Q1: This is an odd signal

$a_0 = 0, \quad a_n = 0$

$$b_n = \frac{4}{T_0} \int_0^{\frac{T_0}{2}} f(t) \sin n\omega_0 t \, dt$$

$$f(t) = \frac{2t}{T_0} \quad -\frac{T_0}{2} \leq t \leq \frac{T_0}{2} \quad , T_0 = 2\text{ msec}$$

$$\Rightarrow b_n = \frac{4}{T_0} \int_0^{\frac{T_0}{2}} \frac{2t}{T_0} \sin n\omega_0 t \, dt$$

$$b_n = \frac{8}{T_0^2} \int_0^{\frac{T_0}{2}} \frac{2t}{T_0} \sin n\omega_0 t \, dt$$

Integrating by parts, we get

$$b_n = \frac{8}{T_0^2} \left[-\frac{1}{n\omega_0} t \cos n\omega_0 t \Big|_0^{\frac{T_0}{2}} + \frac{1}{n\omega_0} \int_0^{\frac{T_0}{2}} \cos n\omega_0 t \, dt \right]$$

$$b_n = \frac{-8}{n T_0^2 \omega_0} \times \frac{T_0}{2} \cos n \frac{\omega_0 T_0}{2} + 0$$

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Noticing that $\omega_o T_o = 2\pi$, we have

$$b_n = \frac{-4}{n * 2\pi} \cos n\pi$$

$$b_n = \frac{2}{n\pi} (-1)^{n+1} \longleftarrow \text{(a)}$$

$$D_n = \frac{a_n - j b_n}{2} = \frac{j(-1)^n}{n\pi}, \quad D_o = a_o = 0$$

To get signal power, we have

$$P = \frac{1}{T_o} \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} |f(t)|^2 \, dt$$

$$P = \frac{1}{T_o} \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} \frac{4t^2}{T_o^2} \, dt$$

$$P = \frac{1}{T_o^3} \left. \frac{4t^3}{3} \right|_{-\frac{T_o}{2}}^{\frac{T_o}{2}}$$

$$P = \left(\frac{4}{24} + \frac{4}{24} \right) = \frac{1}{3} \text{ Watts}$$

Also, $2\sum_{n=1}^{\infty}|D_n|^2 = 2\sum_{n=1}^{\infty}\frac{1}{n^2\pi^2}$

$$2\sum_{n=1}^{\infty}|D_n|^2 = \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{2}{\pi^2}\cdot\frac{\pi^2}{6}$$

$$\Rightarrow 2\sum_{n=1}^{\infty}|D_n|^2 = \frac{1}{3} = P$$

Q1.c) This circuit is a parallel resonance circuit

$$\omega_c = \frac{1}{\sqrt{LC}} = 3.135 \times 10^3$$
$$\simeq 1000\pi = \omega_0$$

only first harmonic is allowed to pass unattenuated

$$y_1(t) \simeq D_1 e^{j\omega_0 t} + D_{-1} e^{-j\omega_0 t}$$

$$y_1(t) \simeq 2|D_1| \cos(\omega_0 t + \angle D_1)$$

but $|D_1| = \frac{1}{\pi}$, $\angle D_1 = -\frac{\pi}{2}$

$$\Rightarrow y_1(t) \simeq \frac{2}{\pi} \cos(\omega_0 t - \frac{\pi}{2})$$

$$y_1(t) \simeq \frac{2}{\pi} \sin \omega_0 t$$

$$y_1(t) = b_1 \sin(1000\pi t)$$

Q2: $f = \frac{1}{10^{-3}} = 1\,kHz$

Nyquist frequency is $2f = 2\,kHz$

$\Rightarrow$ Sampling frequency $= 20\,kHz$

$\Rightarrow$ 20 Samples per Cycle

$$T = \frac{1}{20 \times 10^{3}} = 5 \times 10^{-5} \text{ sec}$$

$$f_T(t) = \sum_{k=-\infty}^{\infty} \sin(2\pi \times 10^{3} * k * T)\, \delta(t - kT)$$

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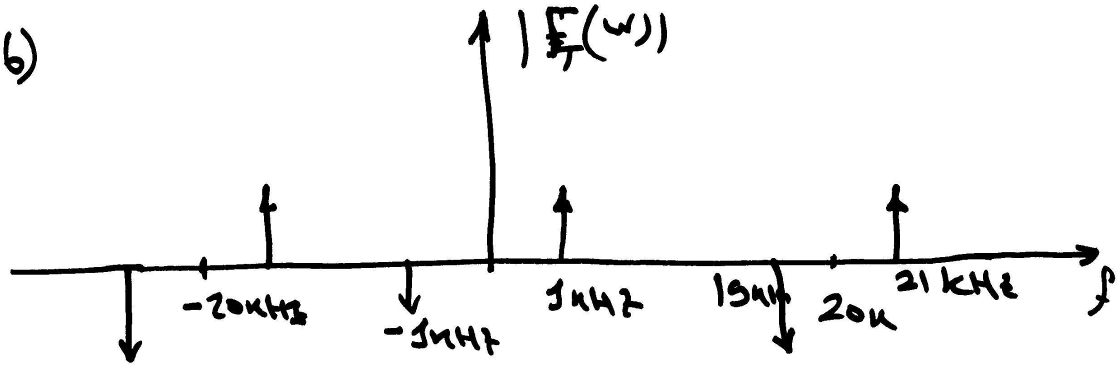
$$f_T(t) = \sum_{k=-\infty}^{k=\infty} \sin(0.1\pi k)\,\delta(t - k*5*10^{-5})$$

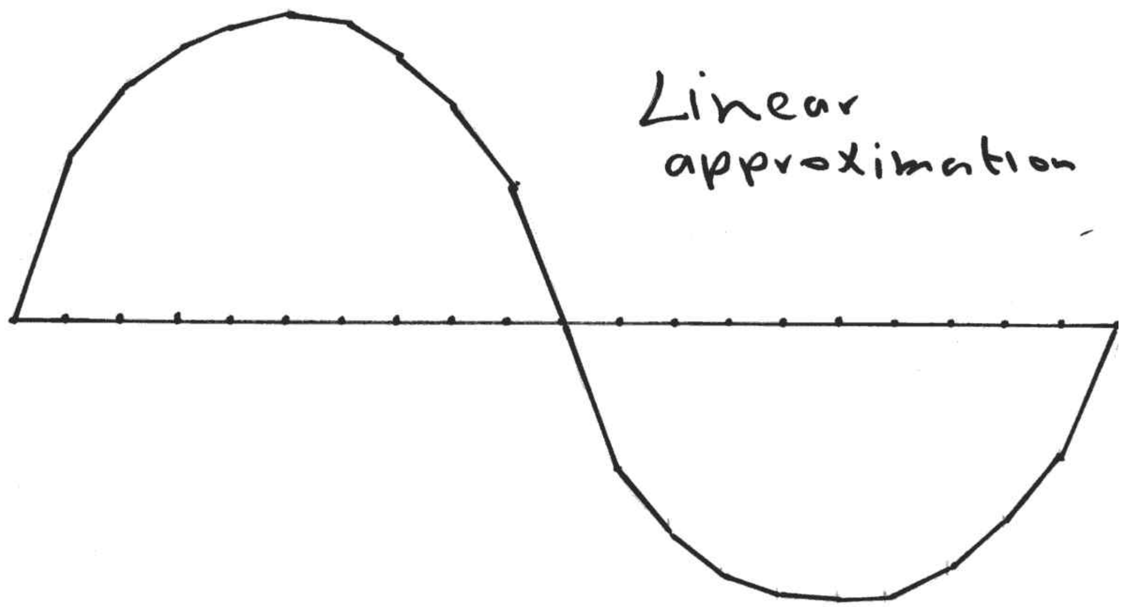
b)



c) The reconstructed signal is

$$f_c(t) = \sum_{k=-\infty}^{\infty} \sin(0.1\pi k)\, h(t - k*5*10^{-5})$$

d) This filter obtains a linear approximation of the signal


Linear approximation

Q3)

$$x(t) = 2e^{-t} \Rightarrow X(w) = \frac{2}{jw+1}$$

$$y(t) = x(2t) = 2e^{-2t}u(t)$$

$$Y(w) = \frac{1}{2}F\left(\frac{w}{2}\right) = \frac{1}{j\frac{w}{2}+1} = \frac{2}{jw+2}$$

$$G(w) = \frac{\frac{1}{jw}Y(w)}{1+\frac{1}{jw}} = \frac{1}{jw+1} \cdot \frac{2}{jw+2}$$

$$G(w) = \frac{2}{jw+1} - \frac{2}{jw+2}$$

$$g(t) = 2e^{-t}u(t) - 2e^{-2t}u(t)$$

$$V(w) = \frac{2}{jw+1} \cdot \frac{1}{1+jw}$$

$$v(t) = 2te^{-t}u(t)$$

$$W(t) = v(2t) = 2(2t)e^{-2t}u(2t)$$

$$W(t) = 4te^{-2t}u(t)$$

$W(t) \neq g(t)$ as order makes a difference

Q4) $T_0 = 5T = 0.5$ sec

$\omega_0 = \frac{2\pi}{T_0} = 4\pi$ rad/sec

$\Omega_0 = 4\pi * 0.1 = 0.4\pi$ rad/sample

$$F_r = \sum_{k=0}^{4} f_k e^{-jk\Omega_0 r}$$

$$F_r = T \sum_{k=0}^{4} f(kT) e^{-jk\Omega_0 r}$$

$$F_0 = 0.1 (1 + 2 - 2 - 1 + 0) = 0$$

$$F_1 = T \sum_{k=0}^{4} f(kT) e^{-jk\Omega_0}$$

$$F_1 = 0.1 (1 \cdot e^{-j0} + 2 e^{-j0.4\pi} - 2 e^{-j0.8\pi} - 1 e^{-j1.2\pi} + 0)$$

$$F_2 = 0.1 \sum_{k=0}^{4} f(kT) e^{-jk(0.8\pi)}$$

$$F_3 = 0.1 \sum_{k=0}^{4} f(kT) e^{-jk(1.2\pi)}$$

$$F_4 = 0.1 \sum_{k=0}^{4} f(kT) e^{-jk(1.6\pi)}$$

a5) $\ddot{y} + 2\dot{y} + 5y = \dot{x} = \delta(t)$

$\Rightarrow s^2Y(s) - s\,y(0) - \dot{y}(0) + 2(sY(s) - y($

$+ 5Y(s) = 1$

For ZIR we have

$Y(s)(s^2 + 2s + 5) - 2s - 0 - 2 = 0$

$$Y(s) = \frac{2s+2}{(s^2+2s+5)} = \frac{2(s+1)}{((s+1)^2+4)}$$

$y_{ZIR}(t) = 2e^{-t}\cos 2t$

For ZSR, we have

$Y(s)(s^2 + 2s + 5) = 1$

$$Y(s) = \frac{1}{((s+1)^2+4)}$$

$y_{ZS}(t) = \frac{1}{2}e^{-t}\sin 2t$

Q6) $T(s) = \dfrac{G(s)}{1+3G(s)}$

$$T(s) = \frac{\frac{s}{s+1000}}{1+\frac{3s}{s+1000}} = \frac{s}{4s+1000}$$

$$T(s) = \frac{s}{4(s+250)}$$

* Original filter $\dfrac{s}{s+1000}$ is a high pass filter with gain = 1 & cut-off frequency 1000 rad/sec

* New filter $\dfrac{s}{4(s+250)}$ is also a high pass filter with gain = $\frac{1}{4}$ & cut-off frequency of 250 rad/sec