Lecture 19: Introduction to Small Signal Analysis
From Chapter 13 in Jaeger, Chapter 8 in Spencer

Outline/objective

Understanding the essence of SS modeling
Developing SS model of a diode
Illustration of utilizing SS model of a diode through an example

Suggested problems: 13.16, 13.43, 13.66, 13.101, 13.102

Some Notations

V_D dc voltage across the diode

I_D dc current through the diode where
$I_D = I_s(\exp(V_D/V_T)-1)$

$v_d(t)$ small signal voltage across the diode

$i_d(t)$ small signal current through the diode

v_D total voltage across the diode where
$v_D = V_D + v_d(t)$

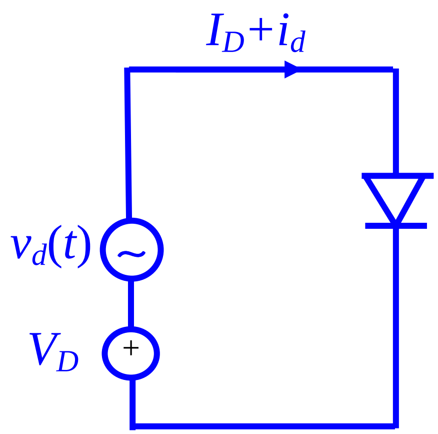
i_D total current through the diode where
$i_D = I_D + i_d(t)$

the total voltage and total current are related by

$I_D + i_d(t) = I_s(\exp((V_D + v_d(t))/V_T) - 1)$

our target is to relate $i_d(t)$ and $v_d(t)$ through a linear model

Derivation of SS model for the diode



$I_D + i_d(t) = I_s(\exp((V_D + v_d(t))/V_T) - 1)$

$I_D + i_d(t) = I_s(\exp(V_D/V_T)\exp(v_d(t)/V_T) - 1)$

for very small values $v_d \ll V_T$ we have

$I_D + i_d(t) = I_s(\exp(V_D/V_T)[1+(v_d/V_T)]-1)$

it follows that we have

$i_d(t) = I_s \exp(V_D/V_T)(v_d/V_T)$

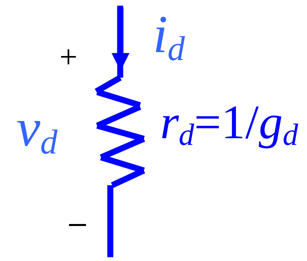
but as $I_s \exp(V_D/V_T) = I_D + I_s$ we get the final result

$i_d(t) = ((I_D + I_s)/V_T) v_d(t) = g_d v_d(t)$

small signal variations in the current are related to small signal variations of the voltage through the small signal conductance

SS Model of the diode

SS model of a diode is a resistor whose value depends on the dc bias conditions

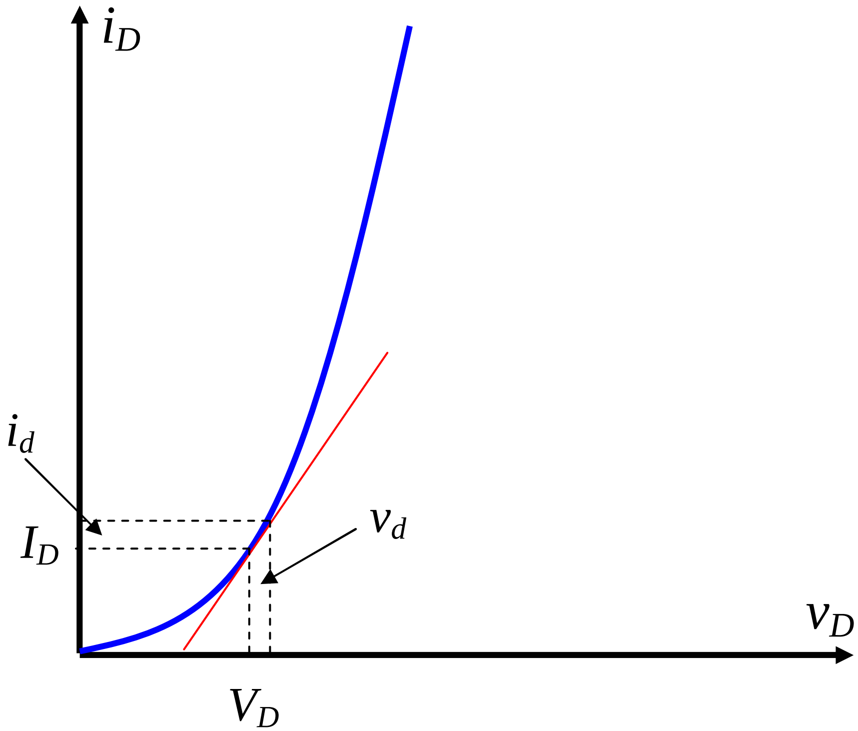


Superposition allows us to separate the dc analysis and SS analysis

Two separate analyses may be carried out: a dc analysis to obtain the operating point and a SS analysis whose parameters are determined from the dc analysis

Remember that the total current and voltages are the sum of the dc and SS values

Graphical Interpretation of the SS model of the Diode



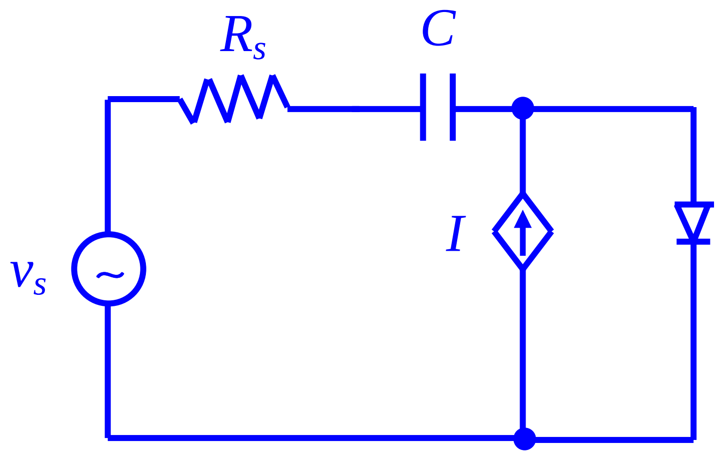
$$g_d = \frac{\partial i_D}{\partial v_D}\bigg)_{Q-point} = \frac{\partial}{\partial v_d}\left\{ I_s \left[\exp\left(\frac{v_D}{v_T}\right) - 1 \right] \right\}\bigg|_{Q-Point}$$

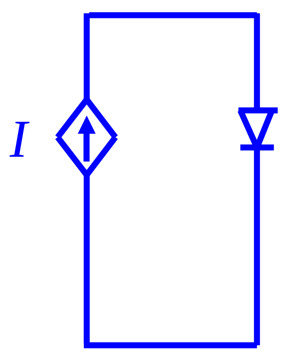
$$g_d = \frac{I_s}{V_T} \exp\left(\frac{V_D}{V_T}\right) = \frac{I_D + I_s}{V_T}$$

General Procedure for SS analysis of any device

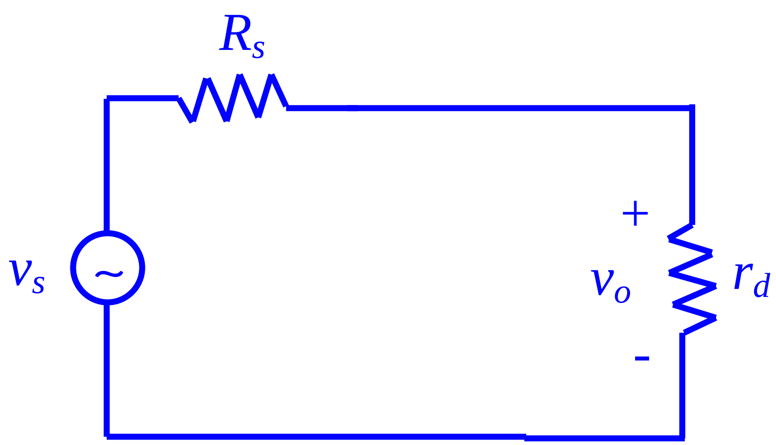
1. Carry out a dc analysis of the circuit. Determine the operating point of the device
2. Determine the values of the parameters of the small signal model
3. Replace the device by its small signal model to determine the small signal gain, input resistance, output resistance, etc.

The Current-Controlled Attenuator





dc



SS

Using dc anaylsis $I_D = I \Rightarrow g_d = (I_D + I_s)/V_T = 1/r_d$

$v_o = v_s\,(r_d/(r_d + R_s))$

for R_s=1.0KΩ and I_s=1.0e-5 A we have

I	r_d	v_o
0	25×10^{12} Ω	v_s
1 μA	25KΩ	0.96v_s
10 μA	2.5KΩ	0.71 v_s
100 μA	250 Ω	0.20 v_s
1.0 mA	25 Ω	0.024 v_s