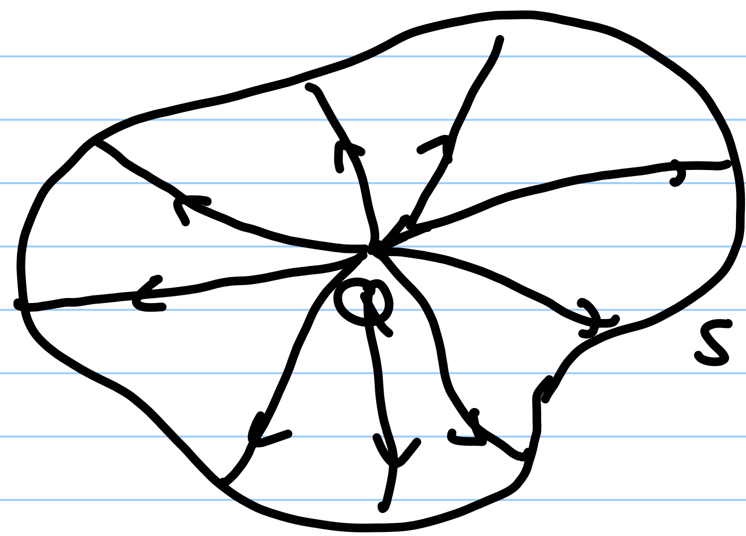
Lecture 2

From sections 4.5 - 4.8

Solve 4.16, 4.18, 4.21, 4.22, 4.25, 4.28, 4.31, 4.33, 4.35, 4.40, 4.41.

Solve All practical exercises

Gauss Law for Electrostatics



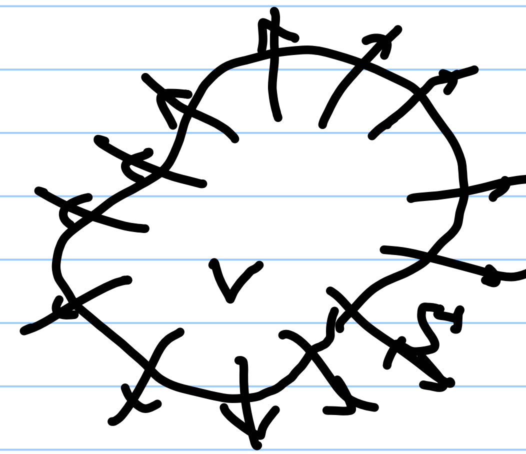
$$\oiint_S \underline{D} \cdot \underline{ds} = Q$$

The total electric flux out of a closed surface is equal to the charge enclosed by that surface

Differential form

*Using Divergence theorem

$$\oint_S \underline{D} \cdot d\underline{s} = \iiint_V (\nabla \cdot \underline{D})\, dv,$$

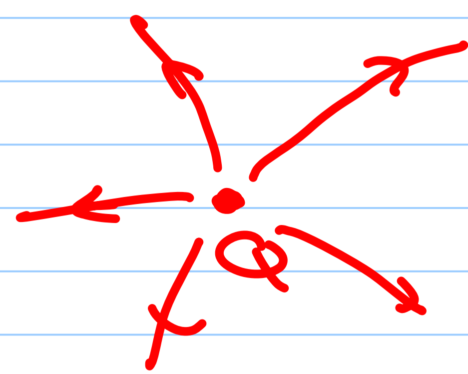


It follows that we have

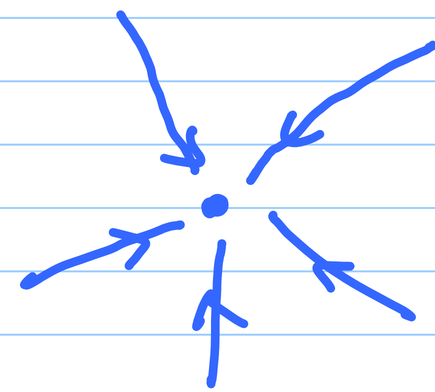
$$\iiint_V (\nabla \cdot \underline{D})\, dv = Q = \iiint_V \rho_v \, dv$$

$$\Rightarrow \boxed{\nabla \cdot \underline{D} = \rho_v}$$

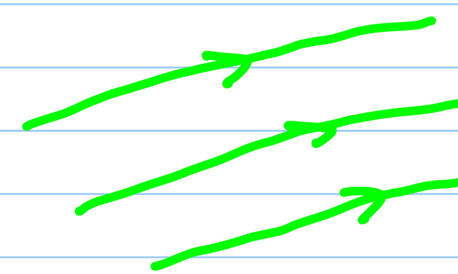
Differential Form (Cont'd)



$(\nabla\cdot\underline{D})_{\underline{r}} = +ve$



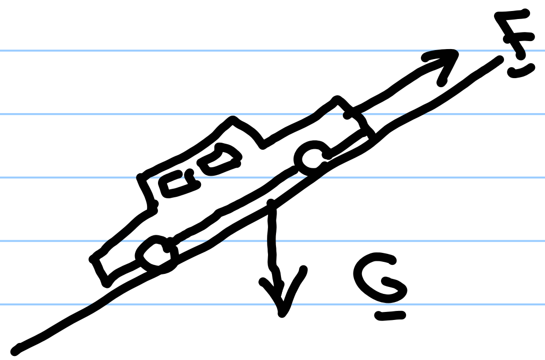
$(\nabla\cdot\underline{D})_{\underline{r}} = -ve$

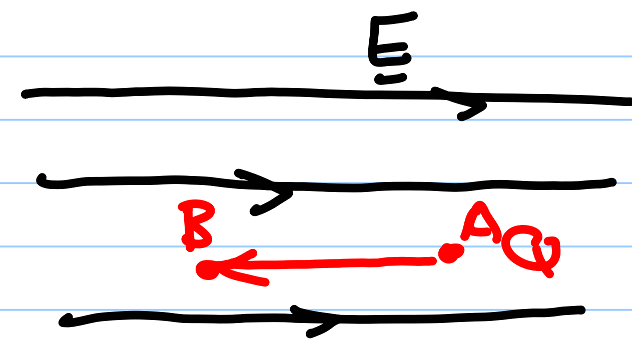


$(\nabla\cdot\underline{D})_{\underline{r}} = 0$

* The divergence of $\underline{D}$ indicates whether there are <u>Free</u> electric charges or not.

Electric Potential





* Moving against field in both cases require exerting work

$$W = -Q\int_A^B \underline{E}\cdot d\underline{l} \quad (\text{Remember } \underline{F} = Q\underline{E})$$

* -ve sign means that the work is exerted when moving against electric field

Electric Potential (cont'd)

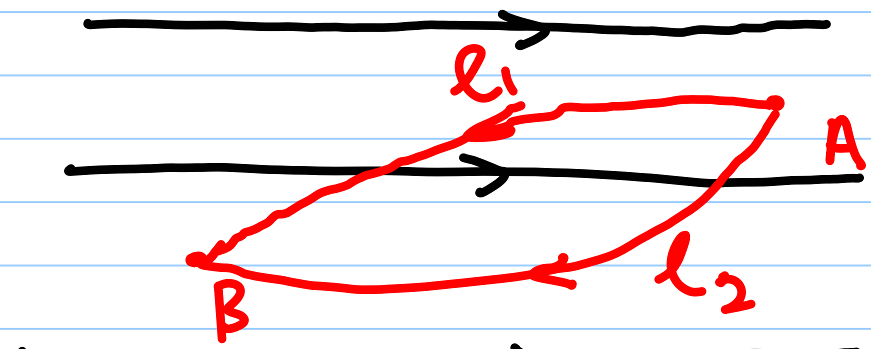
* The potential difference is the work done in moving a unit charge against the electric field

$$V_B - V_A = -\int_A^B \underline{E} \cdot d\underline{l} = V_{BA}$$ (boon ??)

* Only in certain cases, we can define a reference point with zero potential

$$V_B - 0 = V_B \equiv -\int_A^B \underline{E} \cdot \underline{dl}$$

Propertier of Electrostatic Field



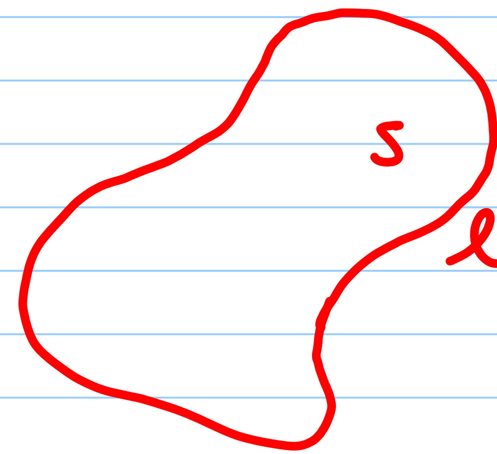
$$V_B - V_A = -\left.\int_A^B \underline{E}\cdot d\underline{l}\right)_{l_1} = -\left.\int_A^B \underline{E}\cdot d\underline{l}\right)_{l_2}$$

$$-\int_{l_1} \underline{E}\cdot d\underline{l} = -\int_{l_2} \underline{E}\cdot d\underline{l} \Rightarrow \oint \underline{E}\cdot d\underline{l} = 0$$

Propertier of Electrostatic Field (Cont'd)

* Using Stokes theorem

$$\oint \underline{E} \cdot \underline{dl} = \iint_S (\nabla \times \underline{E}) \cdot \underline{ds}$$



$$\Rightarrow \iint_S (\nabla \times \underline{E}) \cdot \underline{ds} = 0$$

$$\Rightarrow \nabla \times \underline{E} = \underline{0}$$

* Electrostatic field is Conservative and irrotational

electric field and potential

* $dV = -\int \underline{E} \cdot d\underline{\ell}$

  $\Delta \ell$

  $dV = -E_x\,dx - E_y\,dy - E_z\,dz$

* Using Taylor's expansion we have

  $dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$

* It follows that $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$,

$E_z = -\frac{\partial V}{\partial z} \implies \underline{E} = -\nabla V$