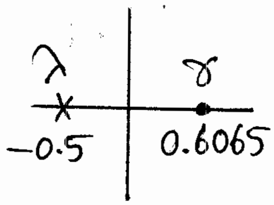
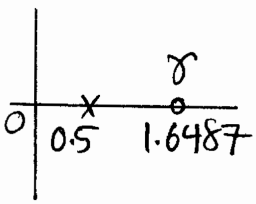
Chapter 8 (Answers)

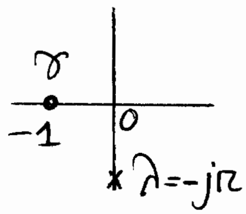
Prepared by: Payam Abolghasem (email: payam@grads.ece.mcmaster.ca)

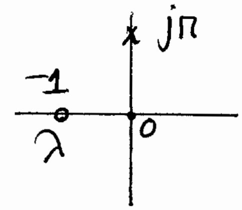
8.2-1:

(a) $(0.6065)^k$ (b) $(1.6487)^k$ (c) $(-1)^k$ (d) $(-1)^k$









8.2-2:

(a) $(-\frac{1}{e})^k$ (b) $(-\frac{1}{e})^k$ (c) $(-e)^k$ (d) $(-e)^k$

(e) $(\frac{1}{e})^k[\cos\frac{\pi}{3}k - j\sin\frac{\pi}{3}k]$ (f) $(e^1)^k e^{-j\frac{\pi}{3}k} = e^k(\cos\frac{\pi}{3}k - j\sin\frac{\pi}{3}k)$

8.2-3

(a) Periodic (b) Aperiodic (c) Aperiodic (d) Periodic

8.2-4

(a) periodic (b) Periodic (c) Periodic

8.2-5

(a) $\Omega_f = 0.8\pi$, $|\Omega_f| = 0.8\pi$ (b) $\Omega_f = 1.2\pi - 2\pi = -0.8\pi$, $|\Omega_f| = 0.8\pi$

(c) $\Omega_f = 6.9 - 2\pi = 0.6168$, $|\Omega_f| = 0.6168$ (d) $\Omega_f = 3.7\pi - 4\pi = -0.3\pi$, $|\Omega_f| = 0.3\pi$

(e) $\Omega_f = 22.9\pi - 22\pi = 0.9\pi$, $|\Omega_f| = 0.9\pi$

8.2-6

$1.4\pi = 2\pi - 0.6\pi \longrightarrow \cos(1.4\pi k + \frac{\pi}{3}) = \cos(-0.6\pi k + \pi/3) = \cos(0.6\pi k - \pi/3).$

$$\cos(0.6\pi k + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}\cos(0.6\pi k) - \frac{1}{2}\sin(0.6\pi k)$$

$$\cos(0.6\pi k - \frac{\pi}{3}) = \frac{1}{2}\cos(0.6\pi k) + \frac{\sqrt{3}}{2}\sin(0.6\pi k)$$

$$\rightarrow \cos(0.6\pi k + \frac{\pi}{6}) + \sqrt{3}\cos(1.4\pi k + \frac{\pi}{3}) = \sqrt{3}\cos(0.6\pi k) + \sin(0.6\pi k) = 2\cos(0.6\pi k - \frac{\pi}{6})$$

8.2-7 (a) $e^{j(0.2\pi k+\theta)}$ (b) $e^{j(0)k} = 1$

(c) $e^{j(4.333k)}$ (d) $e^{j(1.3\pi k)}$

8.2-8
$$E_f = \sum_0^{\infty}(0.8)^2 = 2.7778 \quad ; \quad E_{-f} = E_f = 2.7778 \quad ; \quad E_{cf} = c^2 E_f = 2.7778c^2$$

8.2-9

(a) 19 (b) 19 (c) 252 (d) 40

8.2-10

(a) 1 (b) 1 (c) 0.5 (d) 0.5 (e) 0.5

8.2-11

(a) 19/6 (b) 7/3

8.2-12

Let $\frac{2\pi}{N_0} = \Omega_0$. Because $De^{j\Omega_0 k}$ is periodic with period N_0:
$$P_f = \frac{1}{N_0}\sum_{k=0}^{N_0-1}\left|De^{j\Omega_0 k}\right|^2 = \frac{1}{N_0}\sum_{k=0}^{N_0-1}|D|^2 = |D|^2$$

Interchanging the order of summation yeilds:
$$P_f = \frac{1}{N_0}\sum_{r=0}^{N_0-1}\sum_{m=0}^{N_0-1} D_r D_m^* \left[\sum_{k=0}^{N_0-1} e^{j(r-m)\Omega_0 k}\right]$$

From equation (5.43) in Appendix 5.1 the sum inside the parenthesis is N_0 wher $r=m$ and is zero otherwise. Hence $P_f = \sum_{r=0}^{N_0-1}|D_r|^2$

8.3-1

$F_s = \frac{1}{T} = 2$ MHz. Therefore equation (8.21) yeilds $2\times10^6 = 0 + 2\times10^6$. Hence the reduced frequency is 0 (dc).

8.3-2:

For $10\cos\left(\frac{11}{10}\pi k + \frac{\pi}{6}\right) \xrightarrow{\text{Samples Sinusoid}} 10\cos\left(\frac{9}{10}\pi k - \frac{\pi}{6}\right)$

" $5\cos\left(\frac{29}{10}\pi k - \frac{\pi}{6}\right) \longrightarrow 5\cos\left(\frac{9}{10}\pi k - \frac{\pi}{6}\right)$

8.3-3

(a) Sampled signal : $10\cos\frac{\pi}{2}k + \sqrt{2}\sin\frac{3\pi}{4}k + 2\cos\left(\frac{3\pi}{4}k - \frac{\pi}{4}\right)$

The last two terms can be combined to yeild:

$$\sqrt{2}\sin\left(\frac{3\pi}{4}k\right) + 2\cos\left(\frac{3\pi}{4}k - \pi/4\right) = \sqrt{10}\cos\left(\frac{3\pi}{4}k - 1.107\right)$$

The frequency $5\pi/4$ has been reduced to $3\pi/4$ which indicates aliasing.

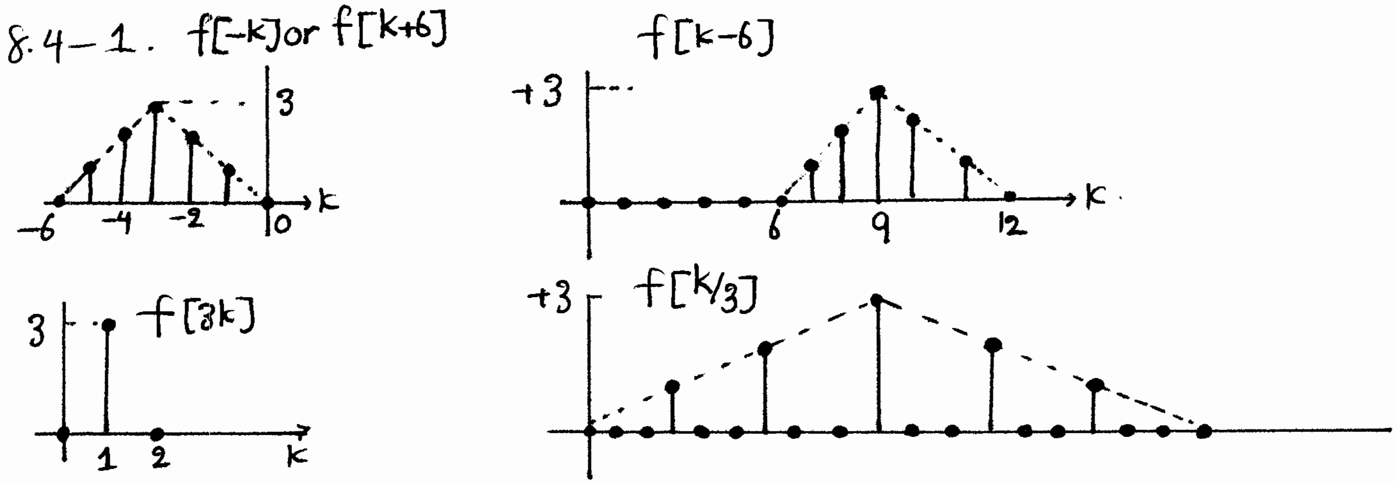
(b) Highest frequency, $\omega = 5000\pi$ $= 2500$ Hz $\longrightarrow$ Maximum value of T without aliasing $= 1/5000$

8.3-4

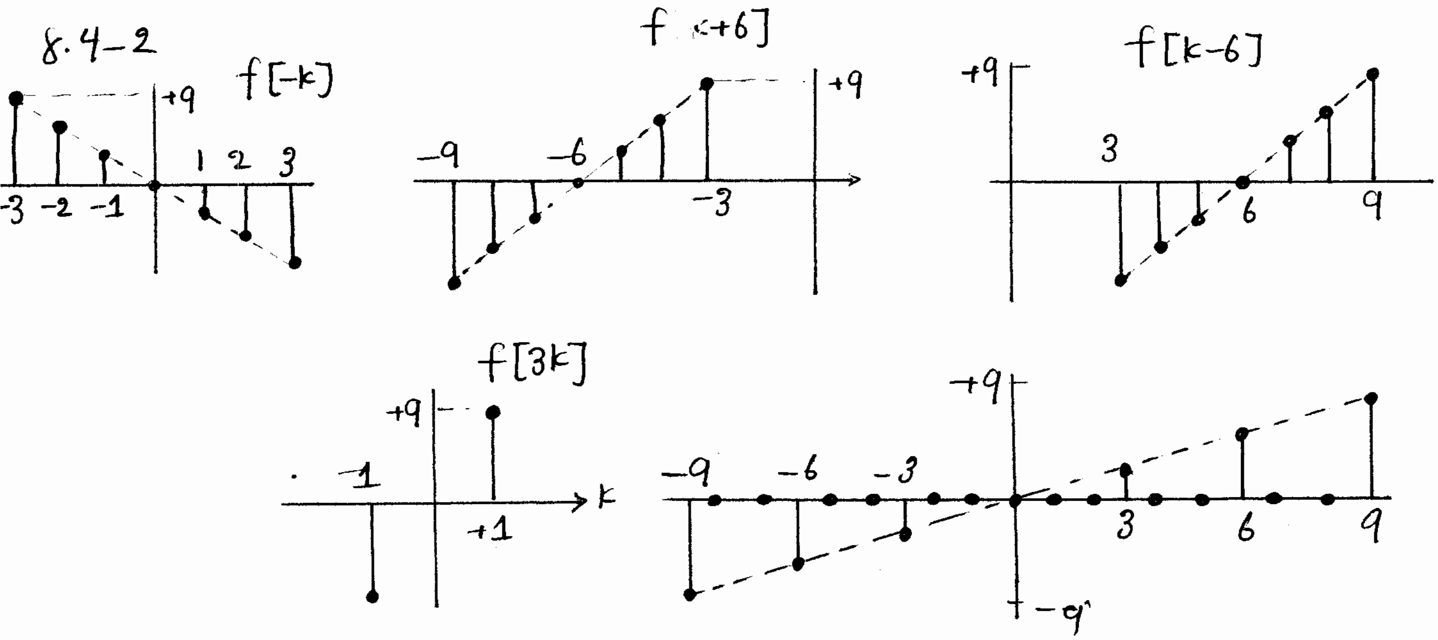
(i) $|F_f| = 1500$, (ii) $|F_f| = 1500$, (iii) $|F_f| = 0$

(iv) $|F_f| = 1500$, (v) $|F_f| = 2000$, (vi) $|F_f| = 600$

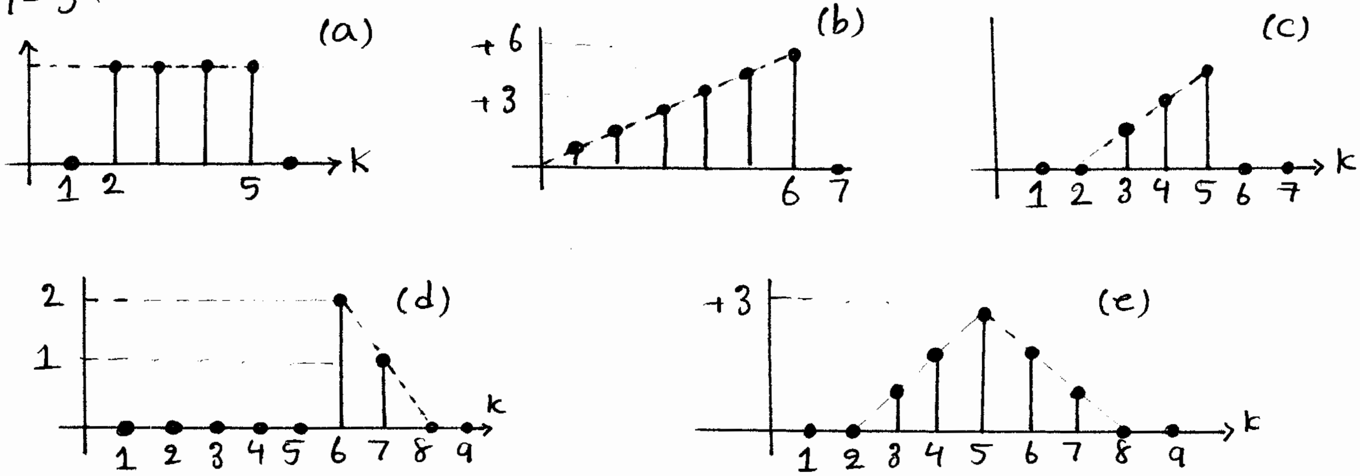
8.4-1.



8.4-2



8.4-3



8.4-4

(a) $f[k] = (k+3)\big(u[k+3]-u[k]\big) + (-k+3)\big(u[k]-u[k-4]\big)$

(b) $f[k] = k\big(u[k]-u[k-4]\big) + (-k+6)\big(u[k-4]-u[k-7]\big)$

(c) $f[k] = k\big(u[k+3]-u[k-4]\big)$

(d) $f[k] = -2k\big(u[k+2]-u[k]\big) + 2k\big(u[k]-u[k-3]\big)$

In all four cases $f[k]$ maybe represented by several other (Slightly different) expressions.

8.4-5

$$E_{f[k-m]} = \sum_{k=-\infty}^{+\infty} \left|f[k-m]\right|^2 = \sum_{r=-\infty}^{+\infty} \left|f[r]\right|^2 = E_f .$$

8.4-6

(a) is trivial

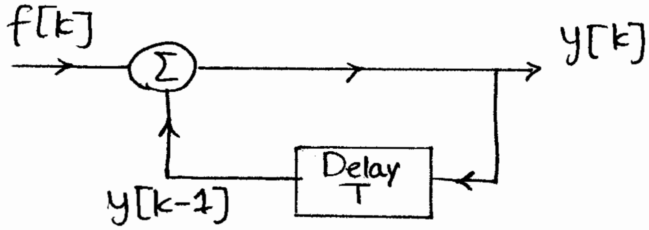
(b) $P_{f[-k]} = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum\limits_{k=-N}^{N} \left|f[-k]\right|^2 = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum\limits_{r=N}^{-N} \left|f[r]\right|^2 = P_f$

(c) $P_{f[k-m]} = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum\limits_{k=-N}^{+N} \left|f[k-m]\right|^2 = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum\limits_{r=-N-m}^{N-m} \left|f[r]\right|^2 = P_f$

(d) $P_{cf} = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum\limits_{k=-N}^{N} c^2 \left|f[k]\right|^2 = c^2 P_f$

8.5-1

$$y[k] = y[k-1] + f[k] \rightarrow y[k] - y[k-1] = f[k]$$



System Realization.

If there were a sales tax of 10% the difference equation becomes:

$$y[k]=y[k-1]+1.1f[k] \quad \text{and} \quad y[k]-y[k-1]=1.1f[k]$$

8.5-2 $\quad P[k+1]-1.02\,P[k]=1.01\,i[k]$

8.5-3

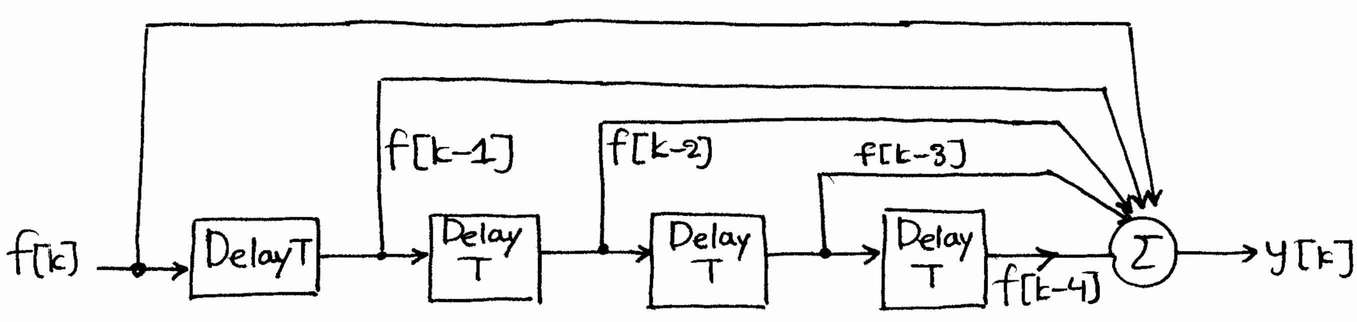
The area under $f(t)$ from 0 to kT is $y(kT)$. Similarly, the area from 0 to $(k-1)T$ is $y[(k-1)T]$. But this area is equal to $Tf[(k-1)T]$ (in the limit $T\to 0$). Now using the notation $y[k]$ to denote $y(kT)$, etc., it follows that (assuming T to be small)

$$y[k]-y[k-1]=Tf[k-1]$$

If the input is $u(t)$, then $f[k]=u[k]$. The equation is $y[k]-y[k-1]=Tu[k-1]$. Setting $k=0$ in this equation and using the fact that $y[-1]=0$ we obtain $y[0]=0$. Setting $k=1$ and using the fact that $y[0]=0$ we obtain $y[1]=T$. Continuing this way, we obtain $y[k]=kTu[k]$.

When the integrator equation is $y[k]-y[k-1]=Tf[k]$, a similar argument shows that $y[0]=T$, $y[1]=2T$ and in general $y[k]=(k+1)T=kTu[k]+Tu[k]$

8.5-4 $\quad y[k]=1/5\,\{f[k]+f[k-1]+f[k-2]+f[k-3]+f[k-4]\}$



8.5.5

The node equation at the k^{th} node is $i_1 + i_2 + i_3 = 0$, or

$$\frac{v[k-1]-v[k]}{R} + \frac{v[k+1]-v[k]}{R} - \frac{v[k]}{aR} = 0$$

Therefore

$$a\left(v[k-1]+v[k+1]-2v[k]\right) - v[k] = 0$$

$$\rightarrow v[k+1] - \left(2+\frac{1}{a}\right)v[k] + v[k-1] = 0$$

which is: $v[k+2] - \left(2+\frac{1}{a}\right)v[k+1] + v[k] = 0$