Dr. Mohamed Bakr, EE2CI5, 2007

Note Title 10/8/2007

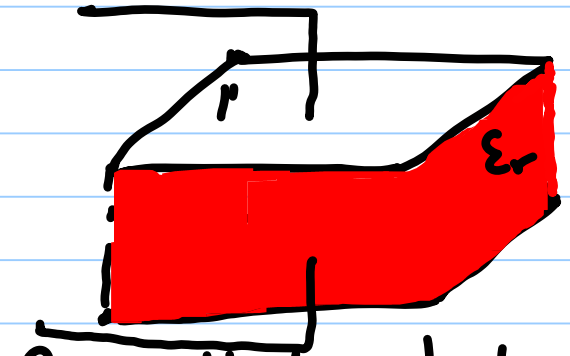
Lecture 15

from section 6.1 of Textboon

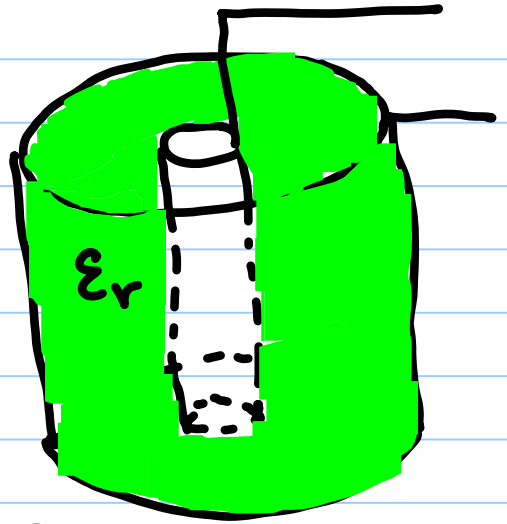
Solve 6.1, 6.4, 6.6, 6.9,
6.10, 6.14, 6.18

Capacitors

* A Capacitor Consists of two conducting electrodes separated by a dielectric (Insulator)



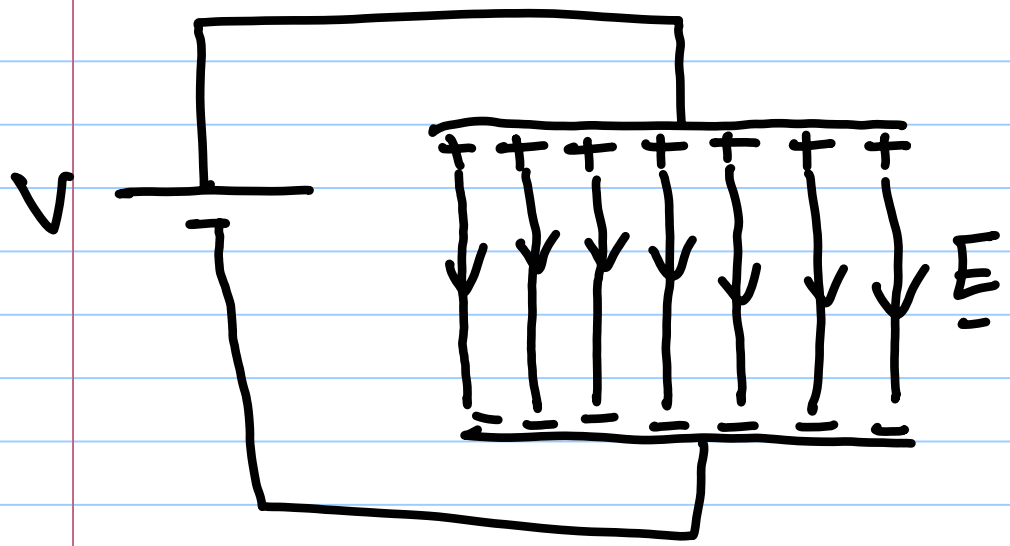
Parallel-plate Capacitor



Cylindrical plate

Capacitor (Cont'd)

* The Capacitor stores energy in the electric field inside the dielectric



The higher the voltage of the Capacitor, the higher the stored charges

Charge and Voltage

* For a Capacitor we have

$$Q = CV$$

* The Constant C is the Capacitance of the Capacitor in Farad (C/V)
* practical Capacitance Valves range between few pico farads to thousands of microfarads

Example

A capacitor has an accumulated charge of 600 μC with 5V across it. What is the value of its capacitance?

Voltage and Current

* Current by definition is the time derivative of the charge

$$\Rightarrow i_C = \frac{dQ}{dt} = \frac{d}{dt}(Cv) = C\frac{dv}{dt}$$

* The Capacitor current is proportional to the rate of change of the voltage

Voltage and Current (cont'd)

* $i = C\frac{dv}{dt} \Rightarrow i(t) = C\frac{dv(t)}{dt}$

$$\Rightarrow dv(t) = \frac{1}{C} i(t)\,dt$$

$$\int_{-\infty}^{t} dv(\tau) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \rightarrow Q(t)$$

$$v(-\infty) = 0\,V$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

Voltage and Current (cont'd)

$$v(t) = \frac{1}{C}\left[\int_{-\infty}^{t_0} i(\tau)\,d\tau + \int_{t_0}^{t} i(\tau)\,d\tau\right]$$

$$\int_{-\infty}^{t_0} i(\tau)\,d\tau = Q(t_0)$$

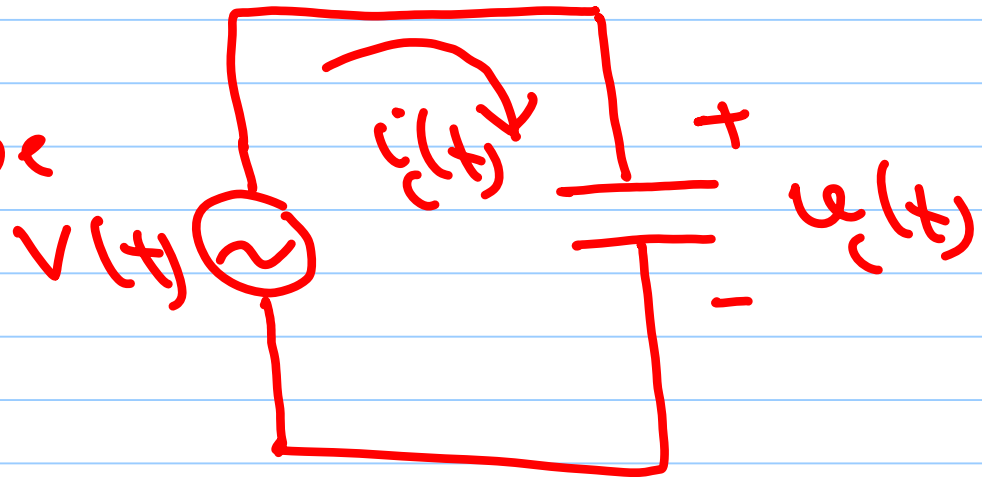
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$$

$$v(t) = v(t_0) + \frac{Q(t_0 \to t)}{C}$$

Charge accumulated from t_0 to t

Example

If the voltage across the shown 50mF capacitor is $v(t) = 50\sin(200t)$, find $i(t)$



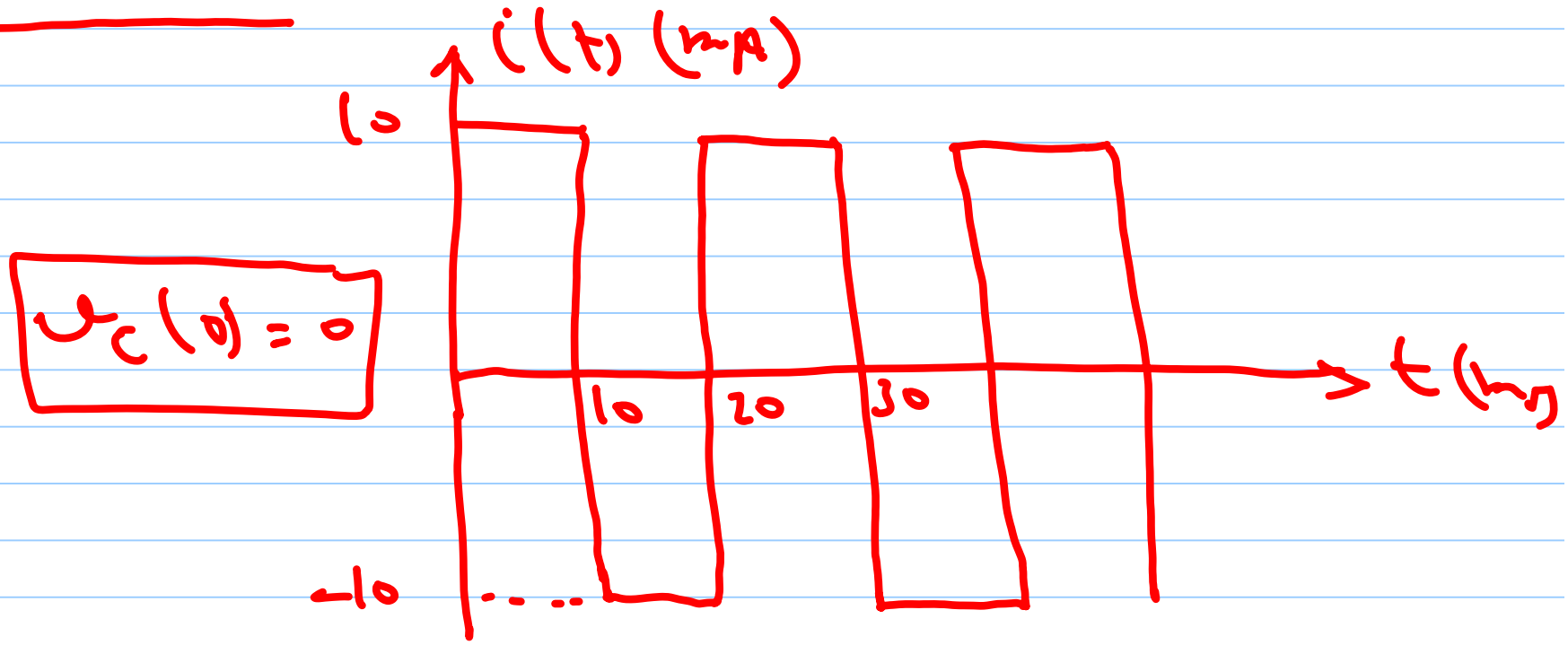
Example

A voltage pulse given by

$$v(t) = \begin{cases} 0\text{V} & t \leq 0 \\ 2t & 0 \leq t \leq 2 \\ 4e^{-(t-2)} & t \geq 2 \end{cases}$$

is applied to a $10\,\mu\text{F}$ capacitor, find the current in the capacitor

Example



$v_c(0) = 0$

Determine the voltage waveform across a 50mF capacitor for the shown current.

Power and Energy

* $p(t) = v(t)\, i(t) = v(t) \times C \frac{dv(t)}{dt}$

$$W = \int_{-\infty}^{t} p(\tau)\, d\tau = C \int_{-\infty}^{t} v(\tau) \frac{dv(\tau)}{d\tau} d\tau$$

$$W = C \int_{-\infty}^{t} v(\tau)\, dv(\tau) = \frac{C}{2} v^2(\tau) \Big|_{-\infty}^{t}$$

$$\boxed{W(t) = \frac{1}{2} C v_c^2(t)}$$

energy stored in Capacitor

Example

* If $C = 50\mu f$ and $v_c(t) = 100 \sin 2\omega t$ V, Determine the energy stored in the capacitor as a function of time