**Trigonometric Fourier Series**

$$f(t) = a_o + \sum_{m=1}^{\infty} a_m \cos(m\omega_o t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$$

$$a_o = \frac{1}{T_o} \int_{T_o} f(t)\, dt$$

$$a_m = \frac{2}{T_o} \int_{T_o} f(t)\, cos(m\omega_o t)\, dt$$

$$b_n = \frac{2}{T_o} \int_{T_o} f(t)\, sin(n\omega_o t)\, dt$$

**Complex Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} D_n \exp(jn\omega_o t)$$

$$D_n = \frac{1}{T_o} \int_{T_o} f(t)\, exp(-jn\omega_o t)\, dt$$

**Parseval's Theorem**

$$P_f = \frac{1}{T_o} \int_{T_o} f^2(t)\, dt = D_o^2 + 2\sum_{n=1}^{\infty} |D_n|^2$$

**Some Useful Series**

$$\sum_{m=0}^{N} r^m = \frac{r^{N+1} - 1}{r - 1},\ r \neq 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

$$\sum_{n\ \text{odd}}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$$

**Some Useful Trigonometric Identities**

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin x \sin \mathrm{y} = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

$$\cos x \cos \mathrm{y} = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$

$$\sin x \cos \mathrm{y} = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$

## Some Useful Discrete-time Convolutions

$$\delta[k-j] * f[k] = f[k-j]$$

$$\gamma^k u[k] * u[k] = \left[\frac{1-\gamma^{k+1}}{1-\gamma}\right] u[k]$$

$$u[k] * u[k] = (k+1)\, u[k]$$

$$\gamma_1^k u[k] * \gamma_2^k u[k] = \left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2}\right] u[k], \quad \gamma_1 \neq \gamma_2$$

$$\gamma^k u[k] * \gamma^k u[k] = (k+1)\gamma^k u[k]$$

$$ku[k] * ku[k] = \frac{1}{6} k(k-1)(k+1)\, u[k]$$

## Some Useful Fourier Transforms:

| Signal | Fourier Transform |
|---|---|
| $e^{-at}u(t),\ a>0$ | $\frac{1}{(a+j\omega)}$ |
| $e^{at}u(-t),\ a>0$ | $\frac{1}{(a-j\omega)}$ |
| $e^{-a\lvert t\rvert},\ a>0$ | $\frac{2a}{(a^2+\omega^2)}$ |
| $te^{-at}u(t),\ a>0$ | $\frac{1}{(a+j\omega)^2}$ |
| $t^n e^{-at}u(t),\ a>0$ | $\frac{n!}{(a+j\omega)^{n+1}}$ |
| $\delta(t)$ | $1$ |
| $1$ | $2\pi\delta(\omega)$ |
| $e^{j\omega_o t}$ | $2\pi\delta(\omega-\omega_o)$ |
| $\cos(\omega_0 t)$ | $\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$ |
| $\sin(\omega_0 t)$ | $j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))$ |
| $u(t)$ | $\pi\delta(\omega)+\frac{1}{j\omega}$ |
| $\mathrm{sgn}(t)$ | $\frac{2}{j\omega}$ |
| $\cos(\omega_0 t)\ u(t)$ | $0.5\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))+\frac{j\omega}{\omega_o^2-\omega^2}$ |
| $\sin(\omega_0 t)\ u(t)$ | $0.5j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))+\frac{\omega_o}{\omega_o^2-\omega^2}$ |
| $e^{-at}\sin(\omega_0 t)\ u(t)$ | $\frac{\omega_o}{(a+j\omega)^2+\omega_o^2}$ |
| $e^{-at}\cos(\omega_0 t)\ u(t)$ | $\frac{a+j\omega}{(a+j\omega)^2+\omega_o^2}$ |
| $\mathrm{rect}(t/\tau)$ | $\tau\,\mathrm{sinc}(\omega\tau/2)$ |
| $\frac{W}{\pi}\sin\mathrm{c}\,(Wt)$ | $\mathrm{rect}\left(\frac{\omega}{2W}\right)$ |
| $\Delta(t/\tau)$ | $\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$ |
| $\frac{W}{2\pi}\mathrm{sinc}^2\left(\frac{W\,t}{2}\right)$ | $\Delta\left(\frac{\omega}{2W}\right)$ |

**Some Useful Fourier Transform Properties:**

| Signal | Fourier Transform |
|---|---|
| $f(at)$ | $\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$ |
| $f(t-t_o)$ | $F(\omega)\, e^{-j\omega t_o}$ |
| $f(t)\, e^{j\omega_o t}$ | $F(\omega-\omega_o)$ |
| $f_1(t) * f_2(t)$ | $F_1(\omega)\, F_2(\omega)$ |
| $f_1(t) f_2(t)$ | $\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$ |
| $df/dt$ | $j\omega F(\omega)$ |
| $(-jt)^n f(t)$ | $d^n F(\omega)/d\omega^n$ |
| $F(t)$ | $2\pi f(-\omega)$ |

**Parseval's Theorem for Nonperiodic Signals**

$$E_f = \int_{-\infty}^{\infty} \lvert f(t) \rvert^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert F(\omega) \rvert^2 d\omega$$

## Some useful Laplace Transforms

| $f(t)$ | $F(s)$ |
|---|---|
| $\delta(t)$ | 1 |
| $u(t)$ | $1/s$ |
| $t\,u(t)$ | $1/s^2$ |
| $t^n\,u(t)$ | $n!/s^{n+1}$ |
| $e^{\lambda t}\,u(t)$ | $1/(s-\lambda)$ |
| $te^{\lambda t}\,u(t)$ | $1/(s-\lambda)^2$ |
| $t^n e^{\lambda t}\,u(t)$ | $n!//(s-\lambda)^{n+1}$ |
| $\cos(bt)\,u(t)$ | $s/(s^2+b^2)$ |
| $\sin(bt)\,u(t)$ | $b/(s^2+b^2)$ |
| $e^{-at}\cos(bt)\,u(t)$ | $\dfrac{s+a}{(s+a)^2+b^2}$ |
| $e^{-at}\sin(bt)\,u(t)$ | $\dfrac{b}{(s+a)^2+b^2}$ |
| $r\,e^{-at}\cos(bt)\,u(t)$<br>$r=\sqrt{\dfrac{A^2c+B^2-2ABa}{c-a^2}}$, $\theta=\tan^{-1}\dfrac{Aa-B}{A\sqrt{c-a^2}}$,<br>$b=\sqrt{c-a^2}$ | $\dfrac{As+B}{s^2+2as+c}$ |

## Some Useful Laplace Transform Properties

| operation | $F(s)$ |
|---|---|
| $df/dt$ | $sF(s)-f(0^-)$ |
| $d^2f/dt^2$ | $s^2F(s)-sf(0^-)-f'(0^-)$ |
| $\int_{0^-}^{t} f(\tau)d\tau$ | $F(s)/s$ |
| $\int_{-\infty}^{t} f(\tau)d\tau$ | $\frac{F(s)}{s}+\frac{\int_{-\infty}^{0^-} f(\tau)d\tau}{s}$ |
| $f(t-t_o)u(t-t_o)$ | $F(s)e^{-st_o}$ |
| $f(t)e^{s_o t}$ | $F(s-s_o)$ |
| $-t\,f(t)$ | $dF(s)/ds$ |
| $f(at)$, $a > 0$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| $f_1(t)*f_2(t)$ | $F_1(s)\,F_2(s)$ |
| $f_1(t)f_2(t)$ | $\frac{1}{2\pi j}F_1(s)*F_2(s)$ |
| $f(0^+)$ | $\lim_{s\to\infty} sF(s)$, $n>m$ |
| $f(\infty)$ | $\lim_{s\to 0} sF(s)$ |

## Some Useful Z-transforms

| $f[k]$ | $F[z]$ |
|---|---|
| $\delta[k-j]$ | $z^{-j}$ |
| $u[k]$ | $\frac{z}{(z-1)}$ |
| $ku[k]$ | $\frac{z}{(z-1)^2}$ |
| $k^2\,u[k]$ | $\frac{z(z+1)}{(z-1)^3}$ |
| $k^3\,u[k]$ | $\frac{z(z^2+4z+1)}{(z-1)^4}$ |
| $\gamma^{k-1}u[k-1]$ | $\frac{1}{(z-\gamma)}$ |
| $\gamma^k u[k]$ | $\frac{z}{(z-\gamma)}$ |
| $k\,\gamma^k u[k]$ | $\frac{\gamma z}{(z-\gamma)^2}$ |
| $k^2\gamma^k u[k]$ | $\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$ |
| $\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$ | $\frac{z}{(z-\gamma)^{m+1}}$ |
| $\lvert\gamma\rvert^k\cos(\beta k)u[k]$ | $\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$ |
| $\lvert\gamma\rvert^k\sin(\beta k)u[k]$ | $\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$ |

## Some Useful Z-transform Properties

| $f[k]$ | $F[z]$ |
|---|---|
| $f[k-m]u[k-m]$ | $\frac{F[z]}{z^m}$ |
| $f[k-m]u[k]$ | $\frac{F[z]}{z^m} + \frac{\sum_{k=1}^{m} f[-k]z^k}{z^m}$ |
| $f[k-1]u[k]$ | $\frac{F[z]}{z} + f[-1]$ |
| $f[k-2]u[k]$ | $\frac{F[z]}{z^2} + \frac{f[-1]}{z} + f[-2]$ |
| $f[k-3]u[k]$ | $\frac{F[z]}{z^3} + \frac{f[-1]}{z^2} + \frac{f[-2]}{z} + f[-3]$ |
| $f[k+m]u[k]$ | $z^m F[z] - z^m \sum_{k=1}^{m-1} f[k]z^{-k}$ |
| $f[k+1]u[k]$ | $zF[z]-zf[0]$ |
| $f[k+2]u[k]$ | $z^2 F[z]- z^2 f[0]-zf[1]$ |
| $f[k+3]u[k]$ | $z^3 F[z]- z^3 f[0]-z^2 f[1]-zf[2]$ |
| $\gamma^k f[k]u[k]$ | $F(z/\gamma)$ |
| $f_1[k]*f_2[k]$ | $F_1[z]F_2[z]$ |
| $kf[k]u[k]$ | $-z(dF/dz)$ |