}{z+1}\right) - 3\frac{z}{(z+1)^2}$$

$$h[k] = [2(-1)^k + 3k(-1)^k]u[k] = (2+3k)(-1)^k u[k]$$

c) $$H[z] = \frac{z^2+2z}{z^2-z+0.5} = \frac{z(z+2)}{z^2-z+0.5}$$

Therefore $$\frac{H[z]}{z} = \frac{z+2}{z^2-z+0.5}$$

We use pair 12c with $A=1$, $B=2$, $a=-0.5$ $|\gamma|^2 = 0.5$, $|\gamma| = \frac{1}{\sqrt{2}}$ and

$r = 5.099$ $\beta = \cos^{-1}(0.5\sqrt{5}) = \frac{\pi}{4}$ $\theta = \tan^{-1}\left(\frac{-2.5}{0.5}\right) = -1.373$

$$h[k] = 5.099\left(\frac{1}{\sqrt{2}}\right)^k \cos\left(\frac{\pi}{4} - 1.373\right)u[k]$$

11.3-15 a)

$$\frac{H[z]}{z} = \frac{1}{(z+0.2)(z-0.8)} = \frac{-1}{z+0.2} + \frac{1}{z-0.8}$$

$$H[z] = -\frac{z}{z+0.2} + \frac{z}{z-0.8}$$

$$h[k] = [-(-0.2)^k + (0.8)^k]u[k]$$

b)

$$\frac{H[z]}{z} = \frac{2z+3}{z(z-2)(z-3)} = \frac{1/2}{z} - \frac{7/2}{z-2} + \frac{3}{z-3}$$

$$H[z] = \frac{1}{2} - \frac{7}{2}\cdot\frac{z}{z-2} + 3\frac{z}{z-3}$$

$$h[k] = [\tfrac{1}{2}\delta[k] - 7/2\,(2)^k + 3(3)^k]u[k]$$

c)

$$\frac{H[z]}{z} = \frac{2z-1}{z(z^2-1.6z+0.8)} = \frac{-1.25}{z} + \frac{1.25z}{z^2-1.6z+0.8}$$

For the second fraction on the right-hand side, $A=1.25$, $B=0$, $a=-0.8$

$|\gamma|^2 = 0.8$, $|\gamma| = \frac{2}{\sqrt{5}}$ and

$r = 2.795$ $\beta = \cos^{-1}\left(\frac{0.8\sqrt{5}}{2}\right) = 0.464$ $\theta = \tan^{-1}(-2) = -1.107$

$$h[k] = -1.25\delta[k] + 2.795\left(\frac{2}{\sqrt{5}}\right)^k \cos(0.464k - 1.107)u[k]$$

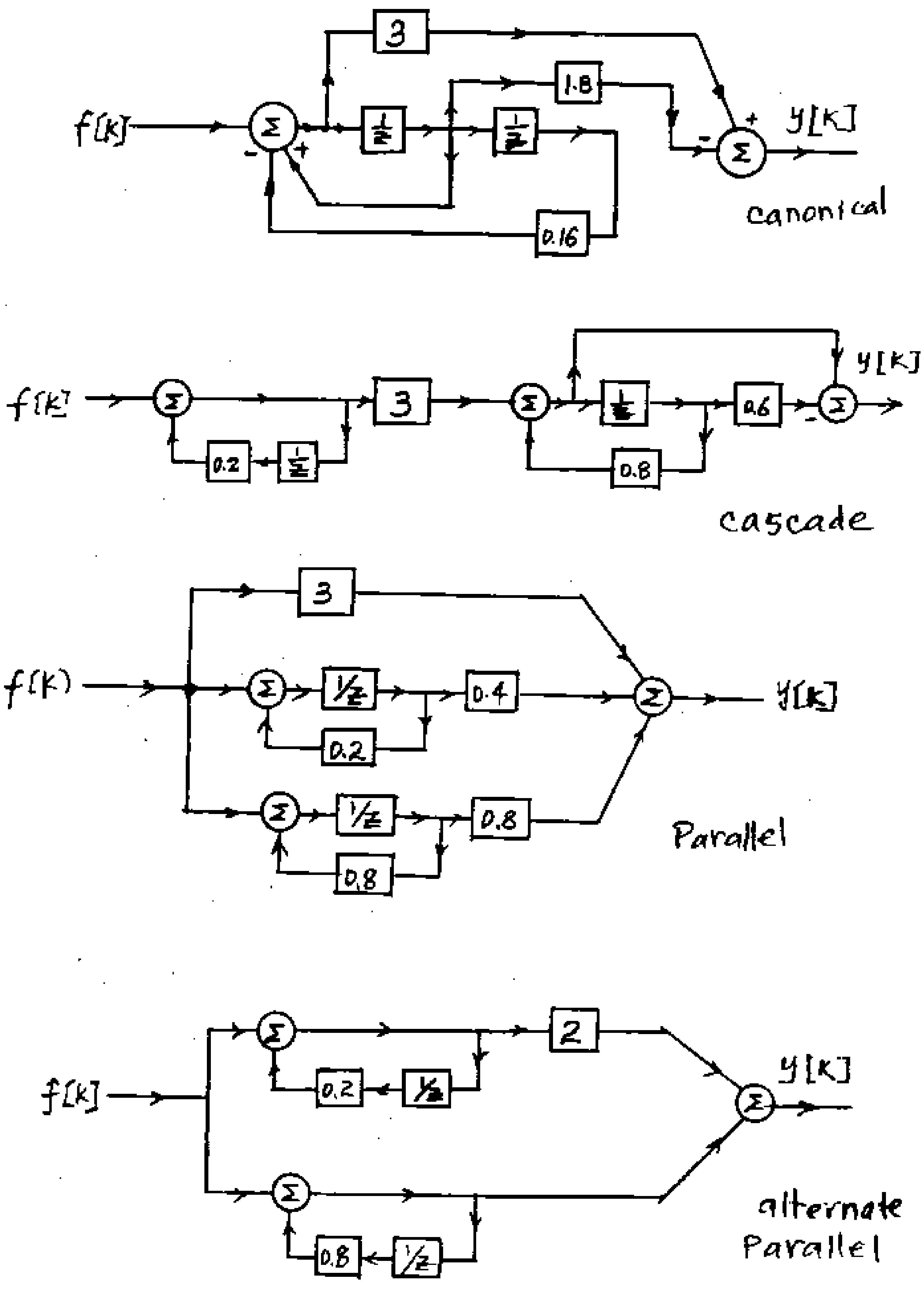
f[k]
3
1.8
1/z
0.16
y[k]
canonical
0.2
0.6
0.8
cascade
0.4
Parallel
2
alternate
Parallel

Fig 11.4-1 (a)

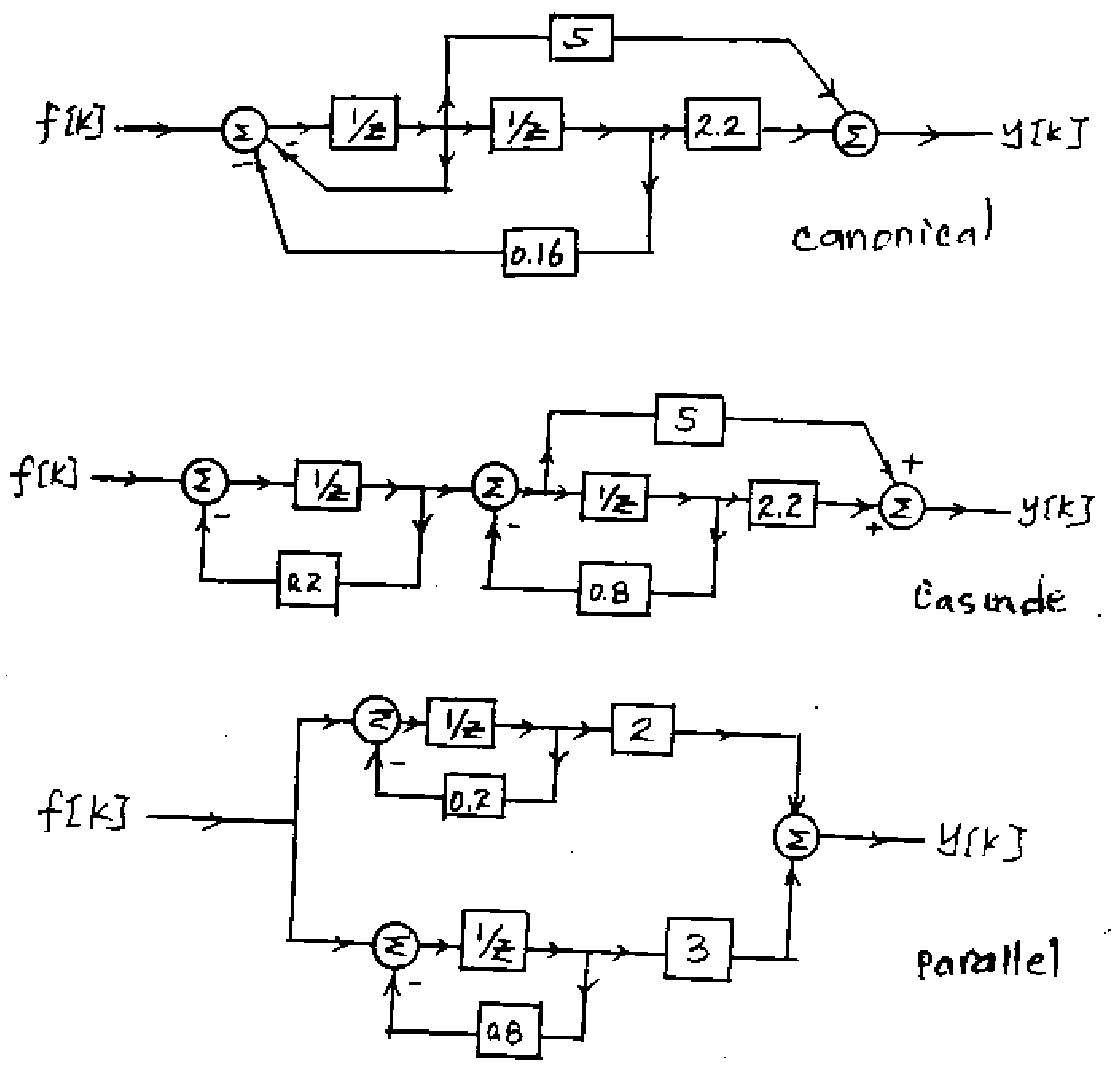

Fig 11.4-1b

11.4-1

a) $H[z] = \dfrac{3z^2 - 1.8z}{z^2 - z + 0.16} = \dfrac{3z(z-0.6)}{(z-0.2)(z-0.8)} = \left(\dfrac{3z}{z-0.2}\right)\left(\dfrac{z-0.6}{z-0.8}\right)$

Parallel form: To realize parallel form, we could expand $H[z]$ or $H[z]/z$ into partial fractions In our case:

$$H[z] = 3 + \frac{1.2z - 0.48}{(z-0.2)(z-0.8)} \quad \text{So} \quad H[z] = 3 + \frac{0.4}{(z-0.2)} + \frac{0.8}{(z-0.8)}$$

Alternatively we could expand $H[z]/z$ into partial fractions as:

$$\frac{H[z]}{z} = \frac{3(z-0.6)}{(z-0.2)(z-0.8)} = \frac{2}{z-0.2} + \frac{1}{z-0.8} \quad \text{So} \quad H[z] = \frac{2z}{z-0.2} + \frac{z}{z-0.8}$$

See realization in Fig 11.4-1a

b) $$H[z] = \frac{5z+2.2}{(z+0.2)(z+0.8)} = \frac{5z+2.2}{z^2+z+0.16}$$

$$= \left(\frac{1}{z+0.2}\right)\left(\frac{5z+2.2}{z+0.8}\right) = \frac{2}{z+0.2} + \frac{3}{z+0.8}$$

All the realizations are shown in Fig 11.4-1 b) in previous page.

c) $$H[z] = \frac{3.8z-1.1}{z^3-0.8z^2+0.37z-0.05}$$

For a cascade form, we express H[z] as:

$$H[z] = \left(\frac{1}{z-0.2}\right)\left(\frac{3.8z-1.1}{z^2-0.6z+0.25}\right)$$

For a parallel form, we express H[z] as

$$H[z] = \frac{-2}{z-0.2} + \frac{2z+3}{z^2-0.6z+0.25}$$

All the realizations are shown in fig 11.4-1 c) in next page.

11.4-2 Note: the complex conjugate poles must be realized together as a second order factor a) Cascade form: $H[z] = \left(\frac{z}{z-0.2}\right)\left(\frac{1.6z-1.8}{z^2+z+0.5}\right)$

parallel form: $$H[z] = \frac{-2z}{z-0.2} + \frac{2z^2+4z}{z^2+z+0.5}$$

b) Cascade form: $$H[z] = \left(\frac{z}{z+0.5}\right)\left(\frac{2z^2+1.3z+0.96}{z^2-0.8z+0.16}\right)$$

Parallel form

$$H[z] = \frac{z}{z+0.5} + \frac{z}{z-0.4} + \frac{2z}{(z-0.4)^2}$$

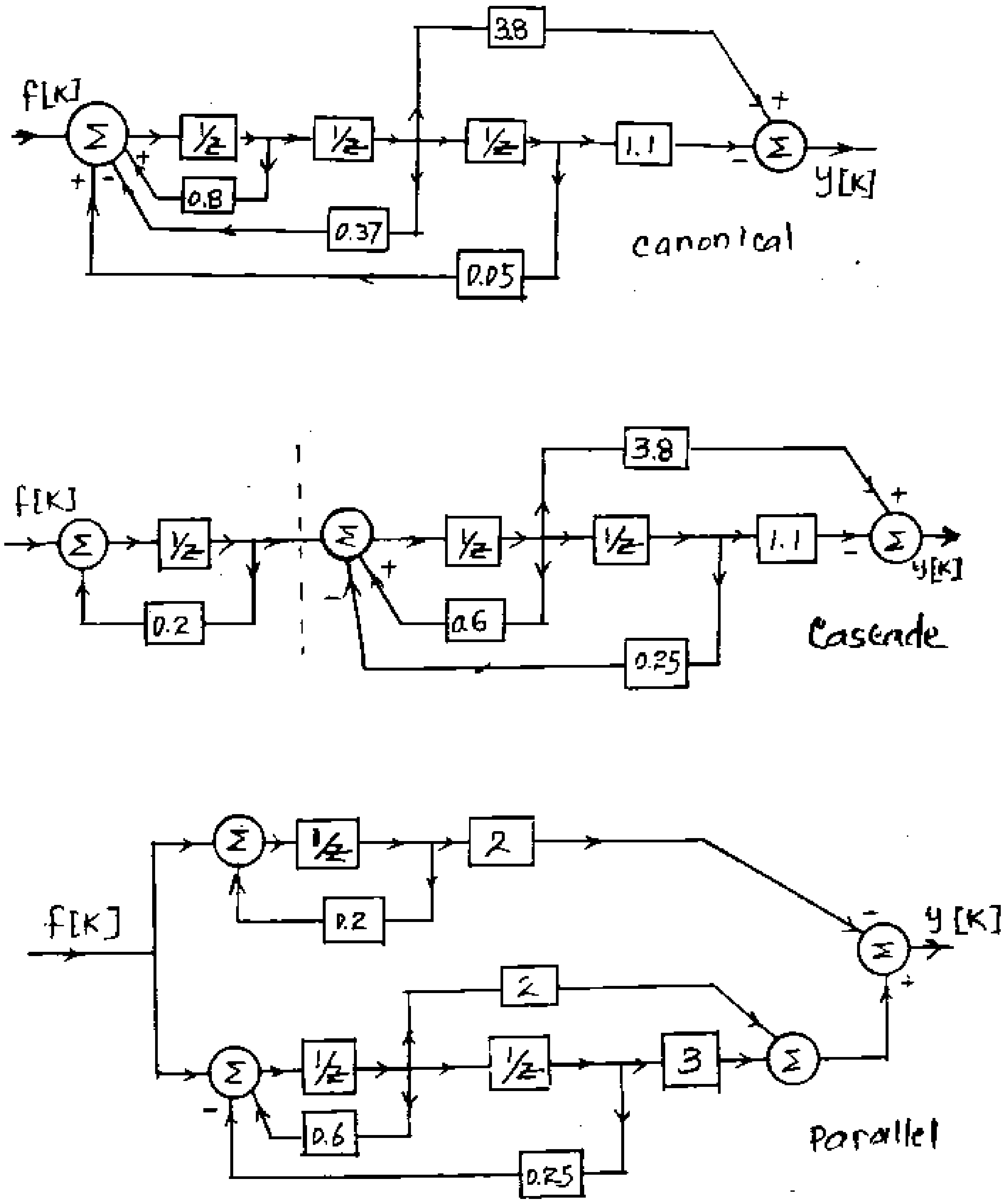
f[K]
y[K]
3.8
1/z
0.8
0.37
0.05
1.1
canonical
0.2
0.6
0.25
Cascade
2
3
Parallel

Fig 11·4-1 (c)

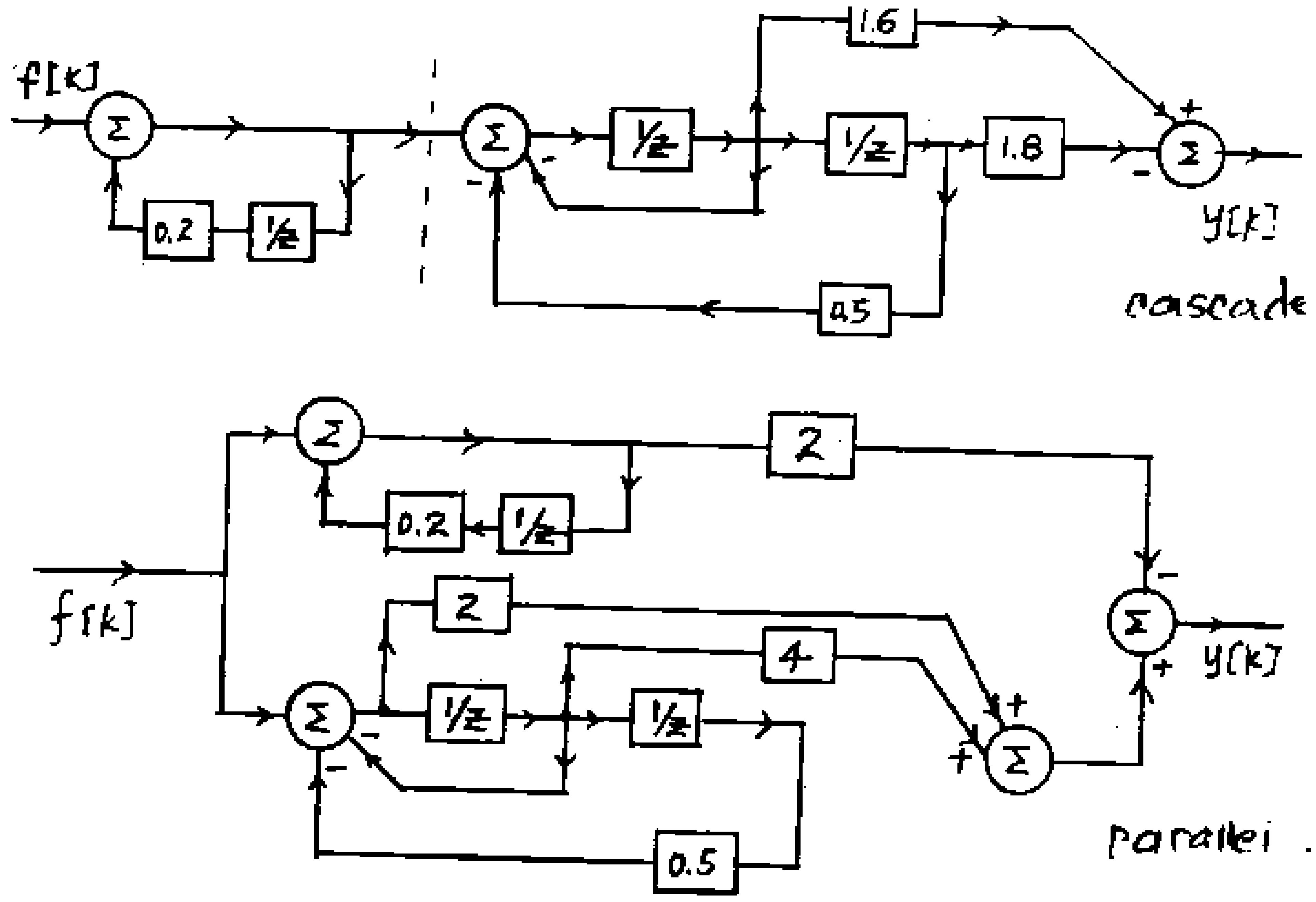

Fig 11.4-1 a

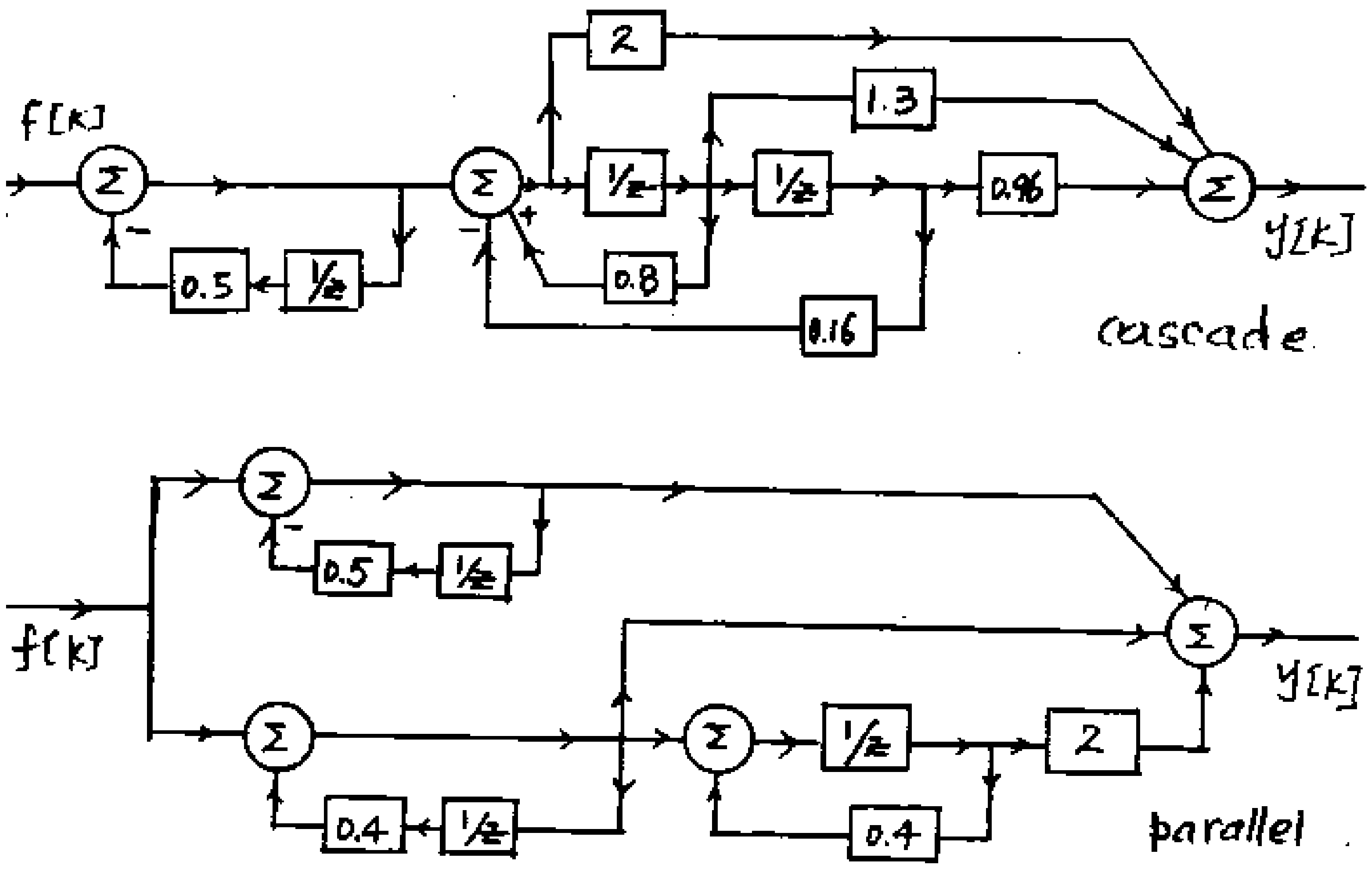

Fig 11.4-2 b'

11.4-3

$$H[z] = 2 + \frac{1}{z} + \frac{0.8}{z^2} + \frac{2}{z^3} + \frac{8}{z^4} = \frac{2z^4 + z^3 + 0.8z^2 + 2z + 8}{z^4}$$

The realization of this transfer function is shown in Fig. 11.4-3. It can be explained in two ways. The realization has 5 paths in parallel, and each path represents one term in the transfer function. The first path (which bypasses all the delays) has transfer function 2. The second path (going through only one delay) has transfer function $1/z$ and so on. Alternatively we observe that this transfer function has $a_0 = a_1 = a_2 = a_3 = 0$ and $b_0 = 8$ and $b_1 = 2$, $b_2 = 0.8$, $b_3 = 1$, $b_4 = 2$. Therefore all the feedback coefficients are zero, and there are no feedback paths. There are 4 feedforward paths with gains 8, 2, 0.8, 1 and 2 as shown below.

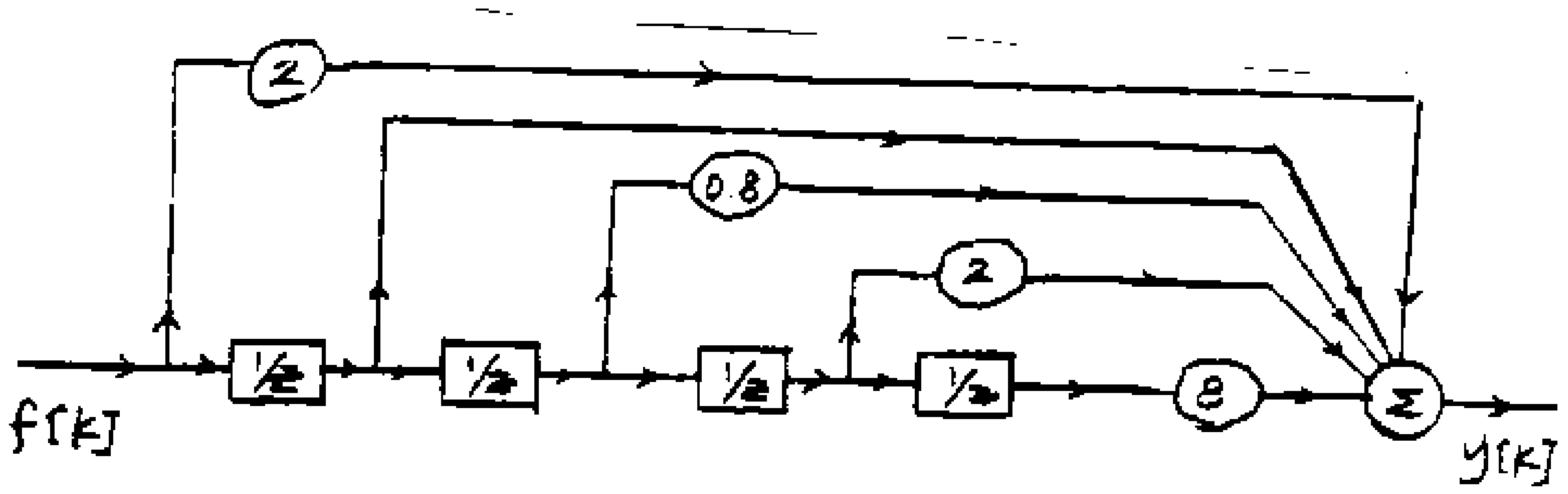

Fig 11.4-3

11.4-4

$$H[z] = \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \frac{4}{z^4} + \frac{5}{z^5} + \frac{6}{z^6}$$

This transfer function is similar to that in Problem 11.4-3. Below see the realization

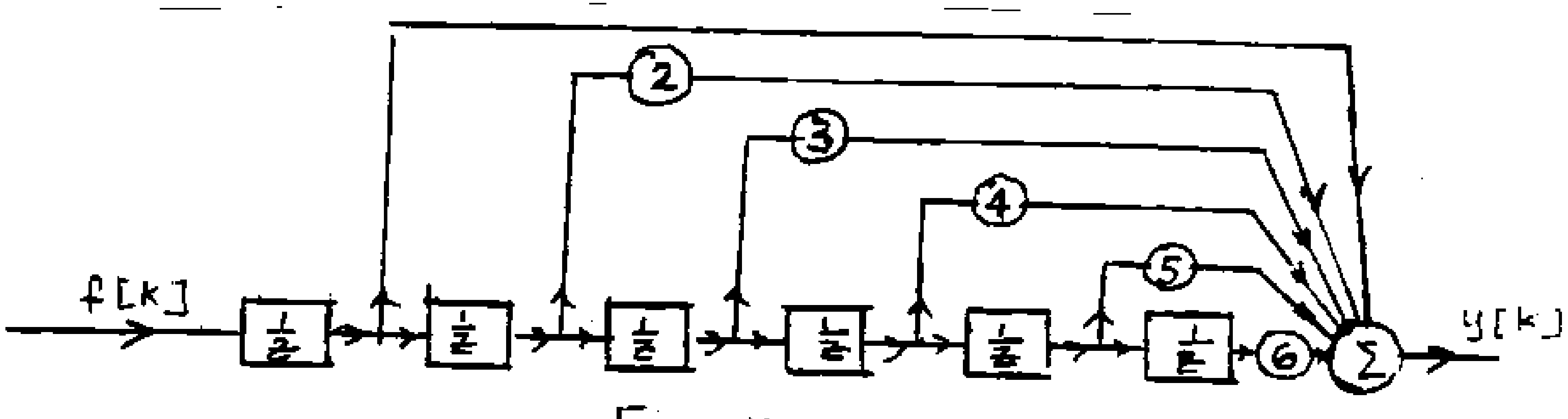

Fig 11.4-4

11.6-1 In this case $Y(z) = F[z]\,G[z]$, where we use table 12.1 to find

$$F[z] = \frac{z}{z-e^{-2T}} \quad \text{and} \quad G[z] = \frac{z}{z-e^{-T}}. \text{ Therefore,}$$

$$Y[z] = \frac{z^2}{(z-e^{-T})(z-e^{-2T})}$$

$$= \frac{1}{1-e^{-T}}\left[\frac{z}{z-e^{-T}} - e^{-T}\frac{z}{z-e^{-2T}}\right]$$

hence

$$y[k] = \frac{1}{1-e^{-T}}\left[e^{-kT} - e^{-T(2k+1)}\right]u[k]$$

11.6-2 We can express $E[z]$ as a sum of two signals; the input and the signal fed back. Therefore

$$E[z] = F[z] - GH[z]E[z]$$

hence
$$E[z] = \frac{1}{1+GH[z]}F[z]$$

and
$$Y[z] = G[z]E[z] = \frac{G[z]}{1+GH[z]}F[z]$$

so
$$T[z] = \frac{Y[z]}{F[z]} = \frac{G[z]}{1+GH[z]}$$

11.6-3 In this case $Y[z]$ (the output of the dotted sampler) has 2 components 1) due to input $F[z]$ and 2) the component resulting from the feedback of $Y[z]$. Hence $Y[z] = FG[z] - GH[z]Y[z]$ and $Y[z] = \frac{FG[z]}{1+GH[z]}$ we cannot seperate $F[z]$ from $FG[z]$. Hence, it is not possible to write the z-transform function relating $y[k]$ to $f[k]$. Analysis and synthesis of such systems involving only one sampler, which is located in the feedback path is little more difficult.

11.7-1 a)

$$f[k] = \underbrace{(0.8)^k u[k]}_{f_1[k]} + \underbrace{2^k u[-(k+1)]}_{f_2[k]}$$

$f_1[k] \Longleftrightarrow \frac{z}{z-0.8}$ $|z| > 0.8$ and $f_2[k] \Longleftrightarrow \frac{-z}{z-2}$ $|z| < 2$

Therefore

$$F[z] = \frac{z}{z-0.8} - \frac{z}{z-2} \quad 0.8 < |z| < 2$$

$$= \frac{-1.2z}{z^2 - 2.8z + 1.6} \quad 0.8 < |z| < 2$$

b)

$$F_1[z] = \frac{z}{z-2} \quad |z| > 2$$

$$F_2[z] = \frac{z}{z-3} \quad |z| < 3$$

Therefore

$$F[z] = \frac{z}{z-2} + \frac{z}{z-3} \quad 2 < |z| < 3$$

$$= \frac{z(2z-5)}{z^2 - 5z + 6} \quad 2 < |z| < 3$$

c) $F_1[z] = \frac{z}{z-0.8}$ $|z| > 0.8$ $F_2[z] = \frac{-z}{z-0.9}$ $|z| < 0.9$

Therefore $F[z] = \frac{z}{z-0.8} - \frac{z}{z-0.9} = \frac{-z}{10(z^2 - 1.7z + 0.72)}$ $0.8 < |z| < 0.9$

d) $[(0.8)^k + 3(0.4)^k] u[-(k+1)] \Longleftrightarrow \left(\frac{-z}{z-0.8} - \frac{3z}{z-0.4}\right)$ $|z| < 0.4$

$$= \frac{-4z(z-0.7)}{(z-0.4)(z-0.8)} \quad |z| < 0.4$$

e) $[(0.8)^k + 3(0.4)^k] u[k] \Longleftrightarrow \frac{z}{z-0.8} + \frac{3z}{z-0.4} = \frac{4z(z-0.7)}{(z-0.4)(z-0.8)}$ $|z| > 0.8$

f) $(0.8)^k u[k] + 3(0.4)^k u[-(k+1)]$

The region of Convergence for $(0.8)^k u[k]$ is $|z|>0.8$. The region of Convergence for $(0.4)^k u[-(k+1)]$ is $|z|<0.4$. The common region doesnot exist. Hence the z-transform for this function doesnot exist.

11.7-2

$$\frac{F[z]}{z} = \frac{e^{-2}-2}{(z-e^{-2})(z-2)} = \frac{1}{z-e^{-2}} - \frac{1}{z-2} \quad \text{so} \quad F[z] = \frac{z}{z-e^{-2}} - \frac{z}{z-2}$$

a) The region of Convergence is $|z|>2$. Both terms are casual. And

$$f[k] = (e^{-2k} - 2^k)u[k]$$

b) The region of Convergence is $e^{-2}<|z|<2$. In this case the 1ST Term is casual and the second is anticasual.

$$f[k] = e^{-2k}u[k] + 2^k u[-(k+1)]$$

c.) The region of Convergence is $|z|<e^{-2}$. Both terms are anticasual in this case. $f[k] = (-e^{-2k}+2^k)u[-(k+1)]$

11.7-3) For Casual signals, the region of Convergence may be ignored. we'll consider it only for noncasual inputs

a) $$Y[z] = F[z]H[z] = \frac{z^2}{(z-e)(z+0.2)(z-0.8)}$$

Modified partial fraction expansion of $Y[z]$ yields

$$Y[z] = 0.477\frac{z}{z-e} - 0.068(-0.2)^k - 0.412(0.8)^k]u[k]$$

b)

$$F[z] = \frac{-z}{z-2} \qquad |z|<2$$

$$H[z] = \frac{z}{(z+0.2)(z-0.8)} \qquad |z|>0.8$$

$$Y[z] = \frac{-z^2}{(z+0.2)(z-0.8)(z-2)} \qquad 0.8<|z|<2$$

and

$$\frac{Y[z]}{z} = \frac{-z}{(z+0.2)(z-0.8)(z-2)} = \frac{1/11}{z+0.2} + \frac{2/3}{z-0.8} - \frac{0.758}{z-2}$$

Therefore $\quad Y[z] = \frac{1}{11}\frac{z}{z+0.2} + \frac{2}{3}\frac{z}{z-0.8} - 0.758\frac{z}{z-2} \qquad 0.8<|z|<2$

and $\quad y[k] = [\frac{1}{11}(-0.2)^k + 2/3(0.8)^k]u[k] + 0.758(2)^k u[-(k+1)]$

c) The input in this case is the sum of the inputs in parts <u>a</u> and <u>b</u>

therefore the response will be the sum of the responses in part <u>a</u> and <u>b</u>

11.7-4 $\quad f[k] = \underbrace{2^k u[k]}_{f_1[k]} + \underbrace{u[-(k+1)]}_{f_2[k]}$

$$F_1[z] = \frac{z}{z-2} \qquad |z|>2$$

$$F_2[z] = \frac{-z}{z-1} \qquad |z|<1$$

There is no region of convergence common to both $F_1[z]$ and $F_2[z]$

$$H[z] = \frac{z}{(z+0.2)(z-0.8)}$$

The region of convergence of $H[z]$ is $|z|>0.8$ (assuming a casual system)

We should find the response to $f_1[k]$ and $f_2[k]$ seperately.

$$Y_1[z] = \frac{z^2}{(z-2)(z+0.2)(z-0.8)} \qquad |z|>2$$

The modified partial fractions of $Y[z]$ yield

$$Y_1[z] = -\frac{1}{11}\frac{z}{z+0.2} - \frac{2}{3}\frac{z}{z-0.8} + 0.758\frac{z}{z-2}$$

and $$y_1[k] = \left[-\frac{1}{11}(-0.2)^k - \frac{2}{3}(0.8)^k + 0.758(2)^k\right]u[k]$$

Similarly
$$Y_2[z] = -\frac{25}{6}\frac{z}{z-1} + \frac{1}{6}\frac{z}{z+0.2} + 4\frac{z}{z-0.8} \qquad 0.8<|z|<1$$

and $$y_2[k] = \left[\frac{1}{6}(-0.2)^k + 4(0.8)^k\right]u[k] + \frac{25}{6}u[-(k+1)]$$

$$y[k] = y_1[k] + y_2[k] = \left[\frac{5}{66}(-0.2)^k + \frac{10}{3}(0.8)^k + 0.758(2)^k\right]u[k] + \frac{25}{6}u[-(k+1)]$$

11.7-5

$$F[z] = \frac{-z}{z-e^2} \qquad |z|<e^2$$

and $$H[z] = \frac{z}{(z+0.2)(z-0.8)} \qquad |z|>0.8$$

No common region of convergence for $F[z]$ and $H[z]$ exists Hence

$$y[k] = \infty$$