Chapter 9 (Answers)

Prepared by: Payam Abolghasem (email: payam@grads.ece.mcmaster.ca)

9.1-1	$y[0] = 5$,	$y[1] = 2.5$,	$y[2] = 1.25$
9.1-2	$y[0] = -15$,	$y[1] = -13$,	$y[2] = -10.2$
9.1-3	$y[0] = 100$,	$y[1] = 75$,	$y[2] = 75$
9.1-4	$y[0] = -12$,	$y[1] = 36$,	$y[2] = -63$
9.1-5	$y[0] = -5$,	$y[1] = 7.667$,	$y[2] = -5.445$

9.2-1 $\quad y[k] = 2(-1)^k - 4(-2)^k \quad k \geq 0$

9.2-2 $\quad y[k] = -(3+2k)(-1)^k$

9.2-3 $\quad y[k] = 2(\sqrt{2})^k \cos\left(\frac{\pi}{4}k\right)$

9.2-4 $\quad v[k] = \dfrac{100}{2^N - (0.5)^N}\left[2^N (0.5)^k - (0.5)^N (2)^k\right] \quad k = 0, 1, \ldots N.$

9.3-1 $\quad h[k] = \frac{1}{2}\delta[k] - \frac{1}{2}(-2)^k u[k]$

9.3-2 $\quad h[k] = (-2)^k u[k]$

9.3-3 $\quad h[k] = \frac{1}{3} k (3)^k u[k]$

9.3-4 $\quad h[k] = 2.061(5)^k \cos(0.923k - 0.244) u[k].$

9.3.5

a) $E^n y[k] = (b_n E^n + b_{n-1} E^{n-1} + \cdots + b_0) f[k]$

or $y[k] = b_n f[k] + b_{n-1} f[k-1] + \cdots + b_0 f[k-n]$

when $f[k] = \delta[k]$, $y[k] = h[k]$. Therefore

$$h[k] = b_n \delta[k] + b_{n-1} \delta[k-1] + \cdots + b_0 \delta[k-n]$$

b) Here $n=3$, $b_3 = 3$, $b_2 = -5$, $b_1 = 0$, $b_0 = -2$. Therefore

$$h[k] = 3\delta[k] - 5\delta[k-1] - 2\delta[k-3]$$

9.4- $y[k] = e^{-k} u[k] * (-2)^k u[k] = (\frac{1}{e})^k u[k] * (-2)^k u[k]$

$$= \frac{(\frac{1}{e})^{k+1} - (-2)^{k+1}}{(1/e)+2} u[k] = \frac{e}{2e+1}\left[e^{-(k+1)} - (-2)^{k+1}\right] u[k].$$

9.4-2 $y[k] = \left\{\frac{1}{2} e^{-k} - \frac{e}{2(2e+1)}\left[e^{-(k+1)} - (-2)^{k+1}\right]\right\} u[k]$

9.4-3 $y[k] = 9\left[\frac{11}{8}(3)^{k+1} - (2)^{k+1} - \frac{3}{8}(-5)^{k+1}\right] u[k]$

9.4-4 $y[k] = \frac{18}{25}\left[(3)^{-k} - (2)^k + 5k(2)^k\right] u[k]$

9.4-5 $y[k] = \frac{1}{\sqrt{7}}\left\{(3)^{k+1} \cos\left[\frac{\pi}{3}(k+1) - 2.261\right] + 0.637(2)^{k+1}\right\} u[k]$

9.4-6 Total response $= \frac{1}{2e+1}\left[-(38e+20)(-2)^k + (e)^{-k}\right] u[k]$

9.4-7

(a) $y[k] = \frac{2^{k+1} - (0.5)^{k+1}}{2-0.5} u[k] = \frac{2}{3}\left[2^{k+1} - (0.5)^{k+1}\right] u[k]$

(b) $y[k] = \frac{1}{8}\,\frac{2}{3}\left[2^{k+1} - (0.5)^{k+1}\right]u[k] = \frac{1}{12}\left[2^{k+1} - (0.5)^{k+1}\right]u[k]$

(c) $y[k] = \frac{8}{3}\left[2^{k-1} - (0.5)^{k-1}\right]u[k-2]$

9.4-8

$$h[k] = (1.01)^k u[k]$$

$$y[k] = 50000\left[(1.01)^{k+1} - 1\right]u[k] - 1500\,(1.01)^{k-4}u[k-4]$$

9.4-9

This problem is identical to the savings account problem with negative initial deposit (loan). If M is the initial loan, then $y[0] = -M$. If $y[k]$ is the loan balance, then [See equ. (8.25b)]

$$y[k+1] - a\,y[k] = f[k+1] \quad a = 1+r$$

or

$$(E-a)\,y[k] = E f[k]$$

The characteristic root is a and the impulse response for this system is found in prob. 9.4-8 to be:

$$h[k] = a^k u[k]$$

This problem can be solved in two ways.

First method: We may consider the loan of M dollars as an negative input $-M\delta[k]$. The monthly payment of P starting at $k=1$ also is an input. with zero initial conditions Thus the total input is $f[k] = -M\delta[k] + Pu[k-1]$. Because $u[k] = \delta[k] + u[k-1]$ we can express the input in a more convenient form as $f[k] = -(M+P)\delta[k] + Pu[k]$. The loan balance response is:

$$\begin{aligned}
y[k] &= h[k] * f[k] \\
&= -(M+P)a^k u[k] + Pa^k u[k] * u[k] \\
&= -(M+P)a^k u[k] + P\left[\frac{a^{k+1}-1}{a-1}\right]u[k] \\
&= -Ma^k u[k] - P\left[a^k - \frac{a^{k+1}-1}{a-1}\right]u[k] = \left\{-Ma^k + P\left[\frac{a^k-1}{a-1}\right]\right\}u[k]
\end{aligned}$$

also $a = 1+r$ and $a-1=r$ where r is the interest rate per dollar per month. At $k=N$ the loan balance is zero therefore

$$y[N] = -Ma^N + P\left[\frac{a^N-1}{r}\right] = 0$$

$$\rightarrow \quad P = \frac{ra^N}{a^N-1}M.$$

Second method: See the detailed solutions of this chapter which will come later.

9.4-10

Using the result of problem 9.4-9. In this problem $r = 0.015$, $a = 1.015$ $P = 500$, $M = 10\,000$. Therefore

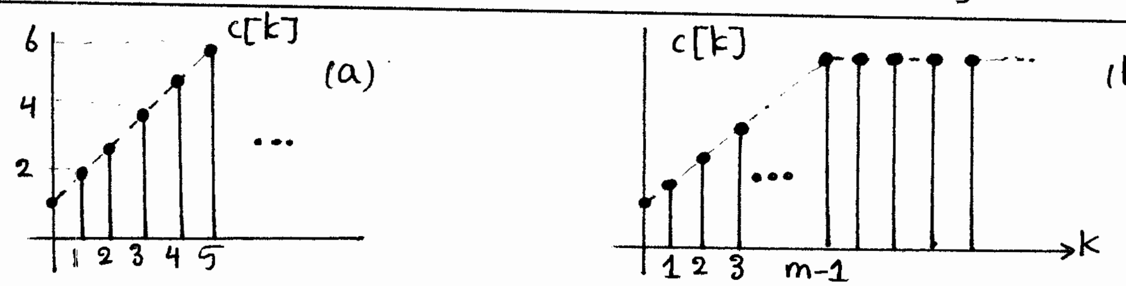
$$500 = 10000\,\frac{(1.015)^N(0.015)}{(1.015)^N - 1} \quad \text{or} \quad (1.015)^N = 1.42857$$

$$\rightarrow N = 23.956$$

Hence $N=23$ payments are needed. The residual balance (remainder) at the 23rd payment is

$$y[23] = -10000(1.015)^{23} + 500\left[\frac{(1.015)^{23}-1}{0.015}\right] = -471.2$$

9.4-11

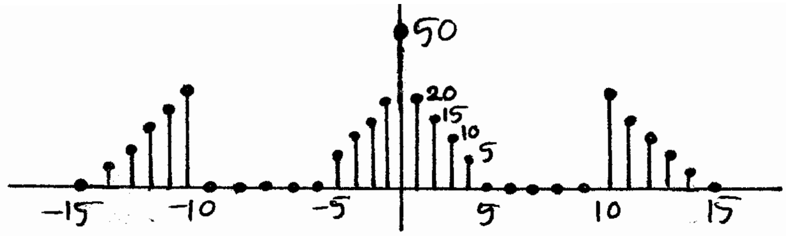


9.4-12

$y[0] = 15$, $y[1] = 15$, $y[2] = 14$, $y[3] = 12$, $y[4] = 9$, $y[5] = 5$, $y[6] = 0$

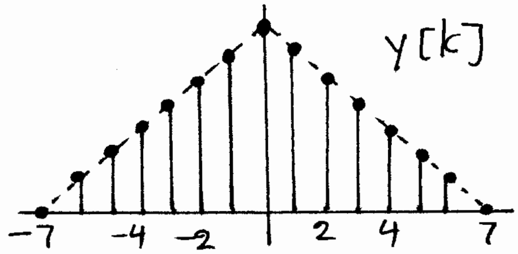
9.4-13

k	y[k]	k	y[k]	k	y[k]
0	50	±6	0	±12	15
±1	20	±7	0	±13	10
±2	15	±8	0	±14	5
±3	10	±9	0	±15	0
±4	5	±10	5	±16	0
±5	0	±11	20	±17	0
				±18	0



9.4-14

k	0	±1	±2	±3	±4	±5	±6	±7	|k|>7
y[k]	7	6	5	4	3	2	1	0	0



9.4-15

(a)

k	0	1	2	3	4	5
h[k]	8	4	2	1	0.5	0.25

(b) Input sequence = $(1, 1/3, 1/9, \cdots)$

9.4-17
When the input to a unit delay is everlasting exponential z^k, the output is also everlasting z^{k-1}. Hence according to equation (9.58) $H[z] = z^{k-1}/z^k = 1/z$

9.5-1. $y[k] = \frac{1}{2e+1}\left[2e(-2)^k + e^{-k}\right] \quad k \geq 0$

9.5-2 $y[k] = \frac{e}{2e+1}\left[-(-2)^k + e^{-k}\right] \quad k \geq 0$

9.5-3 (a) $y[k] = -\frac{1}{4}(-1)^k + \frac{1}{5}(-2)^k + \frac{21}{20}(3)^k \quad k \geq 0$

(b) $y[k] = \frac{11}{4}(-1)^k - \frac{39}{5}(-2)^k + \frac{21}{20}(3)^k \quad k \geq 0$

9.5-4 $y[k] = (33/16 + 9/4\,k)(-1)^k - 1/16\,(3)^{-k} \quad k \geq 0$

9.5-5 $y[k] = -\frac{23}{9}(0.2)^k + \frac{32}{9}(0.8)^k - \frac{5}{3}(0.2)^k \quad k \geq 0$

9.5-6 $y[k] = 0.241(0.2)^k - 0.377(0.8)^k + 0.765\cos\left(\frac{\pi k}{2} - 1.393\right)$

9.6-1 (a). Asymptotically stable

(b) Marginally stable

(c) Unstable

(d) Unstable

(e) Marginally stable.

9.6-2

Assume that a system exist that violates (9.61) and yet produces output for every bounded input. The system response at $k = k_1$ is

$$y[k_1] = \sum_{m=0}^{\infty} h[m] f[k_1 - m]$$

Consider a bounded input $f[k]$ such that

$$f[k_1 - m] = \begin{cases} 1 & \text{if } h[m] > 0 \\ -1 & \text{if } h[m] < 0 \end{cases}$$

In this case $h[m] f[k_1 - m] = |h[m]|$

and $y[k_1] = \sum_{m=0}^{\infty} |h[m]| = \infty$ This violates the assumption.

9.6-3 For marginally stable system $h[k]$ does not decay. For large k, it is either constant or oscillates with constant amplitude. Clearly

$$\sum_{m=0}^{\infty} |h[m]| = \infty$$

The system is BIBO unstable.