Lecture 3

From sections 5.8, 5.9 and 6.2

Solve 5.24, 5.26, 5.27, 5.30, 5.32, 5.33, 5.34

Self Read

review sections 5.5 - 5.7

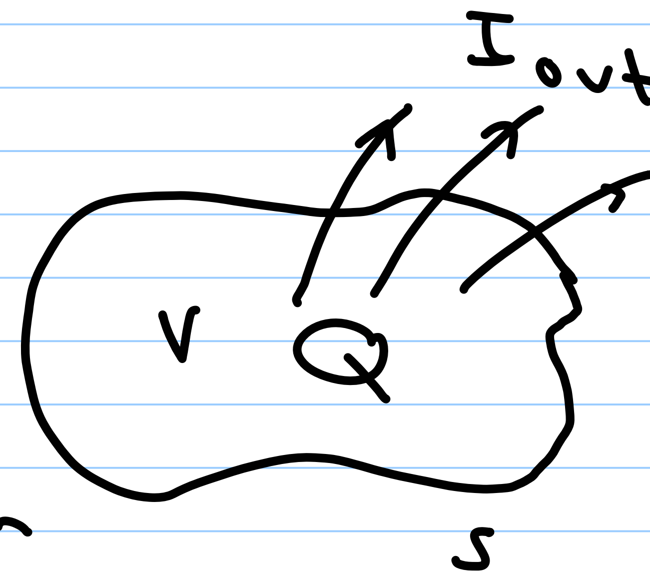
* For air $\underline{D} = \varepsilon_0 \underline{E}$
* for dielectric $\underline{D} = \varepsilon_0 \underline{E} + \underline{P}$
* For a linear dielectric $\underline{P} = \chi_e \varepsilon_0 \underline{E}$

$\Rightarrow$ $\underline{D} = \varepsilon_0 \varepsilon_r \underline{E}$ where

$\varepsilon_r = (1 + \chi_e)$

Continuity Equation

* $I_{out} = \oint\!\!\oint_S \underline{J} \cdot \underline{ds}$



* Flow of current in the outward direction means that Q is decreasing

$$\oint\!\!\oint \underline{J} \cdot \underline{ds} = -\frac{dQ}{dt} = -\frac{d}{dt}\iiint_V \rho_v \, dv$$

Continuity Equation (Cont'd)

* Using Divergence theorem

$$\oiint \underline{J} \cdot d\underline{s} = \iiint_V (\nabla \cdot \underline{J})\, dv = -\frac{\partial}{\partial t} \iiint_V \rho_v \, dv$$

$$\Rightarrow \quad \nabla \cdot \underline{J} = -\frac{\partial \rho_v}{\partial t}$$

* For steady currents $-\frac{\partial \rho_v}{\partial t} = 0$

$\nabla \cdot \underline{J} = 0$ (Kirchhoff's current law)

Relaxation Time

* For a linear medium $\underline{J} = \sigma \underline{E}$

$$\Rightarrow \nabla \cdot (\sigma \underline{E}) = -\frac{\partial \rho_v}{\partial t}$$

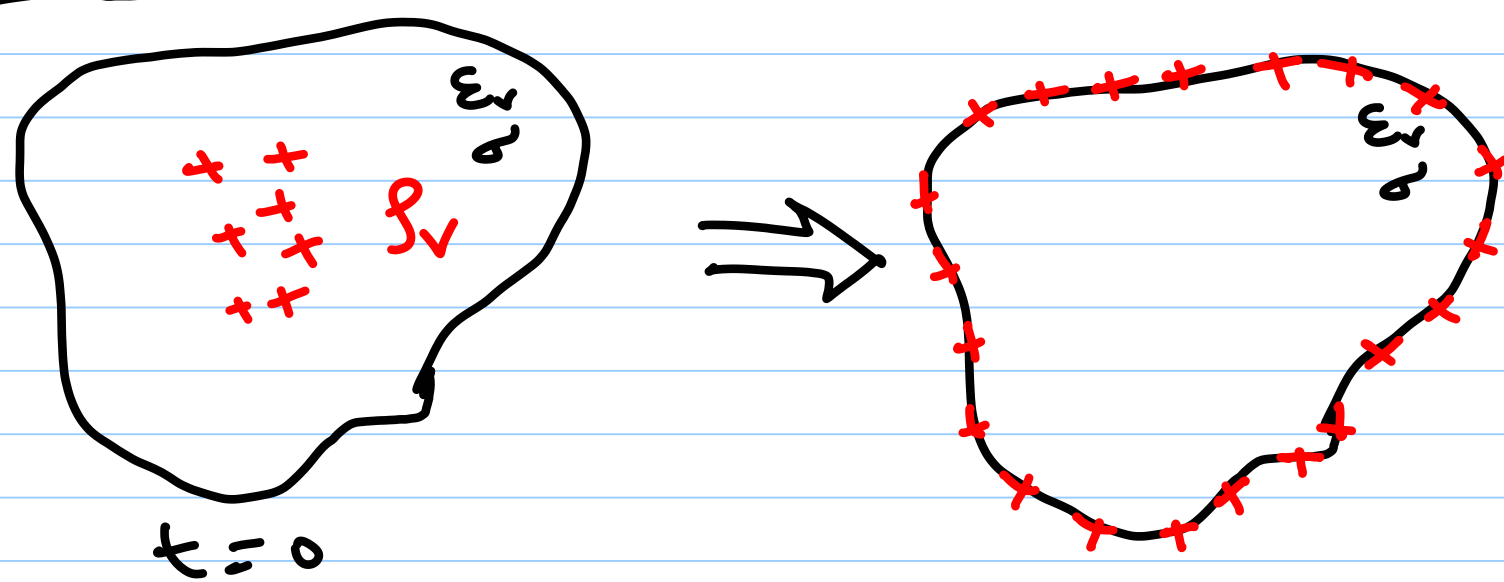
$$\Rightarrow \frac{\sigma}{\varepsilon} \underbrace{(\nabla \cdot \varepsilon \underline{E})}_{\rho_v} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0 \quad \text{(1st order D.E.)}$$

$$\Rightarrow \rho_v = \rho_{v_0} e^{-t/T_r} \quad , \quad T_r = \frac{\varepsilon}{\sigma} =$$

relaxation time

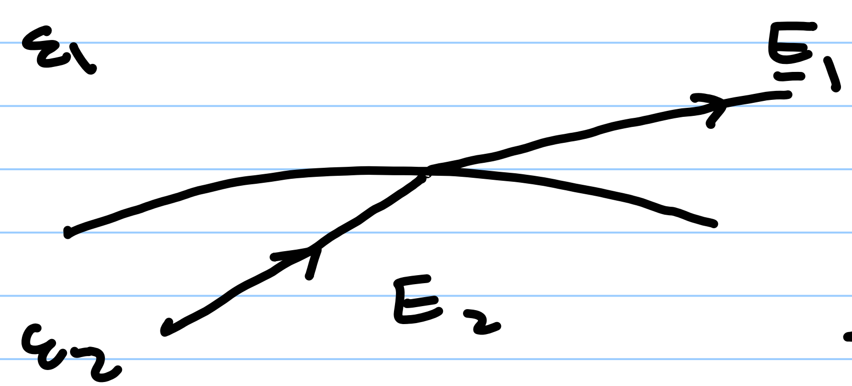
Relaxation Time (Cont'd)



* Charge inside a Conductor moves to the surface
* Charge inside a perfect dielectric does not change

Boundary Conditions

* Dielectric - Dielectric Interface



$$E_{1t} = E_{2t}$$

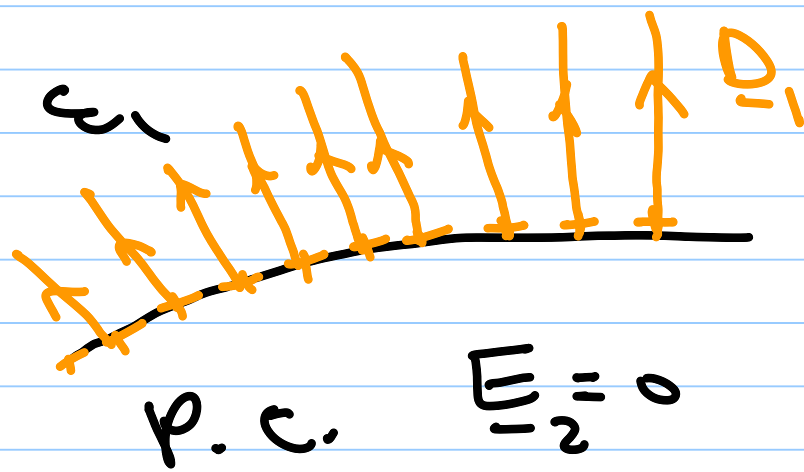
$$D_{1n} - D_{2n} = \rho_r$$

$$\Rightarrow D_{1n} = D_{2n}$$

$$\varepsilon_{r_1} E_{n_1} = \varepsilon_{r_2} E_{n_2}$$

* Tangential component of $\underline{E}$ is continuous, while normal component of $\underline{E}$ is discontinuous

Perfect Conductor-Dielectric Interface



$E_{1n} = E_{2n} = 0$

$D_{1n} - D_{2n} = \rho_s$

$D_{1n} = \rho_s$

* Only a normal $\underline{E}$ field component exists at the surface.

Laplace & Poisson's Equations

$$\nabla \cdot \underline{D} = \rho_v \Rightarrow \nabla \cdot (\varepsilon \underline{E}) = \rho_v$$

$$\Rightarrow \quad \nabla \cdot (\varepsilon \nabla V) = -\rho_v$$

if ε is a constant, we have

$$\varepsilon (\nabla \cdot \nabla V) = -\rho_v$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \leftarrow \text{Laplace equation}$$

if $\rho_v = 0$, $\nabla^2 V = 0$

Laplace & Poisson Equations

* The Laplacian operator is given by

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$