Lecture 15

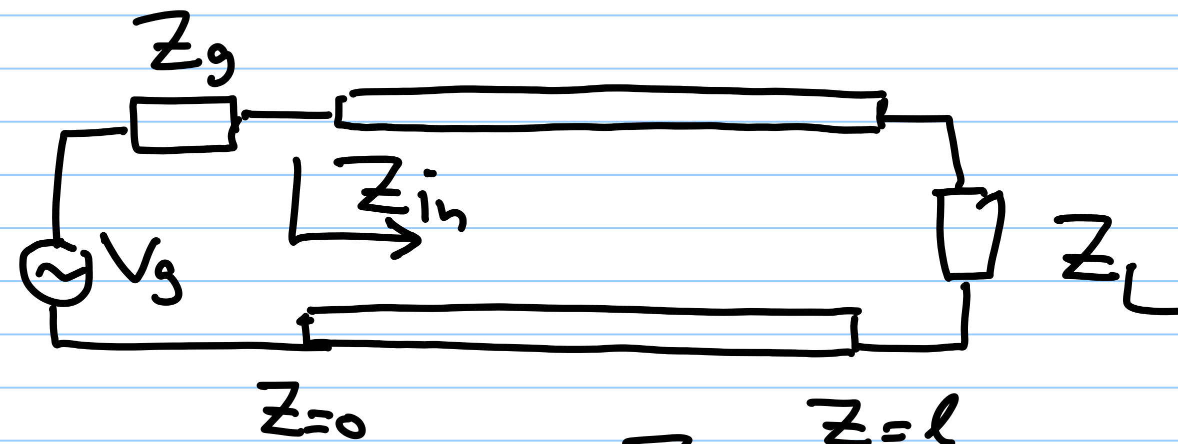
from section 11.4 of textbook

Solve 11.17 - 11.26

Transmission Line Circuits

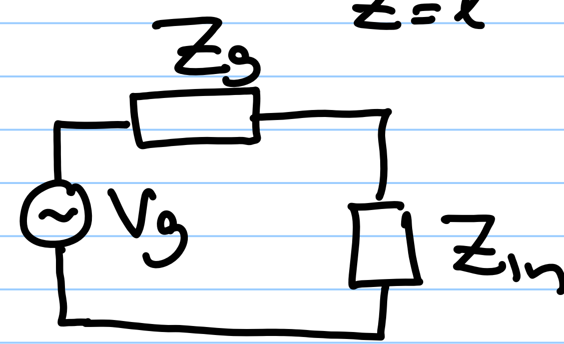
$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$



at $z=0$ (generator side)
we have

$$\tilde{V}(0) = V_0^+ + V_0^-, \quad \tilde{I}(0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$



Input Impedance

$$Z_{in} = \frac{\tilde{V}(0)}{\tilde{I}(0)} = Z_0 \frac{(V_0^+ + V_0^-)}{(V_0^+ - V_0^-)}$$

* Now the circuit looks like a voltage divider

$$\tilde{V}(0) = V_g * \frac{Z_{in}}{Z_{in} + Z_g} \quad \text{and} \quad \tilde{I}(0) = \frac{V_g}{Z_g + Z_{in}}$$

* We can solve for V_0^+ & V_0^- to obtain

$$V_0^+ = \frac{1}{2}\left(\tilde{V}(0) + Z_0 \tilde{I}(0)\right), \quad V_0^- = \frac{1}{2}\left(\tilde{V}(0) - Z_0 \tilde{I}(0)\right)$$

Input Impedance (Cont'd)

* at the load side, we have

$$\tilde{V}(\ell) = V_0^+ e^{-\gamma\ell} + V_0^- e^{\gamma\ell}, \quad \tilde{I}(\ell) = \frac{V_0^+}{Z_0} e^{-\gamma\ell} - \frac{V_0^-}{Z_0} e^{\gamma\ell}$$

but $\dfrac{\tilde{V}(\ell)}{\tilde{I}(\ell)} = Z_L$, $\quad V_L = \tilde{V}(\ell)$, $I_L = \tilde{I}(\ell)$

* we can thus express V_0^+ & V_0^- by

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma\ell}, \quad V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma\ell}$$

Input Impedance (Cont'd)

* at any distance $d < \ell$, we have

$$Z_{in}(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)} = \frac{V_0^+ e^{-\gamma d} + V_0^- e^{\gamma d}}{\frac{V_0^-}{Z_0} e^{-\gamma d} - \frac{V_0^-}{Z_0} e^{\gamma d}}$$

Substituting for V_0^+ & V_0^- and using

$\cosh \gamma d = \frac{e^{\gamma d} + e^{-\gamma d}}{2}$ and $\sinh \gamma d = \frac{e^{\gamma d} - e^{-\gamma d}}{2}$, we get

Input Impedance formula

$$Z_{in}(d) = Z_0 \left[\frac{Z_L + jZ_0 \tanh \gamma d}{Z_0 + jZ_L \tanh \gamma d} \right]$$

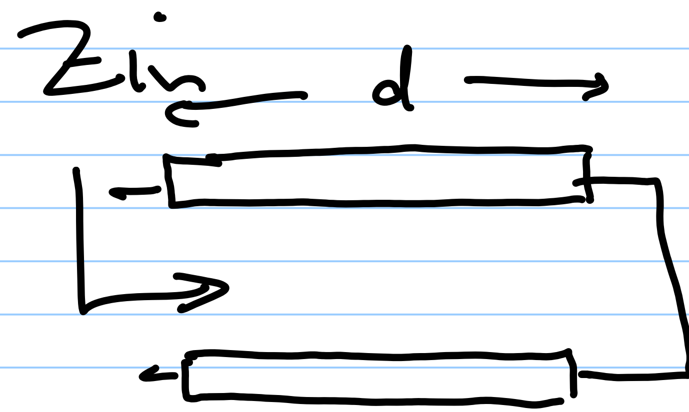
For the lossless case $\gamma = j\beta$

$$Z_{in}(d) = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d} \right]$$

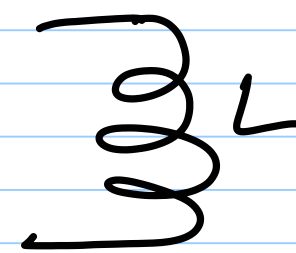
Load Exampler

* short circuit Load $Z_L = 0$

$\Rightarrow \quad Z_{in}(d) = j Z_0 \tan \beta d$

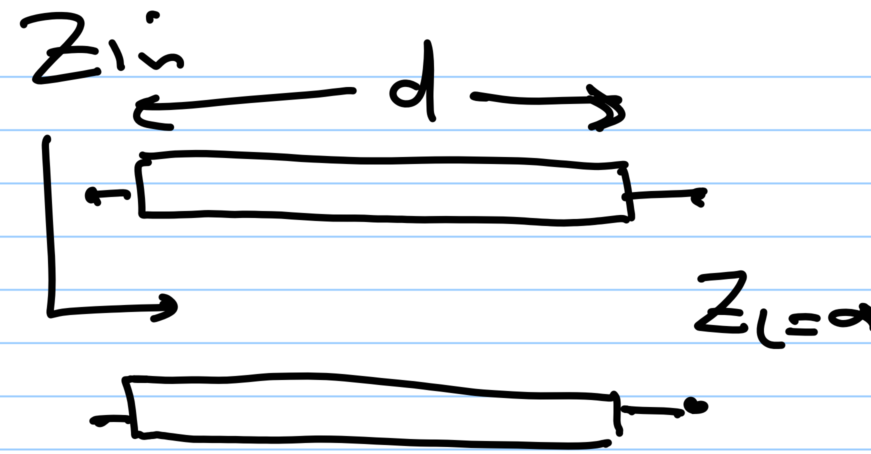


This is equivalent to an inductor L, whose valve is determined by Z_0, β, and d



Load Examples (Cont'd)

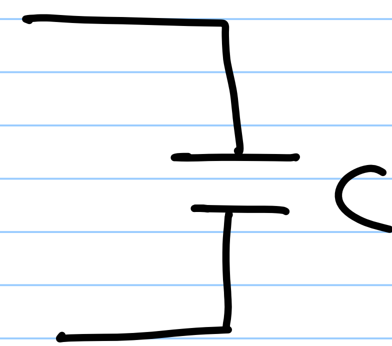
* open circuit Load ($Z_L = \infty$)



* Using the Impedance formula, we have

$$Z_{in} = -jZ_0 \cot\beta d$$

a capacitor of value C where $C = C(Z_0, \beta, d)$



Load Exampler (Cont'd)

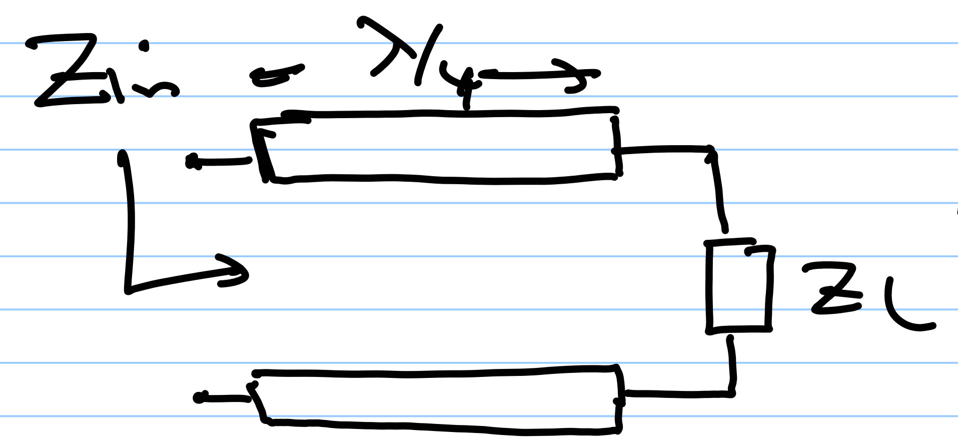
* If $d = \lambda/4$, $\lambda = \frac{2\pi}{\beta}$ = wavelength

$\Rightarrow \beta d = \lambda/4 * \beta = \lambda/4 * \frac{2\pi}{\lambda} = \frac{\pi}{2}$

$\tan \beta d = \infty$

$Z_{in}(d) = \frac{Z_0^2}{Z_L}$

$\Rightarrow$ impedance transformer



Reflection Coefficient

* It is defined as the ratio of reflected to incident voltage waves

$$\Gamma(\ell) = \frac{V_0^- e^{\gamma \ell}}{V_0^+ e^{-\gamma \ell}} \qquad \left\{ \begin{array}{l} V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma \ell} \\ V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma \ell} \end{array} \right\}$$

Substituting, we get $\Gamma(\ell) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Reflection Coefficient (Cont'd)

* At any distance d, we have

$$\Gamma(d) = \frac{V_0^- e^{\gamma d}}{V_0^+ e^{-\gamma d}} = \frac{V_0^-}{V_0^+} e^{2\gamma d} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma \ell} e^{2\gamma d}$$

$$\Gamma(d) = \Gamma_L e^{-2\gamma(\ell - d)}$$

For a lossless case $|\Gamma(d)| = |\Gamma_L|$