Lecture 13

From sections 12.6 - 12.7 of Textbook

Solve 12.24 - 12.35

Power Transmission in WGs

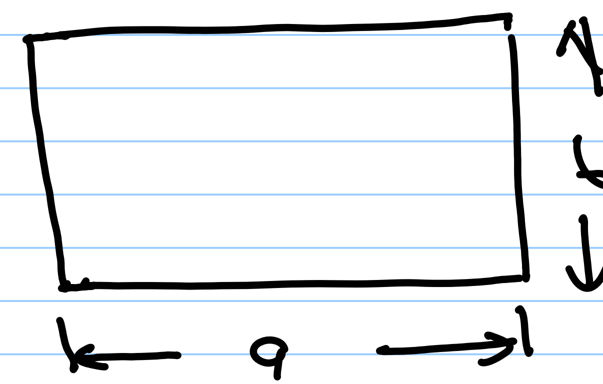
* Average power density is given by

$$\underset{\sim}{P}_{av} = \frac{1}{2}\mathrm{Re}\left(\tilde{\underline{E}} \times \tilde{\underline{H}}^*\right)$$

$$\underset{\sim}{P}_{av} = \frac{1}{2}\mathrm{Re}\left(\tilde{E}_x \tilde{H}_y^* - \tilde{E}_y \tilde{H}_x^*\right)\underline{a}_z$$

$$\underset{\sim}{P}_{av} = \frac{|\tilde{E}_x|^2 + |\tilde{E}_y|^2}{2\eta}\underline{a}_z$$

* This expression can be evaluated for each excited mode



Power Transmission (Cont'd)

$$P_{av} = \iint_S \underset{\sim}{p}_{av} \cdot d\underline{s} = \int_0^a \int_0^b \frac{|\tilde{E}_x|^2 + |\tilde{E}_y|^2}{2\eta} \, dx \, dy$$

* Notice that η is real and positive for propagating modes

Attenuation in WGs

* Attenuation in waveguides is due to losses in the dielectric filling the waveguide or due to losses in conductor walls

* $P_{av}(z) = P_{av}(0)\, e^{-2\alpha z}$

$$\alpha = \underbrace{\alpha_c}_{\text{Conductor losses}} + \underbrace{\alpha_d}_{\text{dielectric losses}}$$

Dielectric Losses

* α_d is obtained by assuming a complex permittivity

$$\gamma = \alpha_d + j\beta_d = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu(\varepsilon' - j\varepsilon''}$$

Squaring and equating real and imaginary parts, we obtain

$$\alpha_d^2 - \beta_d^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon'$$

$$2\alpha_d\beta_d = \omega^2\mu\varepsilon'' = \omega^2\mu\frac{\sigma}{\omega} = \omega\mu\sigma$$

<u>Dielectric Losses (Cont'd)</u>

Solving, we get $(\alpha_d^2 \ll \beta_d^2)$

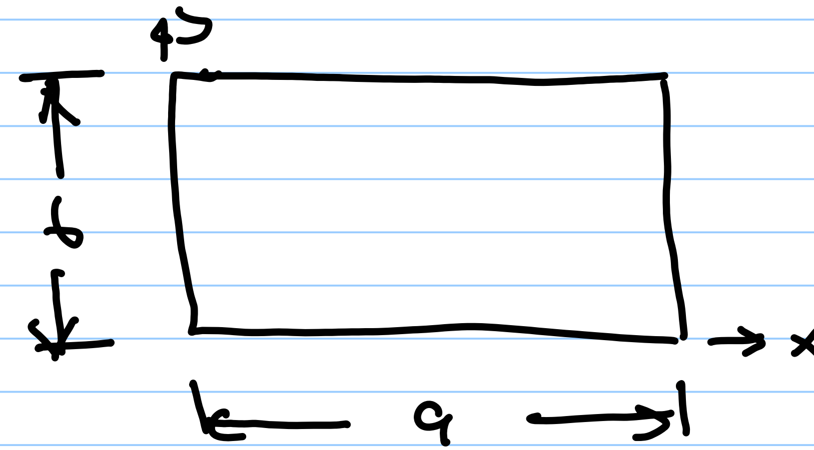
$$\beta_d \simeq \sqrt{\omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad \text{(as in lossless case)}$$

$$\alpha_d = \frac{\sigma \eta'}{2\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

* This is the practical low loss case

Conductor Losses

* The total power loss per unit length due to conductivity of walls is



$$P_L = P_L\big)_{y=0} + P_L\big)_{y=b} + P_L\big)_{x=0} + P_L\big)_{x=a}$$

$$P_L = 2\left(P_L\big)_{y=0} + P_L\big)_{x=0}\right)$$

Conductor Losses (Cont'd)

* Note that current flowing in walls is equal in magnitude to tangential magnetic field

$$P_L\Big)_{y=0} = \frac{1}{2} \mathrm{Re}\left[\eta_c \iint_{\text{wall}} |\tilde{H}_x|^2 + |\tilde{H}_z|^2 \, dx\, dz \right]_{y=0}$$

$$\Rightarrow P_L\Big)_{y=0} = \frac{1}{2} \mathrm{Re}\left[\eta_c \int_0^a \left(|\tilde{H}_x|^2 + |H_z|^2 \right) d\cancel{x} \right)_{y=0}$$

for TE_{10} we have $P_L = \frac{R_s a H_0^2}{4} \left(1 + \frac{\beta^2 a^2}{\pi^2}\right)$

Conductor Losses (Cont'd)

* Similarly, for the other wall (TE10 mode)

$$P_L\big)_{x=0} = \frac{R_s b H_0^2}{2}$$

$$P_L = R_s H_0^2 \left[b + \frac{a}{2}\left(1 + \frac{\beta^2 a^2}{\pi^2}\right)\right]$$

but $\alpha_c = \dfrac{P_L}{2 P_{av}(0)}$ (why)

$\Rightarrow$ for TE10 $\quad \alpha_c = \dfrac{2 R_s}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(0.5 + \dfrac{b}{a}\left[\dfrac{f_c}{f}\right]^2\right)$