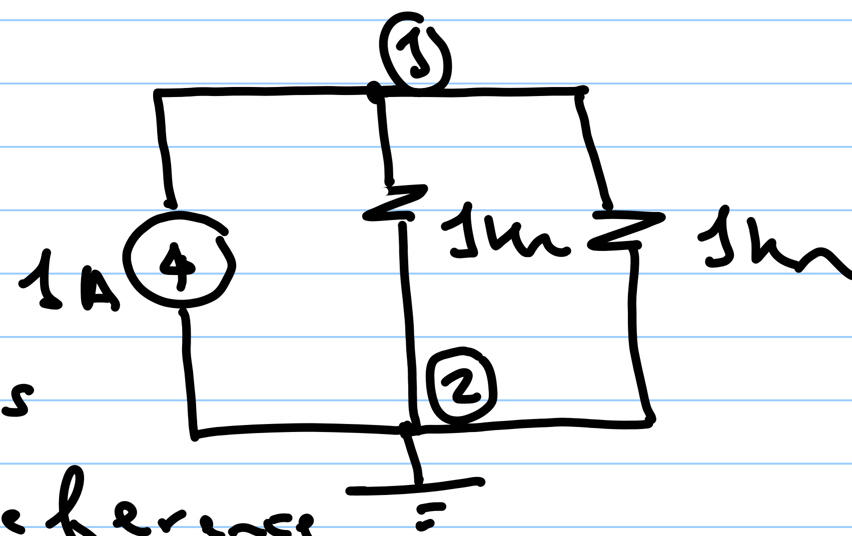
Lecture 6

From Section 3.1 of Textbook

Solve E3.1 - E3.4, 3.2, 3.4

Nodal Analysis

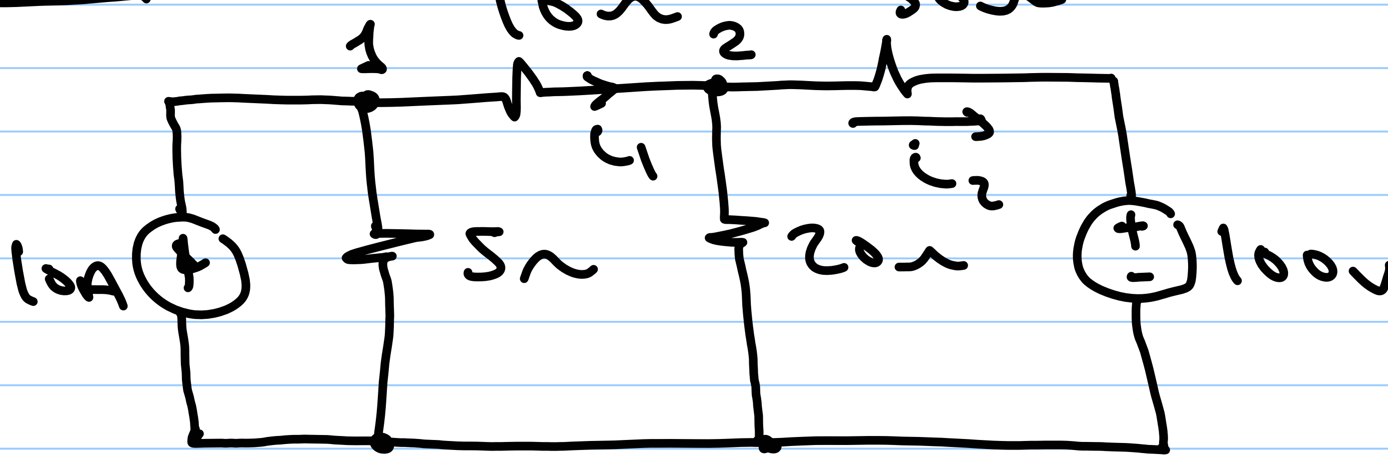
* The shown circuit contains 2 nodes
* The lower node is selected as the reference node (ground node)
* The voltage of node relative to ground ($v_1 - v_2 = v_a$) is sufficient to get all currents



Nodal Analysis (Cont'd)

* for a circuit with N nodes, we pick one node as ground. Our unknowns are the voltages of the N-1 other nodes relative to ground
* KCL is written for each of these nodes to obtain (N-1) equations in the N-1 unknowns

Example

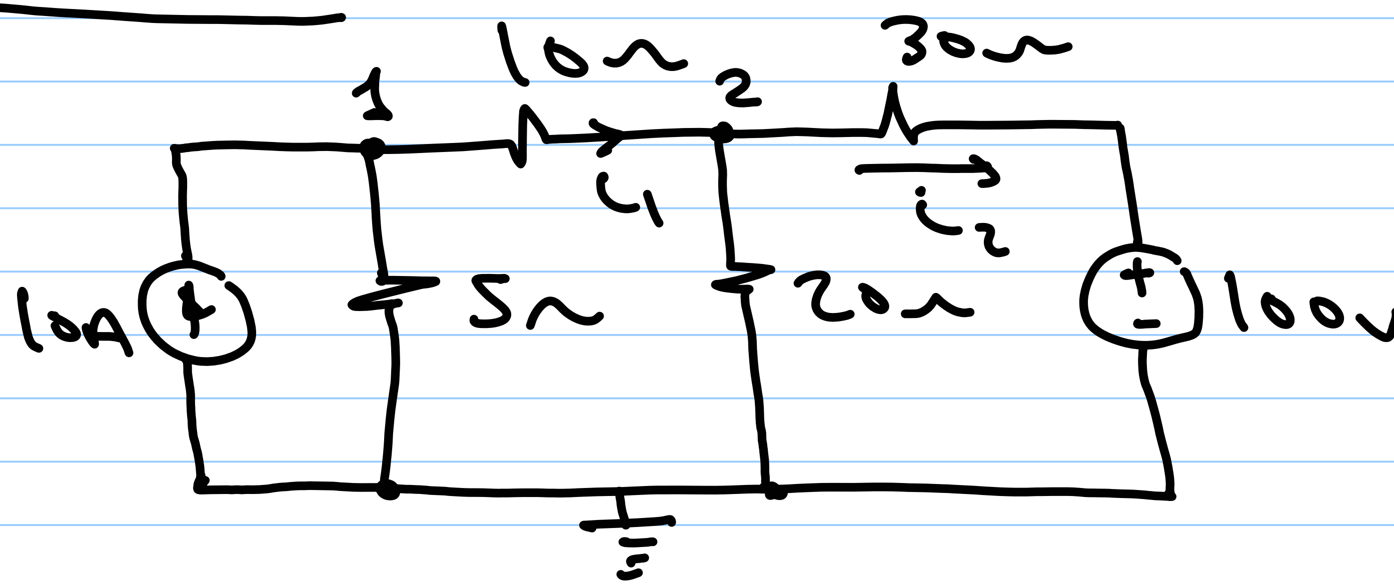


Utilize nodal analysis to solve for the currents in all branches

Example (Cont'd)

* This circuit contains 3 nodes.
* The bottom node is selected as ground
* We need 2 equations to solve for v_1 and v_2, the nodal voltages relative to ground
* KCL is applied to obtain these equations

Example (cont'd)



1st eq: $\frac{v_1}{5} + \frac{(v_1 - v_2)}{10} - 10 = 0$

2nd eq: $\frac{v_2}{20} + \frac{v_2 - 100}{30} - \frac{(v_1 - v_2)}{10} = 0$

Example (Cont'd)

* rearranging, we have

$$\frac{3}{10} v_1 - \frac{1}{10} v_2 = 10 \quad \longleftarrow \textcircled{1}$$

$$-\frac{1}{10} v_1 + \frac{11}{60} v_2 = \frac{10}{3} \quad \longleftarrow \textcircled{2}$$

in matrix form

$$\begin{bmatrix} 3/10 & -1/10 \\ -1/10 & 11/60 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 10 \\ \frac{10}{3} \end{bmatrix} \Longrightarrow \underline{G}\,\underline{v} = \underline{I}$$

Example (Cont'd)

*Using Gaussian elimination

(3/10)	-1/10	10
-1/10	11/60	10/3
1	-1/3	$\frac{100}{3}$
0	9/60	$\frac{20}{3}$

$$9/60\, v_2 = \frac{20}{3} \Rightarrow v_2 = \frac{400}{9}\, v$$

$$v_1 = \frac{100}{3} + \frac{400}{27} = \frac{1300}{27}\, v$$

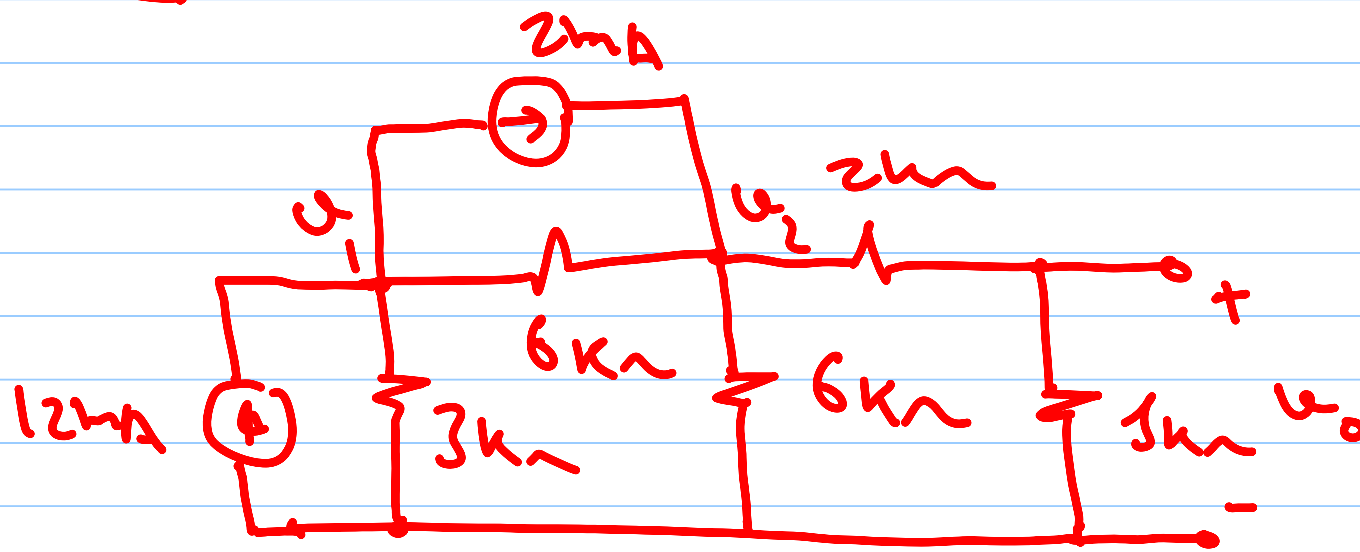
Example (Cont'd)

* all currents can then be evaluated

$$i_1 = \frac{v_1 - v_2}{10} = 0.37\,A$$

$$i_2 = \frac{v_2 - 100}{30} = -1.85\,A$$

Example



Utilize nodal analysis to solve for both v_1 and v_o