Dr. Mohamed Bakr, EE2CI5, 2007

Note Title 11/8/2007

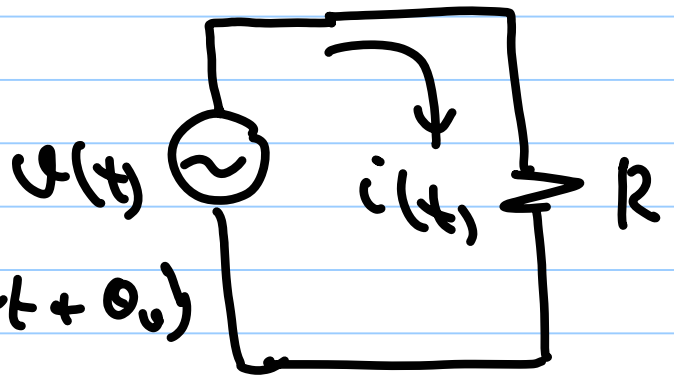
Lecture 23

From Section 8.4 and 8.5

Solve E8.5-E8.10, 8.8, 8.11, 8.13, 8.15, 8.18

Phasor Relation for Resistors

$$v(t) = A_v \cos(\omega t + \theta_v)$$



$$i(t) = \frac{v(t)}{R} = \frac{A_v}{R}\cos(\omega t + \theta_v)$$

$$i(t) = A_i \cos(\omega t + \theta_i)$$

$$\tilde{V} = A_v e^{j\theta_v} \quad , \quad \tilde{I} = A_i e^{j\theta_i}$$

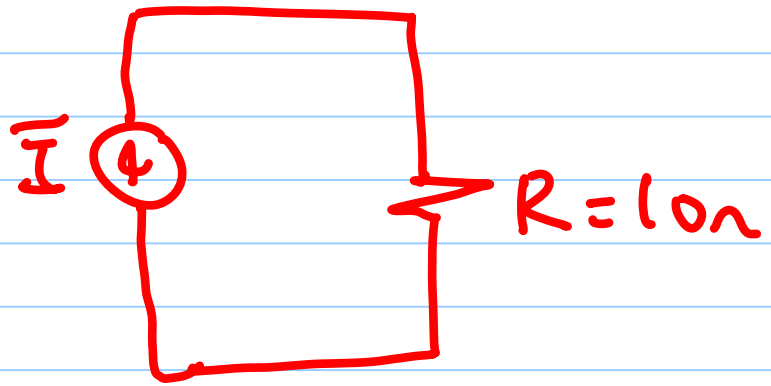
$$\theta_i = \theta_v \quad , \quad A_i = \frac{A_v}{R}$$

$$\boxed{\tilde{V} = R\tilde{I}}$$

In phase

Example

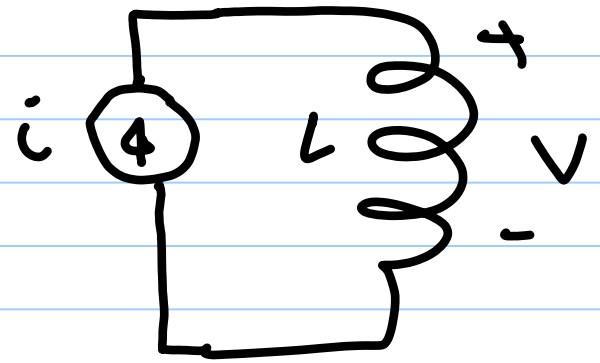
If $\bar{I} = 3\angle 30°$,
$f = 60\,Hz$, find,
$\bar{V}$, $i(t)$, and $v(t)$



Phasor Relation for Inductors

$i(t) = i_o \cos(\omega t + \theta_o)$

$\tilde{I} = i_o \angle \theta_o = i_o e^{j\theta_o}$



$v(t) = L \frac{di}{dt}$

$= L i_o * -\omega \sin(\omega t + \theta_o)$

$v(t) = \omega L i_o \cos(\omega t + \theta_o + \pi/2)$

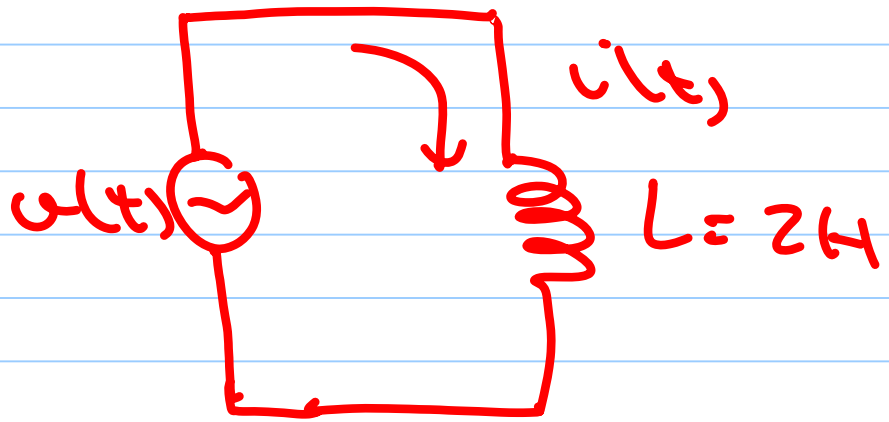
$\tilde{V} = \omega L i_o e^{j(\theta_o + \pi/2)} = j\omega L i_o e^{j\theta_o}$

$\tilde{V} = j\omega L \tilde{I} \rightarrow$ Voltage Leads

Example

If $\bar{V} = 10\angle 60^\circ$,

$f = 50\,Hz$



find $\bar{I}$, $v(t)$,

and $i(t)$. plot $v(t)$, $i(t)$, $\bar{V}$ and

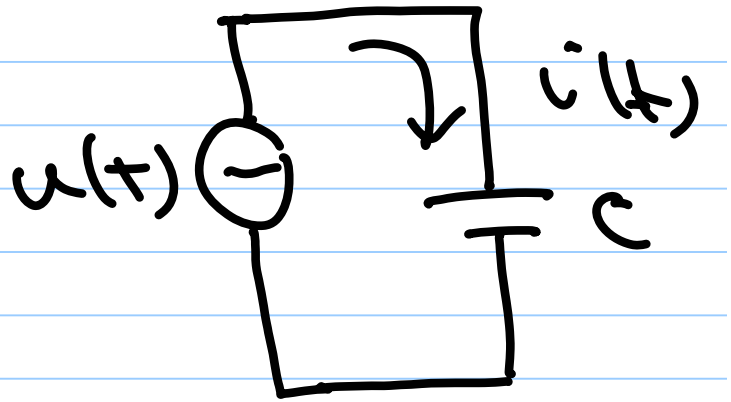
$\bar{I}$.

Phasor Relation for Capacitors

$$v(t) = A_v \cos(\omega t + \theta_v)$$

$$\tilde{V} = A_v \angle \theta_v = A_v e^{j\theta_v}$$

$$i(t) = C \frac{dv(t)}{dt}$$



$$i(t) = A_v * (-\omega \sin(\omega t + \theta_v))$$

$$i(t) = A_v \omega C \cos(\omega t + \theta_v + \pi/2)$$

$$\tilde{I} = \omega C A_v e^{j(\theta_v + \pi/2)} = j\omega C A_v e^{j\theta_v}$$

$$\tilde{I} = j\omega C \tilde{V} \Rightarrow \tilde{V} = \frac{1}{j\omega C}\tilde{I} \rightarrow \text{Current leads}$$

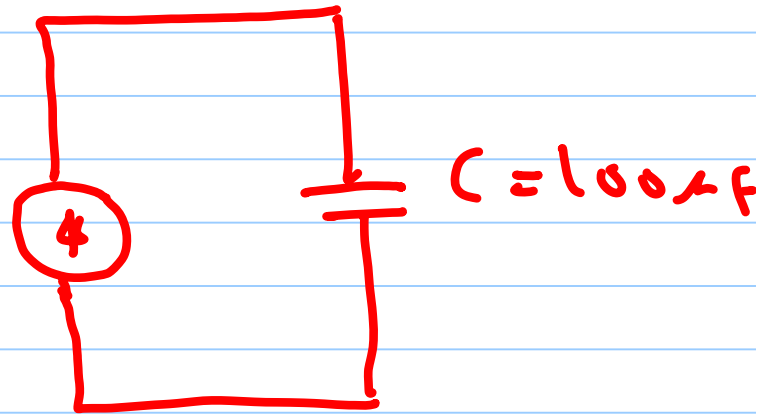
Example

If $\tilde{I} = 0.1 \angle 120°$,

and $\omega = 10\text{kHz}$,

find $\tilde{V}$, $i(t)$, and

$v(t)$. plot $i(t)$, $v(t)$, $\tilde{I}$, and $\tilde{V}$.



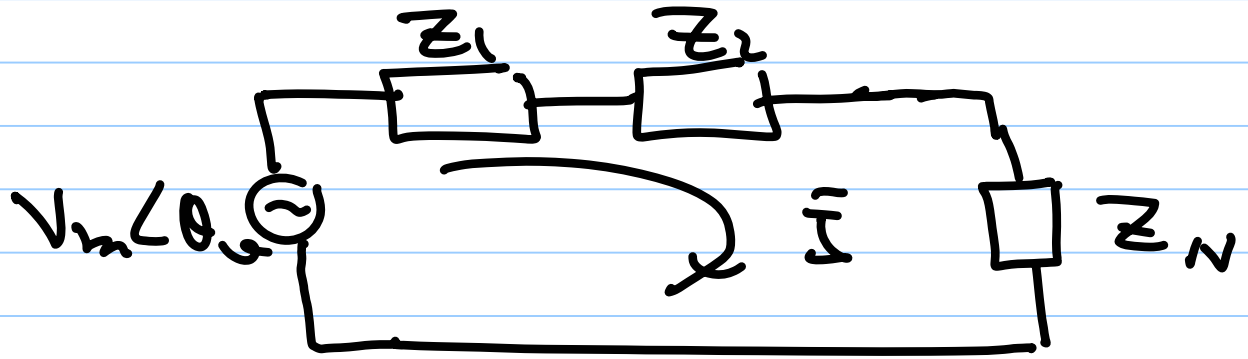
Impedance

* Resistor $\Rightarrow \tilde{V} = R\tilde{I}$, $Z_R = R\angle 0$
* Inductor $\Rightarrow \tilde{V} = j\omega L\tilde{I}$, $Z_L = j\omega L$

  $Z_L = \omega L \angle 90^\circ$
* Capacitor $\Rightarrow \tilde{V} = \frac{1}{j\omega C}\tilde{I}$, $Z_C = \frac{1}{j\omega C}$

  $Z_C = \frac{1}{\omega C}\angle -90^\circ$
* Impedances are added in the same way we add resistors

Impedance (Cont'd)



$$Z = Z_1 + Z_2 + \cdots + Z_N$$

$$Z = R(\omega) + jX(\omega) \quad \Omega$$

$R(\omega)$: resistive part of Impedance

$X(\omega)$: reactive part of Impedance

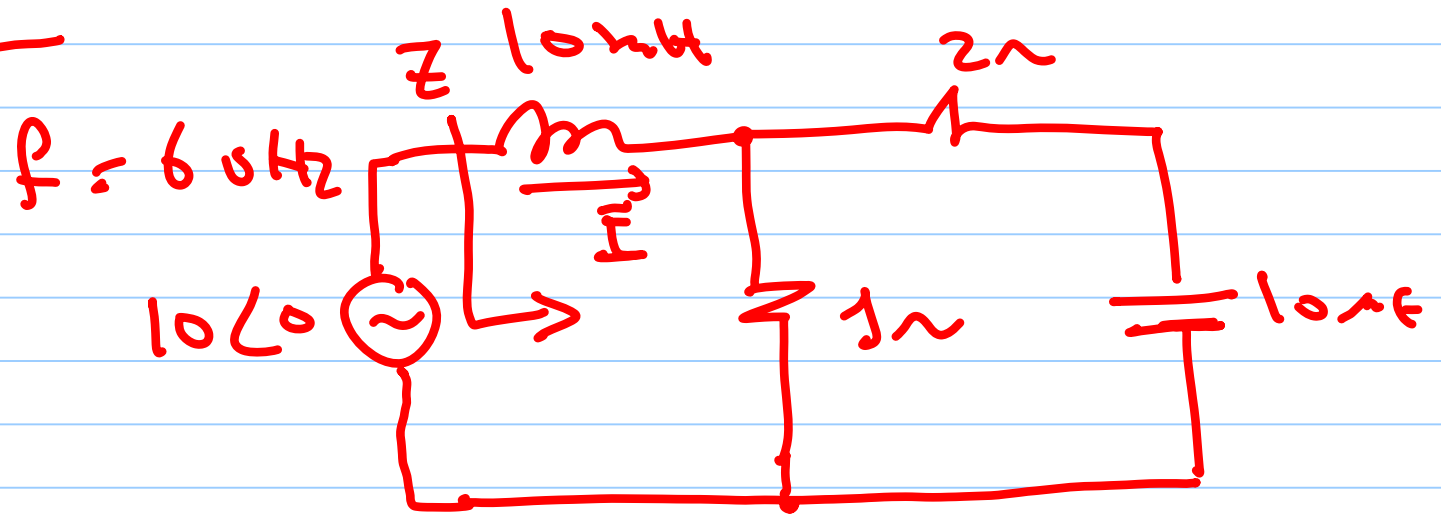
Impedance (cont'd)

$$Z = |Z| \angle \theta_z$$

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{V_m \angle \theta_v}{|Z| \angle \theta_z} = \frac{V_m}{|Z|} \angle \theta_v - \theta_z$$

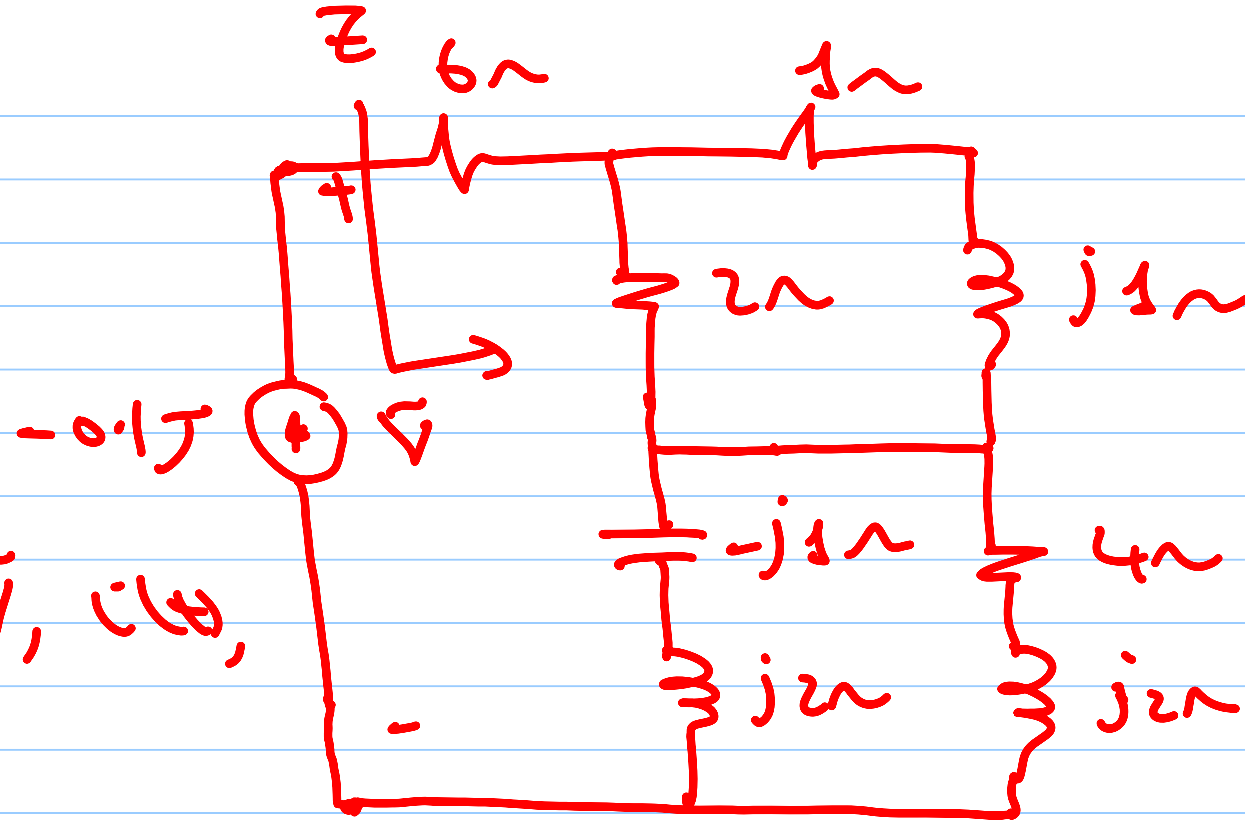
* Impedance changes magnitude and phase of voltage to give the current phasor

Example



Find the impedance Z seen by the source. Find also $\tilde{I}$ and i(t)

Example



find Z, $\tilde{V}$, $i(t)$,

and $v(t)$