Lecture 4

From sections 6.3 – 6.5

Self read 6.6

Solve 6.7, 6.9, 6.12, 6.28, 6.30, 6.36, 6.47

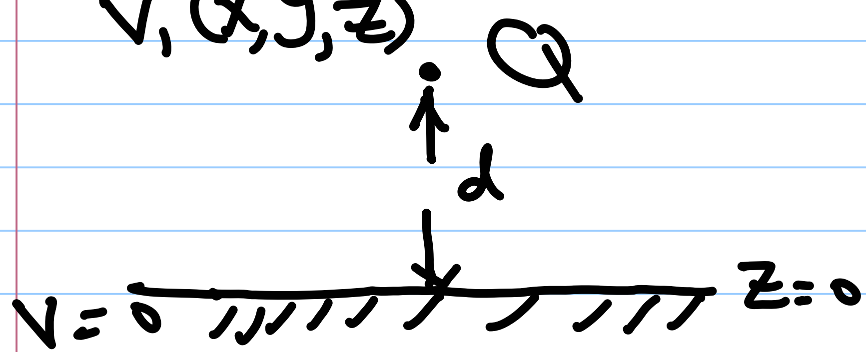
Uniqueness Theorem

* The potential in a certain region $V(x,y,z)$ is unique if it satisfies both Laplace equation and the boundary conditions.

* physical problems allow only one solution

The Uniqueness Theorem

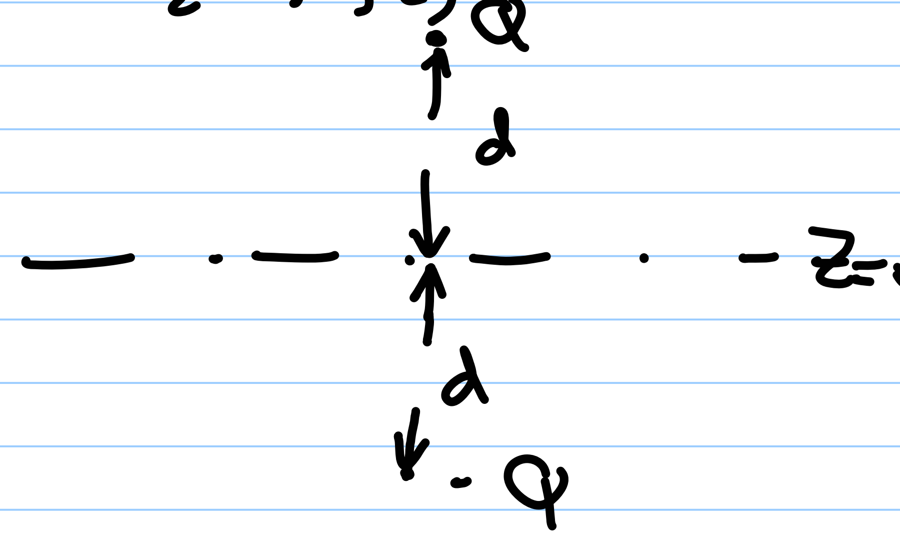
$V_1(x,y,z)$



$\nabla^2 V_1(x,y,z) = 0$

$V_1(z=0) = 0\,V$

$V_2(x,y,z)$



$\nabla^2 V_2(x,y,z) = 0$

$V_2(z=0) = 0$

$\Rightarrow \quad V_1(x,y,z) = V_2(x,y,z) \quad , \; z \geq 0$

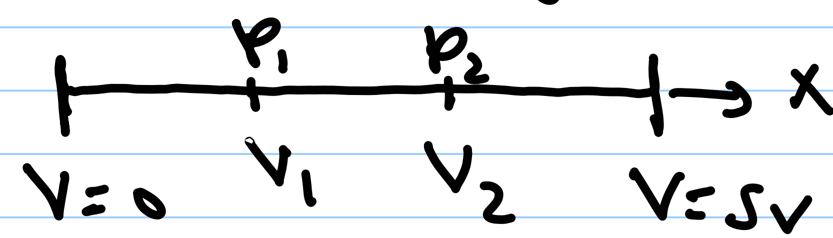
Analytical Solution of Laplace Eqn

* Determine a general solution using separation of variables
* Determine all constants through boundary conditions
* $\underline{E} = -\nabla V$, $\underline{D} = \varepsilon \underline{E}$, $\underline{J} = \sigma \underline{E}$

Read all book examples!

Numerical Solution of Laplace Eqn

* Discretize ~~the~~ domain using a grid



$$\frac{\partial^2 V}{\partial x^2} = 0$$

* Utilize finite difference approximation of 2nd order derivatives

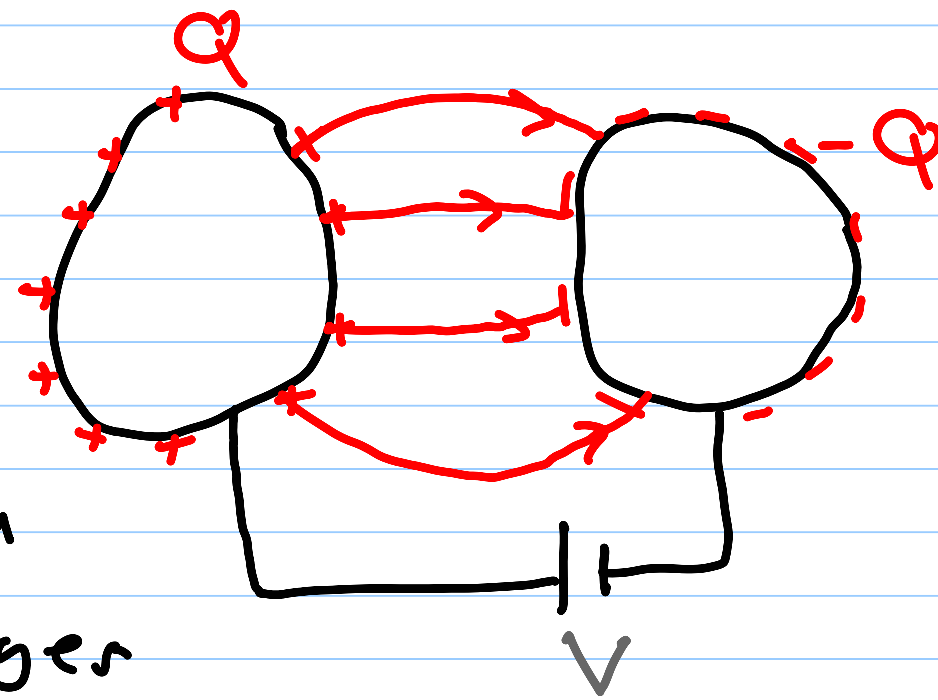
$$\frac{\partial^2 f}{\partial x^2} \approx \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}$$

Numerical Solution (Cont'd)

* Write this 2nd order expression at each unknown node to obtain one equation
* Solve the resulting system of equations

Capacitance

* When two conducting electrodes are connected to a battery, charges of opposite signs accumulate



Capacitance (Cont'd)

* The mutual capacitance is the ratio between the charge Q and the voltage difference V

$$C = \frac{Q}{V} = \frac{\varepsilon \oint_S \underline{E} \cdot d\underline{s}}{\int_1^2 \underline{E} \cdot d\underline{l}}$$

(numerator) ← over a surface S enclosing the +ve electrode

(denominator) ← path from +ve to −ve electrode

Capacitance Calculations

V-method	Q-method
1- Solve $\nabla^2 V = 0$	1- Assume a certain charge Q
2- Get $\underline{E} = -\nabla V$	2- Get $\underline{D}$
3- $\underline{D} = \varepsilon \underline{E}$	3- $\underline{E} = \frac{\underline{D}}{\varepsilon}$
4- Get $\rho_s = D_n$ on electrodes	4- $V_{12} = -\int_2^1 \underline{E} \cdot d\underline{l}$
5- $C = \frac{\iint \rho_s \, ds}{V_{12}}$	5- $C = \frac{Q}{V_{12}}$