Lecture 13

From sections 10.3 - 10.10

self read 10.3b, 10.12

Solve 10.51 - 10.60

Propagation in Coordinate Directions

* $\underline{E} = \underline{E}_o \, e^{-\gamma x}$ (propagation in +ve x)

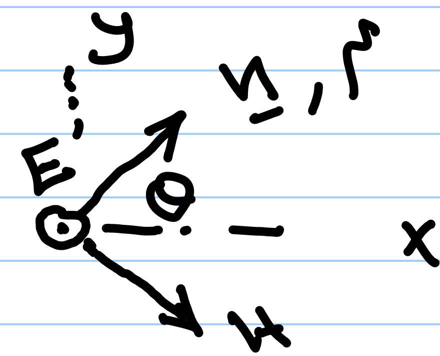
  $\underline{E} = \underline{E}_o \, e^{-\gamma z}$ (propagation in +ve z)

* Note that $\underline{E}_o$ generally have x, y, and z components

* $\gamma = \alpha + j\beta$

for the lossless case $\gamma = j\beta$

Propagation in Arbitrary Direction



* $\underline{E} = \underline{E}_o e^{-j\beta\rho}$

$\underline{E} = \underline{E}_o e^{-j\beta \underline{n} \cdot \rho \underline{n}}$

* $\underline{K} = \beta \underline{n}$ = propagation vector

$\underline{K} = \beta(\cos\theta\, \underline{a}_x + \sin\theta\, \underline{a}_y)$

$\underline{K} = K_x \underline{a}_x + K_y \underline{a}_y$

* $\rho \underline{n}$ = position vector = $\underline{r} = x\, \underline{a}_x + y\, \underline{a}_y$

Arbitrary Directions (Cont'd)

* The general form for a wave travelling in the direction $\underline{n}$ is

$$\underline{E} = \underline{E}_o e^{-j\beta \underline{n} \cdot \underline{r}} = \underline{E}_o e^{-j\underline{k} \cdot \underline{r}}$$

* notice that $\underline{k} = \beta(\cos\alpha\, \underline{a}_x + \cos\beta\, \underline{a}_y + \cos\gamma\, \underline{a}_z)$

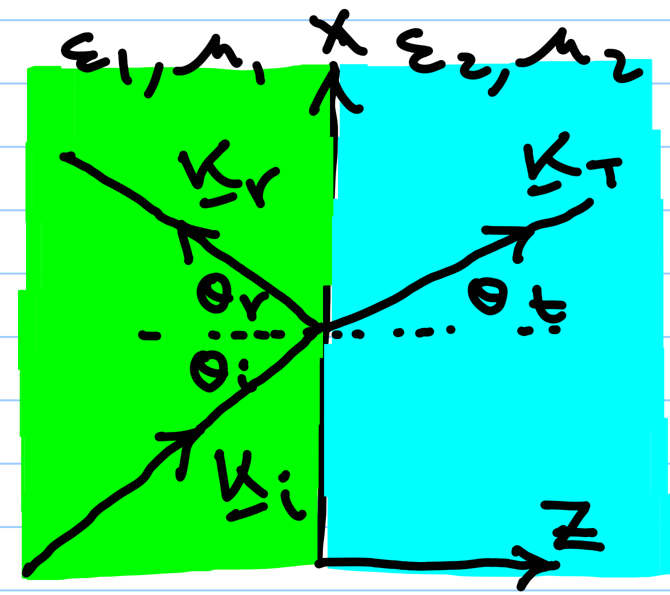
$$\underline{k} = k_x \underline{a}_x + k_y \underline{a}_y + k_z \underline{a}_z, \quad |\underline{k}| = \beta$$

$$\underline{r} = x \underline{a}_x + y \underline{a}_y + z \underline{a}_z$$

Genera Oblique Incidence

* $\underline{E}_i = \underline{E}_{io} \cos(K_{ix}x + K_{iy}y + K_{iz}z - \omega_i t)$

$\underline{E}_r = \underline{E}_{ro} \cos(K_{rx}x + K_{ry}y + K_{rz}z - \omega_r t)$



$\underline{E}_T = \underline{E}_{To} \cos(K_{Tx}x + K_{Ty}y + K_{Tz}z - \omega_T t)$

* Boundary Conditions $\underline{E}_i(z=0) + \underline{E}_r(z=0) = \underline{E}_T(z=0)$

General Oblique (Cont'd)

Boundary condition imply that

$\omega_i = \omega_r = \omega_T$, $K_{ix} = K_{rx} = K_{Tx}$,

$K_{iy} = K_{ry} = K_{Ty}$

$\Rightarrow K_i \sin\theta_i = K_r \sin\theta_r$ (But $K_i = K_r = \omega\sqrt{\mu_1\varepsilon_1}$) $\Rightarrow \boxed{\theta_i = \theta_r}$

* Also, $K_i \sin\theta_i = K_T \sin\theta_T$

$$\frac{\sin\theta_T}{\sin\theta_i} = \frac{K_i}{K_T} = \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}} \Rightarrow \boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

Parallel polarization

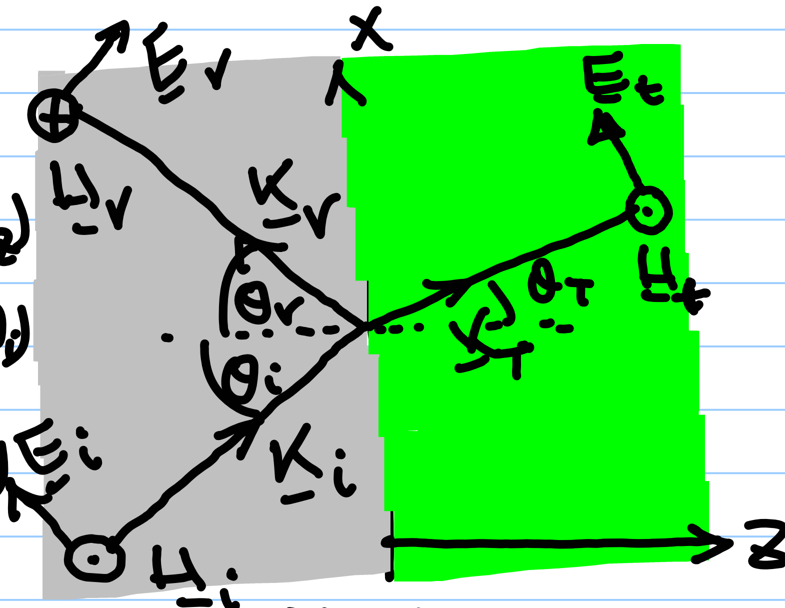
Expression for fields are

$$\underline{E}_i = E_{io}\left(\cos\theta_i\, \underline{a}_x - \sin\theta_i\, \underline{a}_z\right) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\underline{H}_i = \frac{E_{io}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\, \underline{a}_y$$

$$\underline{E}_r = E_{ro}\left(\cos\theta_r\, \underline{a}_x + \sin\theta_r\, \underline{a}_z\right) e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\underline{H}_r = -\frac{E_{ro}}{\eta_2} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$$



Parallel Polarization (Cont'd)

Similarly,

$$\underline{E}_t = E_{t0}\left(\cos\theta_t\, \underline{a}_x - \sin\theta_t\, \underline{a}_z\right) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

$$\underline{\tilde{H}}_t = \frac{E_{t0}}{\eta_2}\, e^{-j(\beta_2 (x\sin\theta_t - z\cos\theta_t)}$$

*Imposing continuity of fields along the interface, we get

Parallel polarization (cont'd)

$$(E_{io} + E_{ro})\cos\theta_i = E_{To}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2}E_{to}$$

Solving for E_{ro} & E_{to} we get

$$E_{ro} = \Gamma_{11} E_{io} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\sin\theta_i}$$

$$E_{To} = \tau_{11} E_{io} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

What is the Brewster angle?