Lecture #29

Small-Signal Modeling and Linear Amplification - Part 2: BJTs

Outline/Learning Objectives:

- Describe the BJT as an amplifier.
- Define and describe the small-signal-model of the BJT.
- Analyze BJT amplifiers using a two-part process:
- dc Analysis: (1) construct the dc equivalent circuit; (2) solve for the Q-point.
- ac Analysis: (3) construct the ac equivalent circuit; (4) replace the BJT by its small-signal model; (5) solve the ac circuit.
- Analyze the BJT common-emitter (C-E) amplifier.
- Use the electronics laboratory to investigate the electrical behavior of simple circuits and devices.
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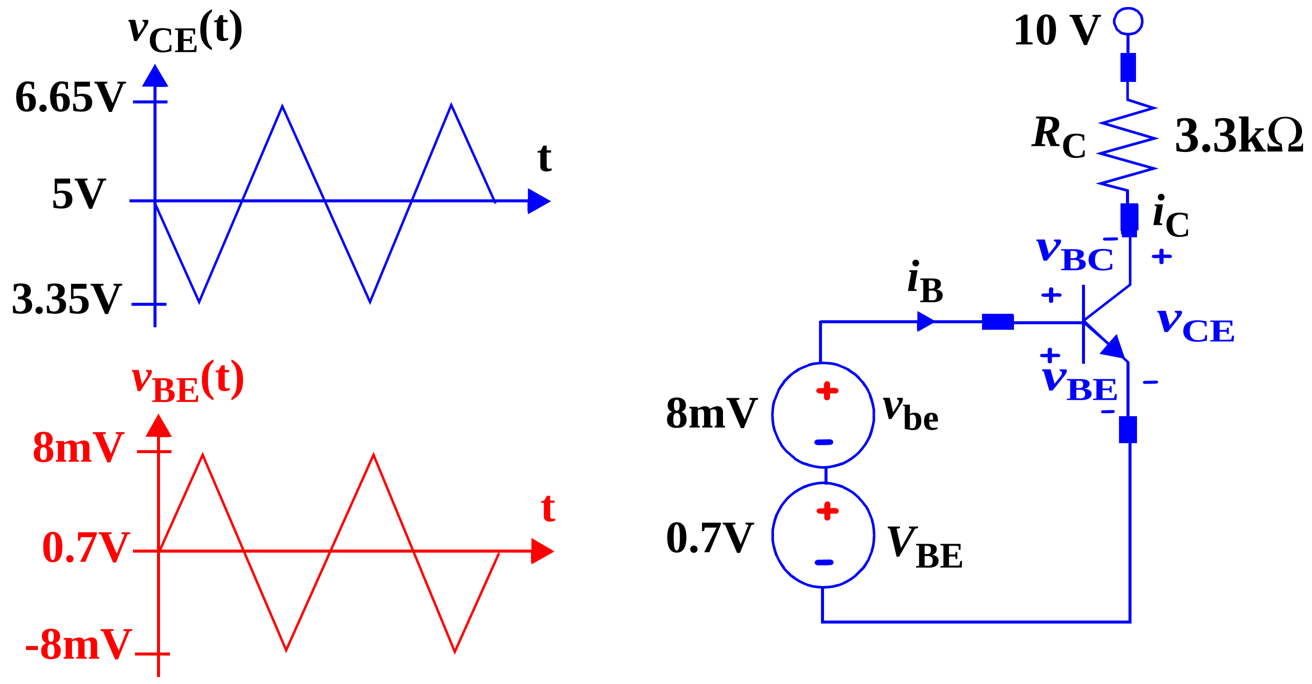
Transistor as an Amplifier

- Small signal parameters of bipolar junction transistor (BJT)
- Voltage gain (A_V), input resistance (R_{IN}), output resistance (R_{OUT})
- Maximum input and output signal amplitudes

The Bipolar Junction Transistor Amplifier

$v_{BE} = V_{BE} + v_{be}$ and $10 = v_{CE} + i_C R_C$ (Load-Line Equation)

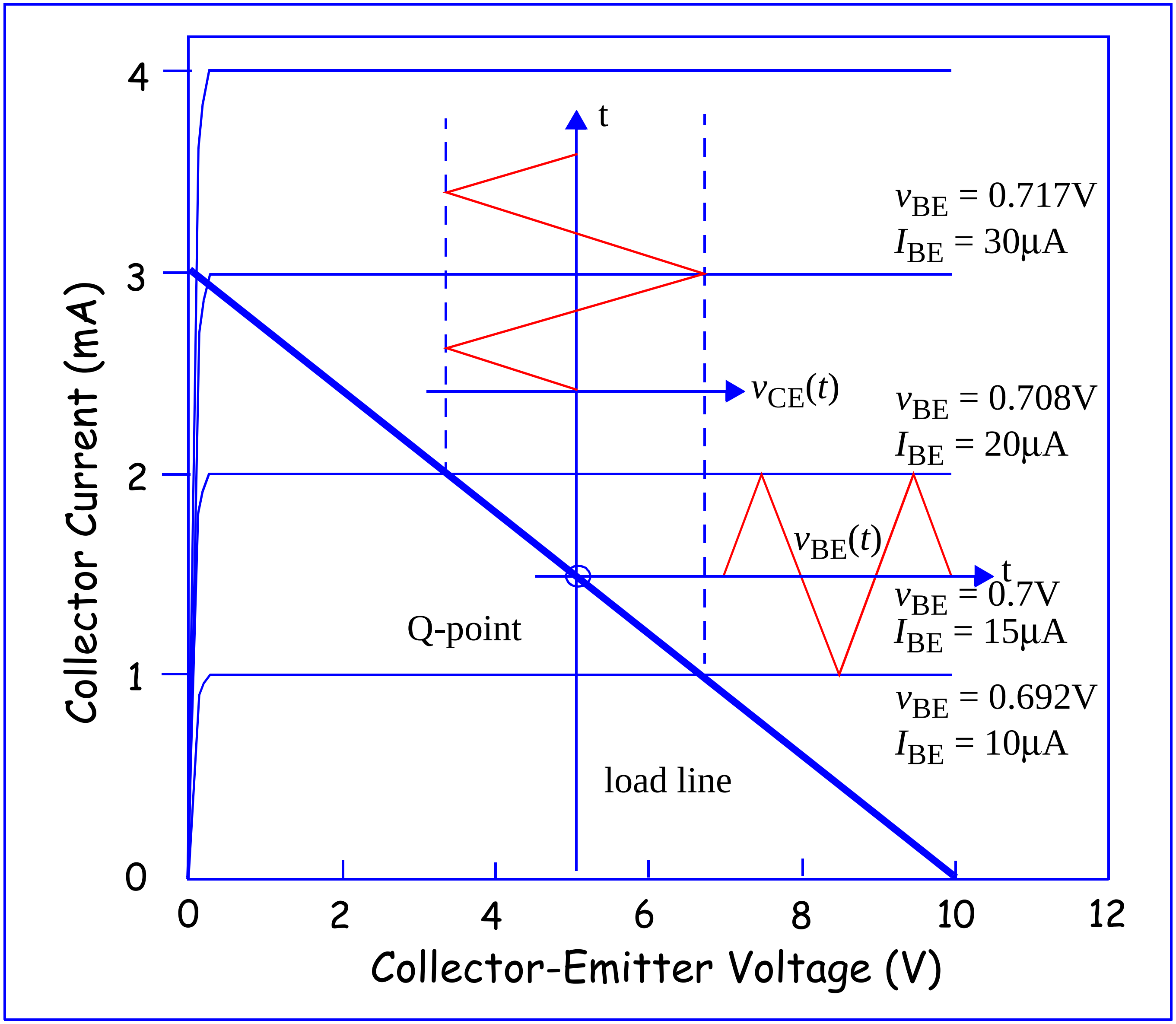


BJT Common-Emitter Amplifier

$i_B = 15\mu A$, $i_C = 1.5mA$ since $\beta = 100$

Q-point is $i_B = 15\mu A$; $V_{BE} = 0.7V$; $i_C = 1.5mA$; $V_{CE} = 5V$

$$A_V = \frac{v_{CE}}{v_{BE}} = \frac{1.65\angle 180}{0.008\angle 0} = 206\angle 180 = -206$$



4
3
2
1
0
0
2
4
6
8
10
12
Collector Current (mA)
Collector-Emitter Voltage (V)
t
$v_{CE}(t)$
$v_{BE}(t)$
t
v_{BE} = 0.717V
I_{BE} = 30μA
v_{BE} = 0.708V
I_{BE} = 20μA
v_{BE} = 0.7V
I_{BE} = 15μA
v_{BE} = 0.692V
I_{BE} = 10μA
Q-point
load line

Small-Signal Modeling

$$\mathbf{i_1} = y_{11}\mathbf{v_1} + y_{12}\mathbf{v_2};\ \mathbf{i_2} = y_{21}\mathbf{v_1} + y_{22}\mathbf{v_2}$$

$$\mathbf{i_b} = y_{11}\mathbf{v_{be}} + y_{12}\mathbf{v_{ce}};\ \mathbf{i_c} = y_{21}\mathbf{v_{be}} + y_{22}\mathbf{v_{ce}}$$

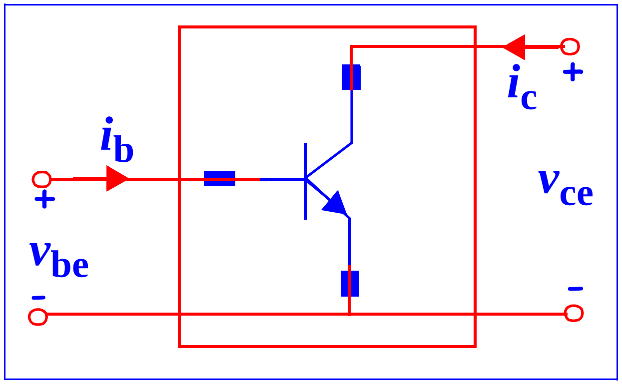
$v_{BE} = V_{BE} + v_{be}$ and $v_{CE} = V_{CE} + v_{ce}$

$i_B = I_B + i_b$ and $i_C = I_C + i_c$.

$v_{be} = \Delta v_{BE} = v_{BE} - V_{BE}$ and

$v_{ce} = \Delta v_{CE} = v_{CE} - V_{CE}$.

$i_b = \Delta I_B = i_B - I_B$ and $i_c = \Delta I_C = i_C - I_C$.



$$y_{11} = \left.\frac{\mathbf{i_b}}{\mathbf{v_{be}}}\right|_{\mathbf{v_{ce}} = 0} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{Q-P} ; \quad y_{12} = \left.\frac{\mathbf{i_b}}{\mathbf{v_{ce}}}\right|_{\mathbf{v_{ce}} = 0} = \left.\frac{\partial i_B}{\partial v_{CE}}\right|_{Q-P}$$

$$y_{21} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{be}}}\right|_{\mathbf{v_{be}} = 0} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q-P} ; \quad y_{22} = \left.\frac{\mathbf{i_c}}{\mathbf{v_{ce}}}\right|_{\mathbf{v_{be}} = 0} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{Q-P}$$

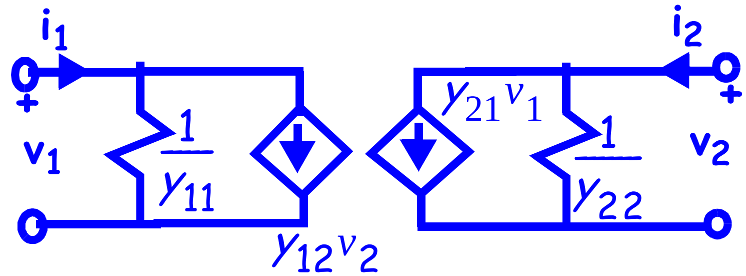
$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)\left[1 + \frac{v_{CE}}{V_A}\right] ; \quad \beta_F = \beta_{F0}\left[1 + \frac{v_{CE}}{V_A}\right] ; \quad i_B = \frac{I_S}{\beta_{F0}} \exp\left(\frac{v_{BE}}{V_T}\right) .$$

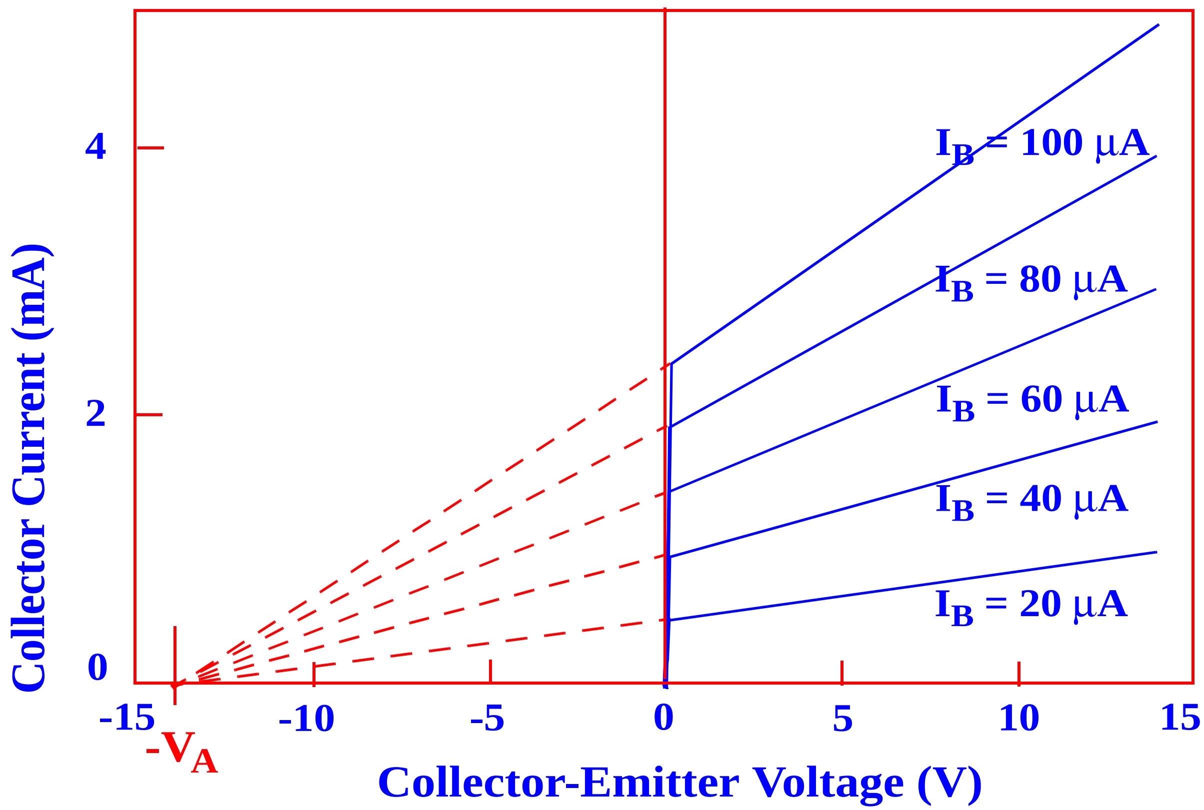
β_{F0} is the value of β_F extrapolated to $v_{CE} = 0$.

Things To Remember

Y parameters have equivalent circuit representation

The current gain is actually a function of both the BE and BC voltages





$$y_{12} = \left.\frac{\partial i_B}{\partial v_{CE}}\right|_{Q-P} = \mathbf{0}$$

$$y_{21} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{Q-P} = \frac{I_S}{V_T}\exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] = \frac{\boldsymbol{I_C}}{\boldsymbol{V_T}} = \boldsymbol{g_m}$$

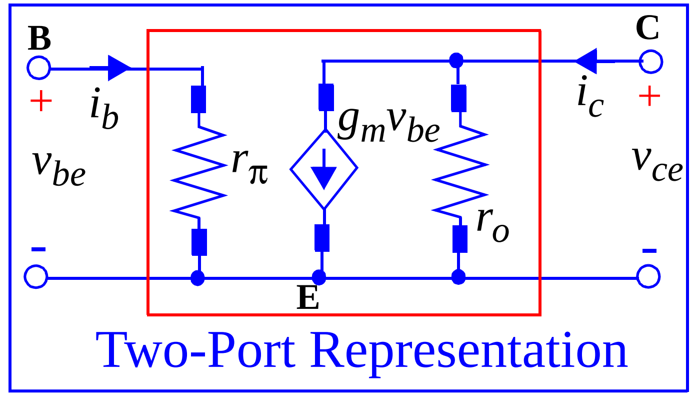
$$y_{22} = \left.\frac{\partial i_C}{\partial v_{CE}}\right|_{Q-P} = \frac{I_S}{V_A}\exp\left(\frac{V_{BE}}{V_T}\right) = \frac{\boldsymbol{I_C}}{\boldsymbol{V_A + V_{CE}}} = \frac{\mathbf{1}}{\boldsymbol{r_o}}$$

$$y_{11} = \left.\frac{\partial i_B}{\partial v_{BE}}\right|_{Q-P} = \frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}} - \frac{i_C}{\beta_F^2}\frac{\partial \beta_F}{\partial v_{BE}} = \frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}} - \frac{i_C}{\beta_F^2}\frac{\partial \beta_F}{\partial i_C}\frac{\partial i_C}{\partial v_{BE}}$$

$$y_{11} = \frac{1}{\beta_F}\frac{\partial i_C}{\partial v_{BE}}\left(1 - \frac{i_C}{\beta_F}\frac{\partial \beta_F}{\partial i_C}\right) = \frac{\boldsymbol{I_C}}{\boldsymbol{\beta_F V_T}}\left(\mathbf{1} - \frac{\boldsymbol{i_C}}{\boldsymbol{\beta_F}}\frac{\boldsymbol{\partial \beta_F}}{\boldsymbol{\partial i_C}}\right) = \frac{\boldsymbol{I_C}}{\boldsymbol{\beta_0 V_T}} = \frac{\mathbf{1}}{\boldsymbol{r_\pi}}.$$

Here, $\beta_0 = \frac{\partial i_C}{\partial i_B}$ is the small-signal common-emitter current gain.

The Hybrid-pi small-signal model of the BJT is as shown.



For $\beta_0 = 100$, $V_A = 75V$, $V_T = 25mV$ and, $V_{CE} = 10V$, we get

I_C	g_m	r_π	r_o	μ_f
1 μA	4x10^{-5} S	2.5 MΩ	85 MΩ	3400
10 μA	4x10^{-4} S	250 kΩ	8.5 MΩ	3400
100 μA	0.004 S	25 kΩ	850 kΩ	3400
1 mA	0.04 S	2.5 kΩ	85 kΩ	3400
10 mA	0.4 S	250 Ω	8.5 kΩ	3400

Summary: $y_{11} = I_C/(\beta_0 V_T) = 1/r_\pi$ and $y_{12} = 0$

$y_{21} = I_C/V_T = g_m$ and $y_{22} = I_C/(V_A + V_{CE}) = 1/r_o$

Small-Signal Current Gain

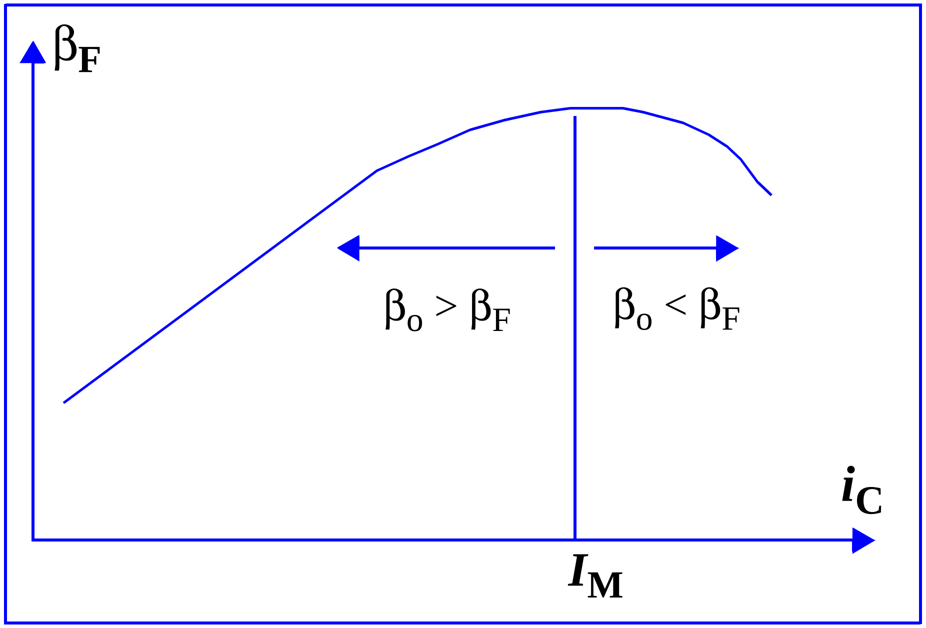
$$\beta_0 = g_m r_\pi$$

$$\frac{\partial \beta_F}{\partial i_C} > 0 \quad \text{for } i_C < I_M$$

$$\frac{\partial \beta_F}{\partial i_C} < 0 \quad \text{for } i_C > I_M$$

where

$$\beta_0 = \frac{\beta_F}{1 - I_C \left(\frac{1}{\beta_F} \frac{\partial \beta_F}{\partial i_C} \right) \Bigg|_{Q-P}}$$



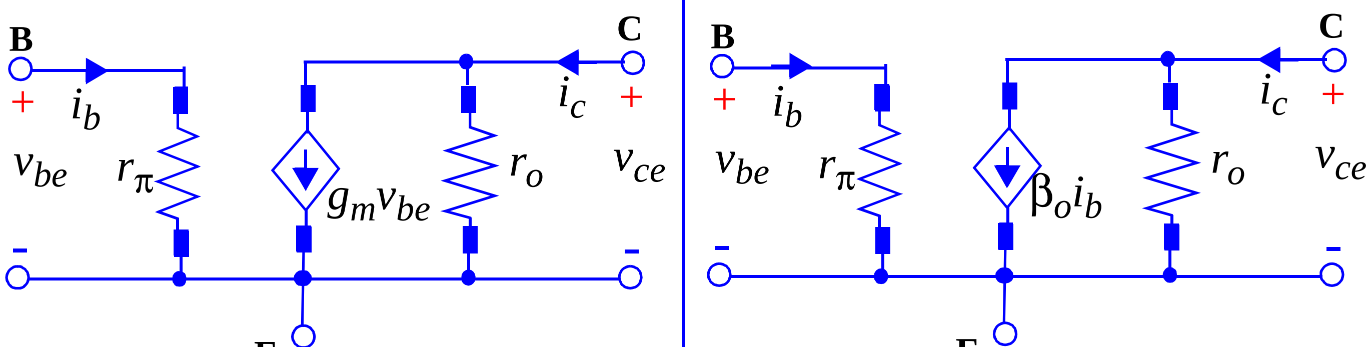
Amplification Factor (p. 600)

$$\mu_f = g_m r_o = \frac{I_C}{V_T} \cdot \frac{V_A + V_{CE}}{I_C} = \frac{V_A + V_{CE}}{V_T} \approx \frac{V_A}{V_T}$$

This is the maximum voltage gain an individual transistor can provide.

Equivalent Forms of Small-Signal Model

$\mathbf{v_{be} = i_b r_\pi}$, $g_m \mathbf{v_{be}} = g_m r_\pi \mathbf{i_b} = \beta_o \mathbf{i_b}$ and $\mathbf{i_c} = \beta_o \mathbf{i_b} + \dfrac{\mathbf{v_{ce}}}{r_o} \approx \beta_o \mathbf{i_b}$



Voltage-controlled current source (VCCS) model

Current-controlled current source (CCCS) model

Definition of Small-Signal for the BJT (p. 602)

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_T}\right).$$

$$I_C + i_c = \left\{ I_S \exp\left(\frac{V_{BE}}{V_T}\right) \right\} \exp\left(\frac{v_{be}}{V_T}\right) = I_C \exp\left(\frac{v_{be}}{V_T}\right).$$

Using $i_C = I_C\left[1 + \frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots\right]$ and $i_c = i_C - I_C$, we get

from $i_c = i_C - I_C$ that $$i_c = I_C\left[\frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots\right].$$

Now linearity requires that $\frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 \ll \frac{v_{be}}{V_T}$ or $v_{be} \ll 2V_T$.

Using a factor of 10 to satisfy the inequality, we get, $|v_{be}| \le 5mV$.

Now, we can write that $$i_C \approx I_C\left[1 + \frac{v_{be}}{V_T}\right] = I_C + g_m v_{be}.$$

$$\frac{i_c}{I_C} = \frac{i_C - I_C}{I_C} = \frac{g_m v_{be}}{I_C} = \frac{v_{be}}{V_T} \le \frac{5mV}{25mV} = 0.20.$$

A 5 mV change in v_{BE} corresponds to a 20% deviation in i_C from its Q-point as well as a 20% change in i_E since $i_E \sim i_C$.

Summary of BJT Small-Signal Parameters

Parameter	BJT
Transconductance g_m	I_C / V_T
Input Resistance R_{IN}	$r_\pi = \beta_o / g_m = \beta_o V_T / I_C$
Output Resistance R_{OUT}	$(V_A + V_{CE}) / I_C \approx V_A / I_C$
Amplification Factor μ_f	$g_m r_o = (V_A + V_{CE}) / V_T$
Small-Signal Requirement	$v_{be} \leq 5mV$

Small-Signal Model for the pnp BJT

This is identical to that of the npn transistor.

Remember that the dc currents in a pnp transistor flow in an opposite direction to that in a npn transistor.