Lecture 6

From sections 7.5-7.7, 8.5-8.7

Self read 8.6

Solve 7.31, 7.33, 7.34, 7.35, 7.37, 7.38, 7.41, 7.44, 7.50, 7.51, 8.26, 8.28, 8.31

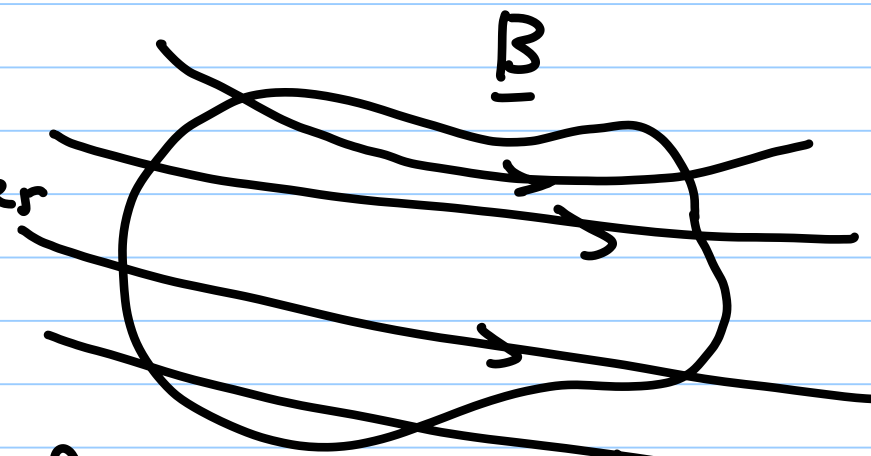
Magnetic Flux

* $\underline{B} = \mu \underline{H}$ is magnetic flux density in webers/m^2
  * μ_0, permeability of free space has the value $4\pi \times 10^{-7}$ H/m
* The magnetic flux through a surface is $\psi_m = \iint_S \underline{B} \cdot d\underline{s}$

Magnetic Flux (Cont'd)

* There are no magnetic charges

$\Rightarrow$ magnetic field



Lines are closed lines

$$\Rightarrow \oiint_S \underline{B} \cdot d\underline{s} = 0 \Rightarrow \nabla \cdot \underline{B} = 0$$

Gauss Law for magnetostatics

Magnetic Vector Potential

* $\nabla \cdot \underline{B} = 0$ but $\nabla \cdot (\nabla \times \underline{A}) = 0$ for any vector $\underline{A}$

$\Rightarrow$ $\underline{B} = \nabla \times \underline{A}$, where $\underline{A}$ is the magnetic vector potential

* In some problems, it is easier to find $\underline{A}$ and then find $\underline{B}$

(note similarity with V and $\underline{E}$)

Equation Governing $\underline{A}$

* $\nabla \times \underline{H} = \underline{J}$ (Currents result in $\underline{H}$)

$\nabla \times \left(\frac{\underline{B}}{\mu}\right) = \underline{J}$

* if μ is constant, we can take it out $\Rightarrow \frac{1}{\mu} \nabla \times \underline{B} = \underline{J}$

$\Rightarrow \frac{1}{\mu} \nabla \times (\nabla \times \underline{A}) = \underline{J}$

* We use the identity

$\nabla \times \nabla \times A = \nabla(\nabla \cdot \underline{A}) - \nabla^2 \underline{A}$

Equation Governing $\underline{A}$ (Cont'd)

* We impose the condition $\nabla \cdot \underline{A} = 0$ in order to uniquely determine $\underline{A}$

* It follows that $\nabla^2 \underline{A} = -\mu \underline{J}$

Note the similarity with Poisson's equation $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$

Analogy Examples

$$V = \frac{1}{4\pi\varepsilon}\int_{C'} \frac{\rho_\ell}{R}\, d\ell' \iff \underline{A} = \frac{\mu}{4\pi}\int_{C'} \frac{I\, d\underline{\ell}'}{R} \quad \text{Wb/m}$$

$$V = \frac{1}{4\pi\varepsilon}\iint_{S'} \frac{\rho_s}{R}\, ds' \iff \underline{A} = \frac{\mu}{4\pi}\iint_{S'} \frac{\underline{J}_s}{R}\, ds'$$

$$V = \frac{1}{4\pi\varepsilon}\iiint_{V} \frac{\rho_v}{R}\, dv \iff \underline{A} = \frac{\mu}{4\pi}\iiint_{V'} \frac{\underline{J}}{R}\, dv'$$

<u>Magnetization</u>

* In a general material

$$\underline{B} = \mu_0(\underline{H} + \underline{M})$$

* Magnetization in magnetic materials is similar to polarization in dielectric materials

* For linear materials, we have

$$\underline{M} = \chi_m \underline{H}$$

magnetic susceptibility

Magnetization (Cont'd)

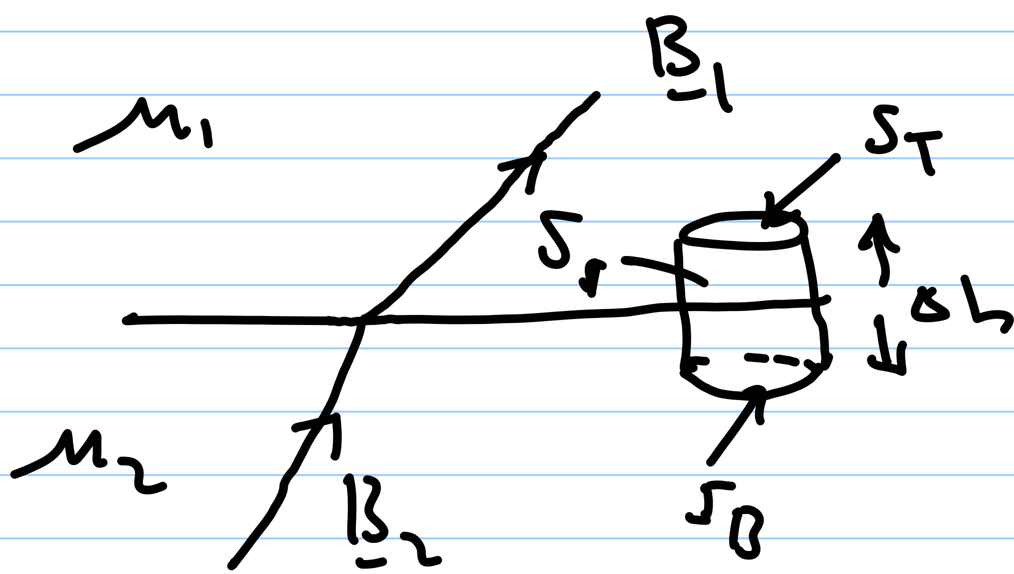
* It follows that we have

$$\underline{B} = \mu_0 (\underline{H} + \underline{M}) = \mu_0 (\underline{H} + \chi_m \underline{H})$$

$$\underline{B} = \mu_0 (1 + \chi_m) \underline{H} = \mu_0 \mu_r \underline{H}$$

* A material is defined by ε_r, μ_r, and σ

Magnetic Boundary Conditions



$$\oint\!\!\!\!\oint \underline{B}\cdot d\underline{s} = 0 = \iint_{S_T} \underline{B}\cdot d\underline{s} + \iint_{S_B} \underline{B}\cdot d\underline{s} + \iint_{S_s} \underline{B}\cdot d\underline{s}$$

If $\Delta h \to 0$, we have

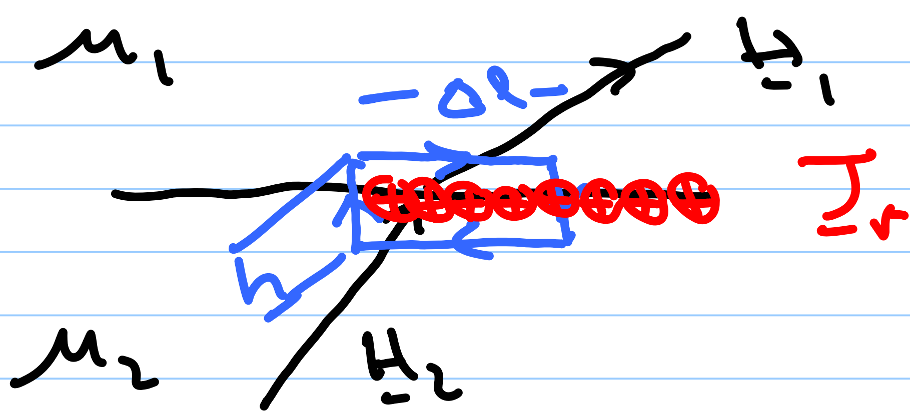
Boundary Conditions (Cont'd)

$$B_{1n}\Delta S - B_{2n}\Delta S = 0 \Rightarrow \boxed{B_{1n} = B_{2n}}$$

$$\Rightarrow \mu_1 H_{1n} = \mu_2 H_{2n}$$

Normal Component of $\underline{H}$ is discontinuous

Boundary Conditions (Cont'd)



$$\oint \underline{H} \cdot \underline{dl} = I_{enc}$$

as $h \to 0$, the two side integrals give zero

Boundary Conditions (Cont'd)

* $H_{1T}\,\Delta l - H_{2T}\,\Delta l = J_s\,\Delta l$

  $H_{1T} - H_{2T} = J_s$

  $(\underline{H}_1 - \underline{H}_2) \times \underline{a}_{n12} = \underline{J}_s$

* Note that a surface current creates a discontinuity in the vector $\underline{H}$ in the normal direction to $\underline{J}_s$