Lecture 12

from sections 10.7 - 10.8

Solve 10.31 - 10.50

EM Power Flow

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad \cdot \underline{H}$$

$$\nabla \times \underline{H} = \underline{J}_c + \frac{\partial \underline{D}}{\partial t} \quad \cdot \underline{E}$$

Subtract

$$\underbrace{\underline{H} \cdot (\nabla \times \underline{E}) - \underline{E} \cdot (\nabla \times \underline{H})}_{\nabla \cdot (\underline{E} \times \underline{H})} = \underline{H} \cdot \left(-\frac{\partial \underline{B}}{\partial t}\right) - \underline{E} \cdot \left(\underline{J}_c + \frac{\partial \underline{D}}{\partial t}\right)$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\underline{H} \cdot \frac{\partial \underline{B}}{\partial t} - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t} - \underline{E} \cdot \underline{J}_c$$

EM Power Flow (Cont'd)

* Typically $\frac{\partial \varepsilon}{\partial t} = 0$, $\frac{\partial \mu}{\partial t} = 0$

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\mu \underline{H} \cdot \frac{\partial \underline{H}}{\partial t} - \varepsilon \underline{E} \cdot \frac{\partial \underline{E}}{\partial t} - \underline{E} \cdot (\sigma \underline{E})$$

$$\Rightarrow \nabla \cdot (\underline{E} \times \underline{H}) = -\frac{1}{2}\mu \frac{\partial |\underline{H}|^2}{\partial t} - \frac{1}{2}\varepsilon \frac{\partial |\underline{E}|^2}{\partial t} - \sigma |\underline{E}|^2$$

$w_e = \frac{1}{2}\varepsilon |\underline{E}|^2$ J/m^3 electric energy density

$w_m = \frac{1}{2}\mu |\underline{H}|^2$ J/m^3 magnetic energy density

$p_\sigma = \sigma |\underline{E}|^2$ W/m^3 power density of Losses

Vector of Poynting

* The vector $\underline{S} = \underline{E} \times \underline{H}$ is called the vector of poynting

* This vector points in the direction of EM power propagation

* Integrating over the volume of interest, we get

Poynting Theorem

$$\iiint_V \nabla \cdot (\underline{E} \times \underline{H})\, dv$$

$$= -\frac{1}{2}\iiint_V \varepsilon \frac{\partial |\underline{E}|^2}{\partial t}\, dv - \frac{1}{2}\iiint_V \mu \frac{\partial |\underline{H}|^2}{\partial t}\, dv$$

$$- \iiint_V \sigma |\underline{E}|^2\, dv$$

$$-\iint_S (\underline{E} \times \underline{H}) \cdot d\underline{s} = \frac{1}{2}\iiint_V \varepsilon \frac{\partial |\underline{E}|^2}{\partial t}\, dv$$

$$+ \frac{1}{2}\iiint_V \mu \frac{\partial |\underline{H}|^2}{\partial t}\, dv + \iiint_V \sigma |\underline{E}|^2\, dv$$

Poynting Theorem (Cont'd)

$$-\iint_S (\underline{E}\times\underline{H})\cdot d\underline{s} = \frac{1}{2}\iiint_V \varepsilon \frac{\partial (\underline{E})^2}{\partial t}\,dv + \frac{1}{2}\iiint_V \mu \frac{\partial (\underline{H})^2}{\partial t}\,dv + \iiint_V \sigma\, |\underline{E}|^2\, dv$$

Energy Flowing in through S

= stored Energy + dissipated Energy

Poynting Theorem (Cont'd)

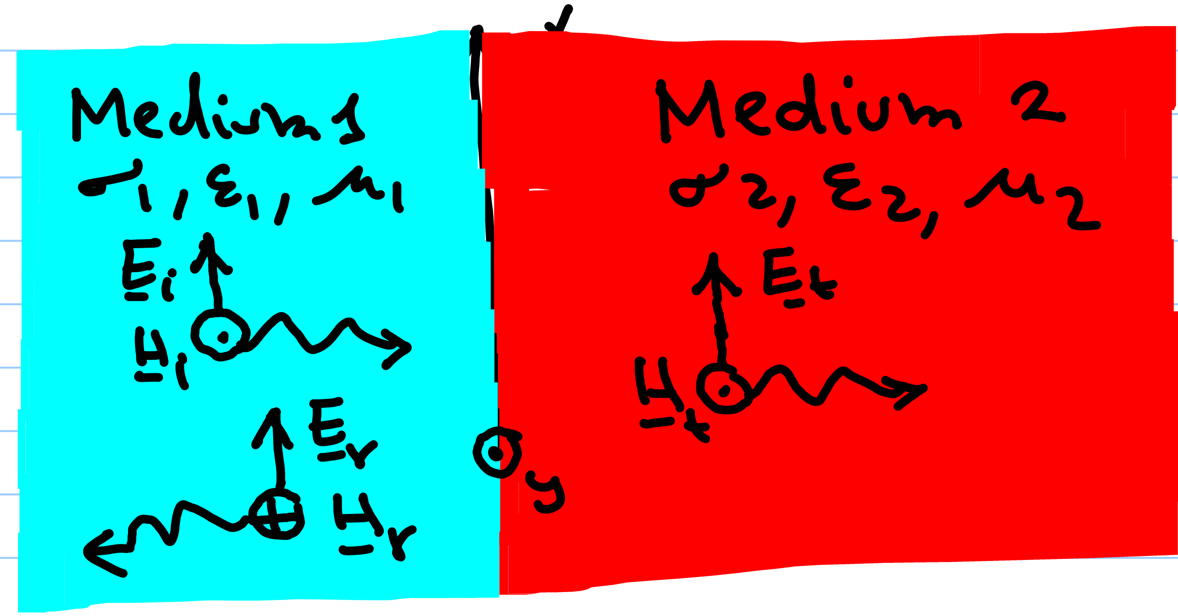
If $\underline{J} = \underline{J}_c + \underline{J}_i$, we get

$$\iiint_V (\underline{E} \cdot \underline{J}_i)\, dv = \iint_S (\underline{E} \times \underline{H}) \cdot d\underline{s} + \frac{1}{2} \iiint_V \varepsilon \frac{\partial (\underline{E})^2}{\partial t}\, dv + \frac{1}{2} \iiint_V \mu \frac{\partial (\underline{H})^2}{\partial t}\, dv + \iiint_V \sigma |\underline{E}|^2\, dv$$

Energy generated = Energy leaving + Energy stored + Energy dissipated

EM energy is Conserved!

Reflection Due to Normal Incidence



The incident wave creates both a transmitted wave and a reflected wave

Reflection (Cont'd)

$$\underline{E}_i = E_{io}\, e^{-\gamma_1 z}\, \underline{a}_x, \quad \underline{H}_i = \frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\, \underline{a}_y$$

$$\underline{E}_r = E_{ro}\, e^{\gamma_1 z}\, \underline{a}_x, \quad \underline{H}_r = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z}\, \underline{a}_y$$

$$\underline{E}_T = E_{To}\, e^{-\gamma_2 z}\, \underline{a}_x, \quad \underline{H}_T = \frac{H_{To}}{\eta_2} e^{-\gamma_2 z}\, \underline{a}_y$$

* To determine E_{ro}, E_{To} in terms of E_{io}, we apply continuity conditions of electric and magnetic fields at interface.

Reflection (Cont'd)

$$\underline{E}_i(0) + \underline{E}_r(0) = \underline{E}_T(0)$$

$$\Rightarrow \quad E_{io} + E_{ro} = E_{To} \quad \longleftarrow \quad 1$$

$$\underline{H}_i(0) + \underline{H}_r(0) = \underline{H}_T(0)$$

$$\Rightarrow \quad \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{To}}{\eta_2} \quad \longleftarrow \quad 2$$

Solving for E_{ro} and E_{To}, we obtain

Reflection (Cont'd)

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0} \Rightarrow E_{r0} = \Gamma E_{i0}$$

reflection Coeff.

$$E_{T0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0} \Rightarrow E_{T0} = \tau E_{i0}$$

transmission Coeff.

$\boxed{\text{Note that } 1 + \Gamma = \tau}$