# Lecture #3

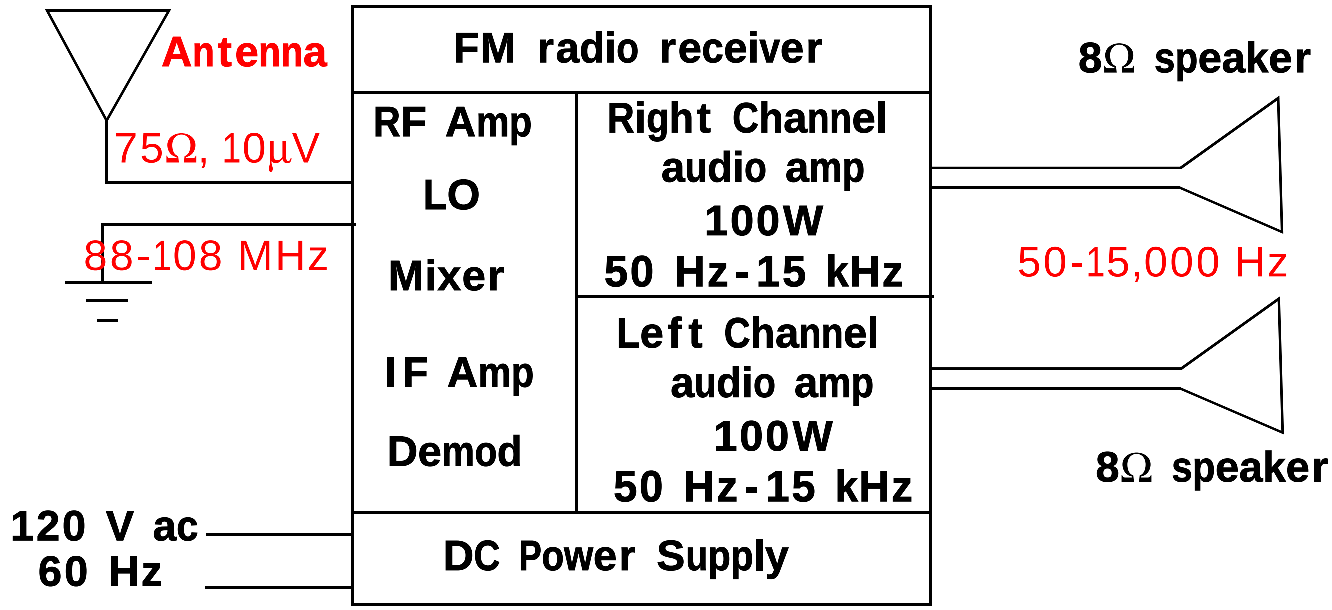
**From Chapter 10 of Jaeger, Section 6.5.1 of Spencer**

**10.11, 10.12, 10.20, 10.21, 10.27, 10.28, 10.40, 10.48, 10.57, 10.58, 10.59**

## Outline/Learning Objectives:

- Describe the concept of linear amplification (and nonlinear distortion).
- Define the quantities used to measure the performance of analog amplifiers, including voltage gain, current gain, power gain, input and output resistances.
- Express the voltage, current, and power gain in terms of the decibel, or dB.
- Describe amplifier biasing for linear operation.
- Model linear amplifiers using simple two-port representations (g-parameters, h-parameters, y-parameters, and z-parameters).
- Define unilateral two-port representations of linear amplifiers.
- Describe mismatched conditions at the input and output ports of an amplifier and the concepts of ideal voltage and current amplifiers.
- Use the electronics laboratory to investigate the electrical behaviour of simple circuits and devices.
- From Chapter 10 in Jaeger

# Example of Analog Electronic System




**FM Stereo Receiver**

**RF = 0.5 - 50 MHz; VHF = 50 - 150 MHz; UHF = 150 - 1000 MHz**

## Table 1: FM Stereo Receiver

| Linear Circuit Function | Non-linear Circuit Function |
|---|---|
| RF amplification | |
| Audio frequency amplification | |
| Frequency selection (tuning) | DC power supply (rectification) |
| Impedance matching (75Ω input) | Frequency conversion (mixing) |
| Tailoring AF response | Detection/demodulation |
| Local oscillator | |

Large V, I and P gains required in going from VHF signal received from antenna to 100W audio signal delivered to speaker.

Receiver input matched to 75Ω impedance of coaxial transmission line coming from antenna.

Receiver requires circuits with high frequency selectivity at input.

LO used to tune receiver. VHF signal mixed down to IF.

Audio information separated from RF carrier by demodulation.

# Sinusoids

1. Many natural phenomena are sinusoidal - vibration of a guitar string, current in oscillating circuits etc.

2. Sinusoids are important in generation and transmission of electric power and in communication of intelligence. Sinusoidal source always produces sinusoids in linear circuits (derivatives and integrals are sinusoids themselves).

3. Periodic waves can be represented by a series of sinusoids using Fourier analysis.

$a = A \cdot \cos(\omega t + \theta)$; a = instantaneous value.

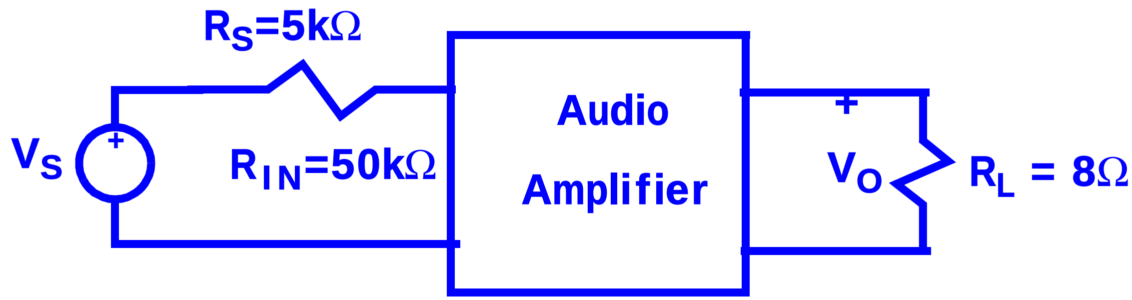
A = maximum value or amplitude; t = time, seconds.

$\omega$ = angular frequency, radians/second; $\theta$ = phase angle in radians.

$f = \frac{\omega}{2\pi}$ = frequency in cycles/second or hertz. $T = \frac{1}{f}$ = period, secs.

# Amplification

$v_O = V_O \sin(\omega_s t + \theta)$; $v_S = V_S \sin(\omega_s t)$, $V_S = 1\text{mV}$




Audio amplifier channel from FM receiver

Amplifier output power is $P_O = \left(\frac{V_O}{\sqrt{2}}\right)^2 \frac{1}{R_L}$, $V_{O,\,RMS} = \left(\frac{V_O}{\sqrt{2}}\right)$.

Output Voltage $V_O = \sqrt{2 P_O R_L}$. $P_O = 100\text{W}$, $R_L = 8\Omega \Rightarrow V_O = 40\text{V}$

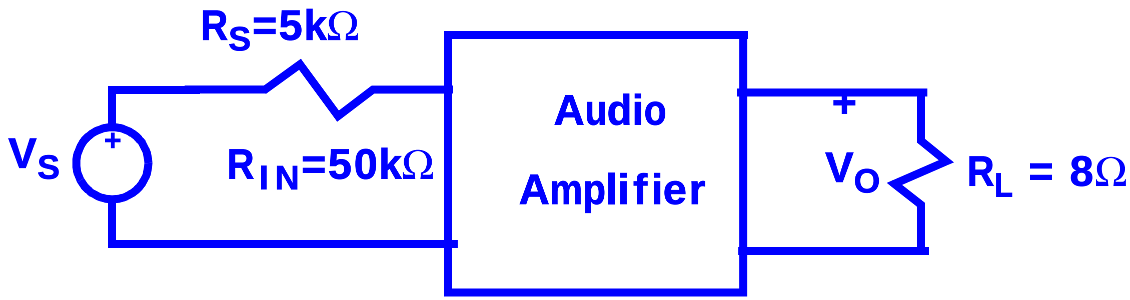
Output Current is $i_O = I_O \bullet \sin(\omega_s t + \theta)$ => $I_O = 5\text{A}$

Voltage gain $A_V = \frac{v_O}{v_S} = \frac{V_O\angle\theta}{V_S\angle 0} = \frac{V_O}{V_S}\angle\theta = |A_V| \bullet \angle A_V$. $|A_V| = 4\text{x}10^4$

Current gain

# Amplification

$v_O = V_O \sin(\omega_s t + \theta)$; $v_S = V_S \sin(\omega_s t)$, $V_S = 1mV$




Audio amplifier channel from FM receiver

$$I_S = \frac{1mV}{55k\Omega} = 18.18nA$$

Amplifier output power is $P_O = \left(\frac{V_O}{\sqrt{2}}\right)^2 \frac{1}{R_L}$, $V_{O,\,RMS} = \left(\frac{V_O}{\sqrt{2}}\right)$.

Output Voltage $V_O = \sqrt{2 P_O R_L}$. $P_O$ = 100W, $R_L$ = 8Ω => $V_O$ = 40V

Output Current is $i_O = I_O \bullet \sin(\omega_s t + \theta)$ => $I_O$ = 5A

Voltage gain $A_V = \frac{v_O}{v_S} = \frac{V_O \angle\theta}{V_S \angle 0} = \frac{V_O}{V_S} \angle\theta = |A_V| \bullet \angle A_V$. $|A_V| = 4\text{x}10^4$

Current gain $A_I = \frac{i_O}{i_S} = \frac{I_O \angle\theta}{I_S \angle 0} = \frac{I_O}{I_S} \angle\theta = |A_I| \bullet \angle A_I$. $|A_I| = 2.75\text{x}10^8$

Power gain $A_P = \frac{P_O}{P_S} = \frac{\left(\frac{V_O}{\sqrt{2}} \bullet \frac{I_O}{\sqrt{2}}\right)\angle\theta}{\left(\frac{V_S}{\sqrt{2}} \bullet \frac{I_S}{\sqrt{2}}\right)} = \frac{V_O}{V_S} \bullet \frac{I_O}{I_S} = |A_V| \bullet |A_I|$

$$A_P = |A_V| \bullet |A_I| = 4x10^4 \bullet 2.75x10^8 = 1.1x10^{13}$$

Decibel Scale $A_{P,\,dB} = 10 \bullet \log A_P$ or $A_{V,\,dB} = 20 \bullet \log|A_V|$

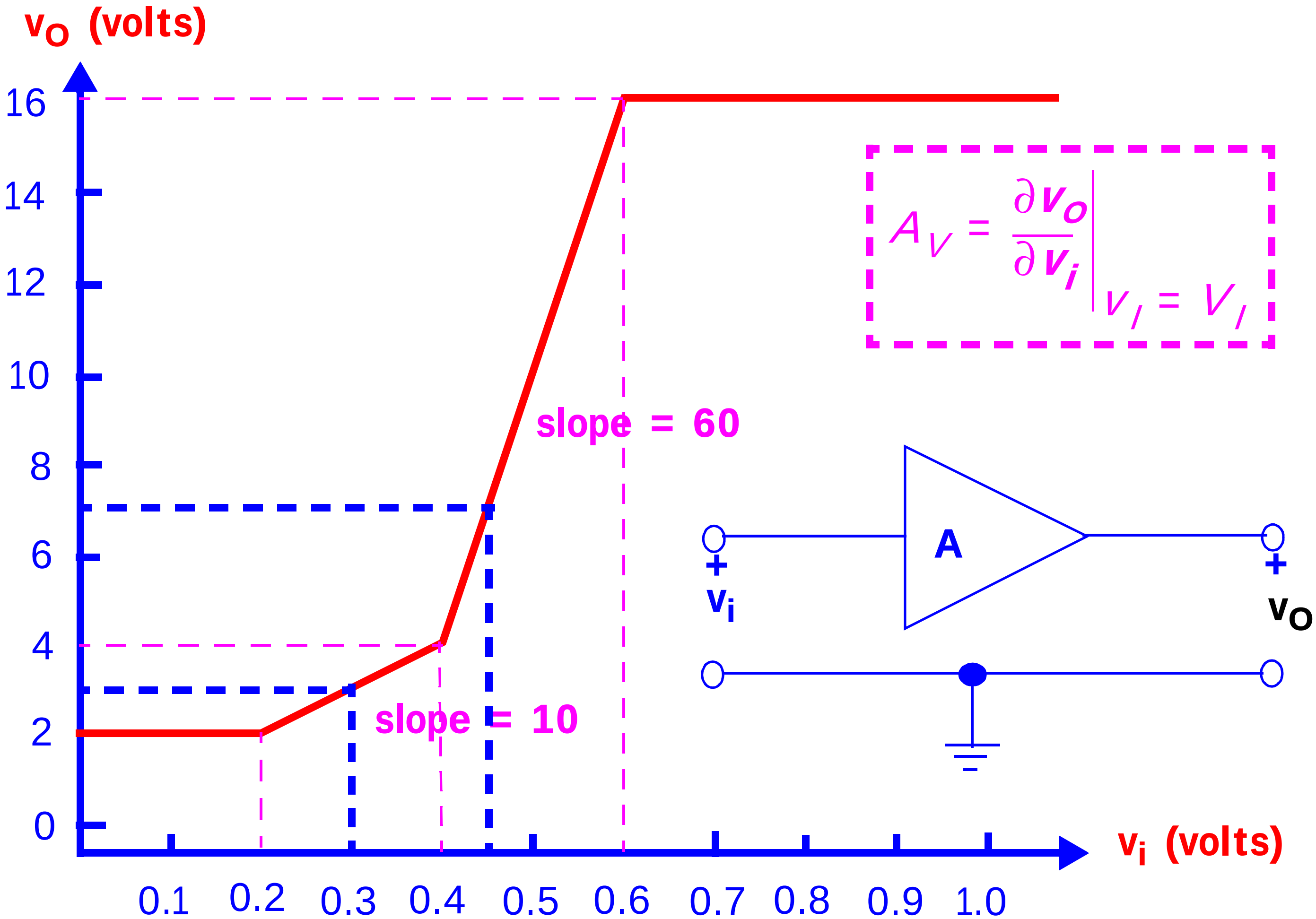
Power - Decibel Scale $A_{P,\,dB} = 10 \bullet \log 1.1 \times 10^{13} = 130.4dB$

Voltage - Decibel Scale $A_{V,\,dB} = 20 \bullet \log|A_V| = 20 \bullet \log 4 \times 10^4 = 92dB$

Current - Decibel Scale $A_{I,\,dB} = 20 \bullet \log|A_I| = 20 \bullet \log 2.75x10^8 = 168.8dB$

# Amplifier Biasing for Linear Operation

Non-inverting amplifier since Input and Output are in phase.




$$A_V = \left.\frac{\partial v_O}{\partial v_i}\right|_{v_I = V_I}$$

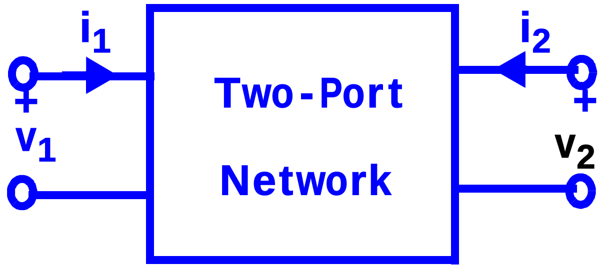
Explain how to set up the bias point and limits on values of input voltage. For example, at bias $V_I = 0.45$ V => $v_i \leq |0.05\,V|$.

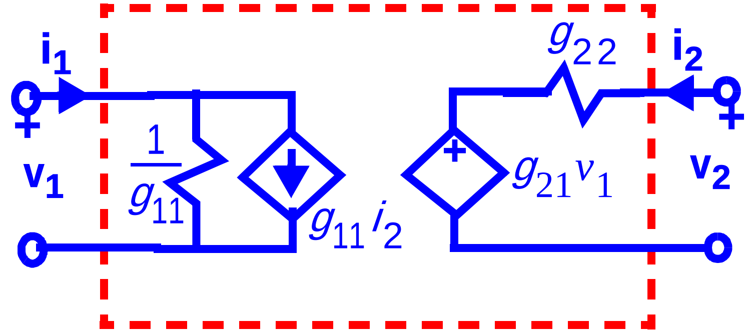
# Two-Port Models for Amplifiers

For each representation, explain how to get the $\mathbf{x}_{ij}$ parameters where **x** is **g**, **h**, **y** or **z.**

Also explain what each of the four components of **x** mean.

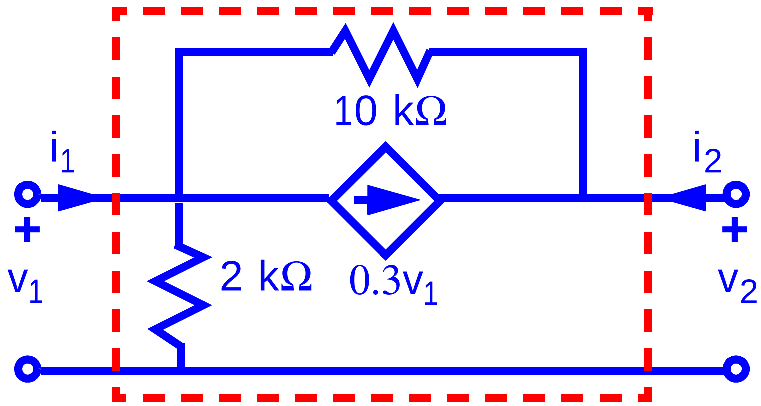
## The g-parameters




$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \bullet \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

Example: Calculate the g-parameters for the following circuit.




$$g_{11} = \left.\frac{i_1}{v_1}\right|_{i_2 = 0} = \frac{1}{2k\Omega} = 0.500mS$$. Since $i_2$ =0, all $i_1$ flows into 2kΩ.

$$g_{12} = \left.\frac{i_1}{i_2}\right|_{v_1 = 0} = -1$$. Since $v_1$=0, $i_1$ = $i_2$. Have a short since $v_1$=0 in // with 2kΩ.

$$g_{21} = \left.\frac{v_2}{v_1}\right|_{i_2 = 0} \quad \therefore v_2 = v_1 + 0.3v_1(10k\Omega) \rightarrow g_{21} = 3001$$. Since $i_2$ =0, 0.3$v_1$ --> 10kΩ.

$$g_{22} = \left.\frac{v_2}{i_2}\right|_{v_1 = 0} = 10k\Omega$$. Since $v_1$=0, have a short // with 2kΩ and 0.3$v_1$ =0.

# Fourier Series (should know from Engineering Math):

For a periodic function f(t) that is

(1) piecewise continuous,

(2) has isolated maxima and minima, and

(3) is absolutely integrable over a period, that is $\int_0^T |f(t)| dt < \infty$,

then

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \bullet \cos\{n\omega_o t\} + b_n \bullet \sin\{n\omega_o t\})$$ with

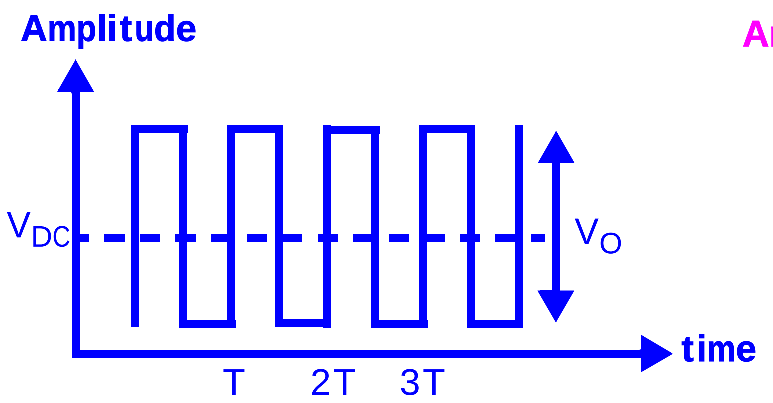
$$a_n = \frac{2}{T}\int_{t_o}^{T+t_o} f(t) \cdot \cos\{n\omega_o t\} dt \ , \ b_n = \frac{2}{T}\int_{t_o}^{T+t_o} f(t) \cdot \sin\{n\omega_o t\} dt \ .$$

## Frequency Spectrum of Electronic Signals

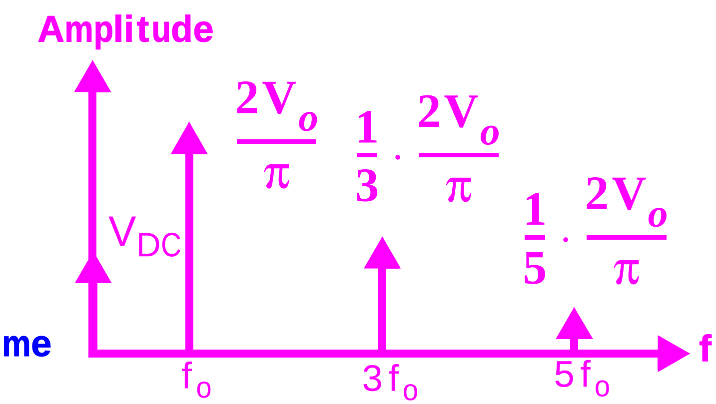
Fourier Series, Fourier Analysis - give an example

**Fourier Theory** - complex signals are actually composed of a continuum of sinusoids, each having a distinct amplitude, frequency and phase.

**Frequency spectrum** - amplitude and phase of the components of the signal versus frequency




**Periodic Signal**




**Amplitude Spectrum**

$$v(t) = V_{DC} + \frac{2V_O}{\pi}\left(\sin\omega_o t + \frac{1}{3}\sin 3\omega_o t + \frac{1}{5}\sin 5\omega_o t + \cdot \quad \cdot \quad \cdot\right)$$

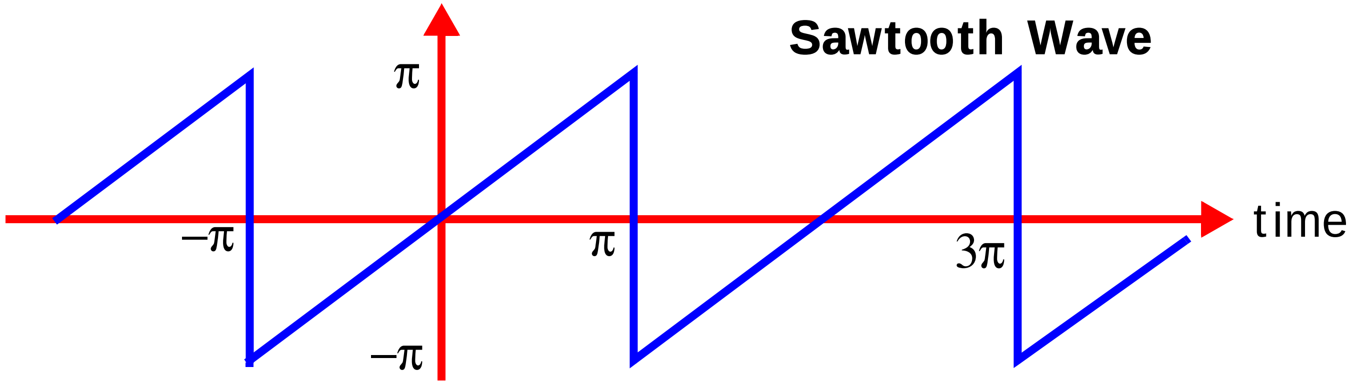
$\omega_o = \frac{2\pi}{T}$ radians/second - fundamental radian frequency

$f_o = \frac{1}{T}$ Hertz - fundamental signal frequency

$2f_o, 3f_o, 4f_o, \ , \ ,$ are 2nd, 3rd, 4th harmonic frequencies

Example: Find the Fourier series for the sawtooth wave

$f(t) = t$ for $-\pi < t < \pi$ and $f(t + 2\pi) = f(t)$




Now, the period $T = 2\pi$, so $\omega_o = 2\pi / T$. Choose $t_o = -\pi$

For n=0, we get $a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} t dt = 0$. Case n=0 had to be considered separately because of appearance of $n^2$ in denominator in the general case. Also, $a_0/2$ is the average value of value of the sawtooth wave, then by inspection, we see that $a_0 = 0$.

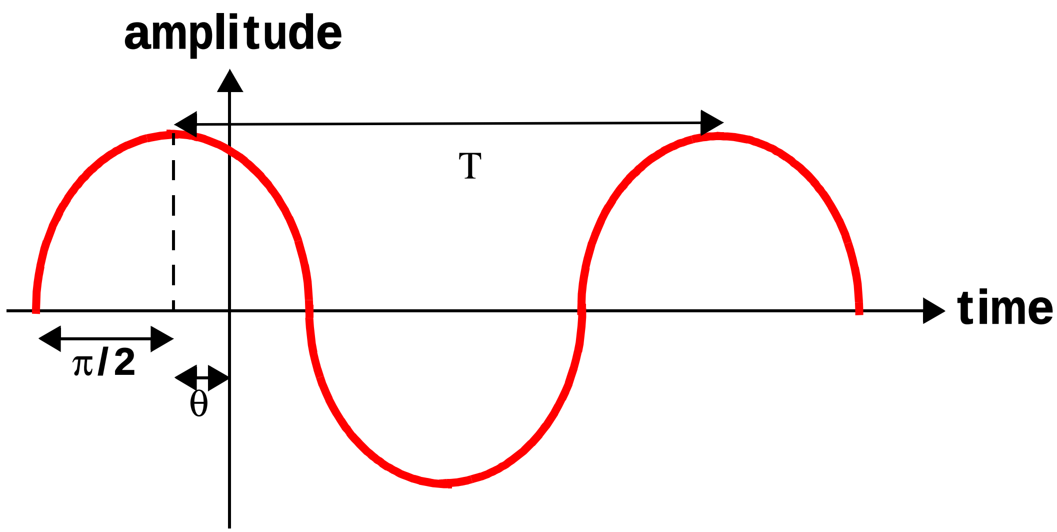
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t \cdot \cos\{nt\} dt = \frac{1}{n^2\pi}\{\cos(nt) + nt \cdot \sin(nt)\}\Big|_{-\pi}^{\pi} = \mathbf{0}.$$ n=1,2,3,...

$$_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t \cdot \sin\{nt\} dt = \frac{1}{n^2\pi}\{\sin(nt) + nt \cdot \cos(nt)\}\Big|_{-\pi}^{\pi}.$$ n=1,2,3,...

$$b_n = -\frac{2\cos n\pi}{n} = \frac{2(-1)^{n+1}}{n}$$

$$\boxed{f(t) = 2\left(\sin t - \frac{\sin 2t}{2} + \frac{\sin 3t}{3} + \ldots\right)}$$

Periodic Signal




From Fourier analysis, can represent any periodic complex signal as

$$v_s = \sum_i V_i \bullet \sin(\omega_i t + \phi_i)$$

Note that this expression is the same as that on page 19 that is composed of a sum of sines and cosines.

$$v_s = \sum_{i=1}^{\infty} (V_i \bullet \sin\phi_i \bullet \cos\{i\omega_o t\} + V_i \bullet \cos\phi_i \bullet \sin\{i\omega_o t\})$$

IP signal $v_s = V_S \bullet \sin(\omega_s t)$; OP signal $v_O = V_O \bullet \sin(\omega_s t + \theta)$