Lecture 18

From sections 12.4 – 12.5 of Textbook

Self read 12.5

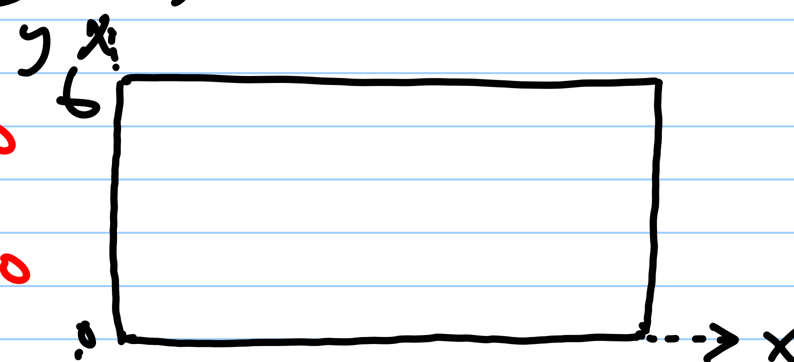
Solve 12.1 – 12.35

TE Modes

In a TE Mode $\tilde{E}_z = 0$, $\tilde{H}_z \neq 0$

$\tilde{E}_x(y=0) = 0$, $\tilde{E}_x(y=b) = 0$

$\tilde{E}_y(x=0) = 0$, $\tilde{E}_y(x=a) = 0$



* The general solution is

$$\tilde{H}_z = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)$$
$$(B_5 e^{-\gamma z} + B_6 e^{-\gamma z})$$

Boundary Conditions

$$\tilde{E}_x = -\frac{j\omega\mu}{h^2}\frac{\partial \tilde{H}_z}{\partial y} \qquad \tilde{E}_x(y=0)=0 \Rightarrow \left.\frac{\partial \tilde{H}_z}{\partial y}\right)_{y=0} = 0$$

$$\tilde{E}_y = +\frac{j\omega\mu}{h^2}\frac{\partial \tilde{H}_z}{\partial x} \qquad \tilde{E}_x(y=b)=0 \Rightarrow \left.\frac{\partial \tilde{H}_z}{\partial y}\right)_{y=b} = 0$$

$$\tilde{H}_x = -\frac{\gamma}{h^2}\frac{\partial \tilde{H}_z}{\partial x} \qquad \tilde{E}_y(x=0)=0 \Rightarrow \left.\frac{\partial \tilde{H}_z}{\partial x}\right)_{x=0} = 0$$

$$\tilde{H}_y = -\frac{\gamma}{h^2}\frac{\partial \tilde{H}_z}{\partial y} \qquad \tilde{E}_y(x=a)=0 \Rightarrow \left.\frac{\partial \tilde{H}_z}{\partial x}\right)_{x=a} = 0$$

Modal Equations

* It follows that for TE_{mn} mode, we have

$$\tilde{H}_z = H_0 \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{-\gamma z} \quad \text{(either } m \text{ or } n \text{ may be 0)}$$

$$\tilde{E}_x = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, e^{-\gamma z}$$

$$\tilde{E}_y = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\tilde{H}_x = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$\tilde{H}_y = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

Modal Equations (Cont'd)

* remember that $K_x = \frac{m\pi}{a}$, $K_y = \frac{n\pi}{b}$
* $h^2 = K_x^2 + K_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$
* $\gamma = \sqrt{h^2 - k^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$
* $\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$
* note that both m and n cannot be zero simultaneously

Cut off frequency

* EACH TE_{mn} does not propagate beyond its cut off frequency

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

<u>Propagation Constant</u>

$$\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{1}{\omega^2\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)}$$

$$\beta = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad , \quad \beta' = \frac{\omega}{u} = \omega\sqrt{\mu\varepsilon}$$

$$\lambda = \frac{2\pi}{\beta} \quad , \quad u_p = \frac{\omega}{\beta}$$

<u>Wave Impedance</u>

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{j\omega\mu}{\gamma} = \frac{\omega\mu}{\beta}$$

$$\eta_{TE} = \frac{\omega\mu}{\frac{\omega}{u}\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\mu * \frac{1}{\sqrt{\mu\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \quad \eta' = \sqrt{\frac{\mu}{\varepsilon}}$$

<u>Dominant Mode</u>

* The TE_{10} mode is the dominant mode as it has the lowest cut off frequency

$$\tilde{H}_z = H_0 \cos\frac{\pi x}{a}\, e^{-\gamma z}$$

$$\tilde{E}_x = 0 \quad , \quad \tilde{H}_y = 0$$

$$\tilde{E}_y = -\frac{j\omega\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\frac{\pi x}{a}$$

$$\tilde{H}_x = \frac{\gamma}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\frac{\pi x}{a}$$

$$f_c = \frac{u'}{2} * \frac{1}{a} = \frac{u'}{2a}$$