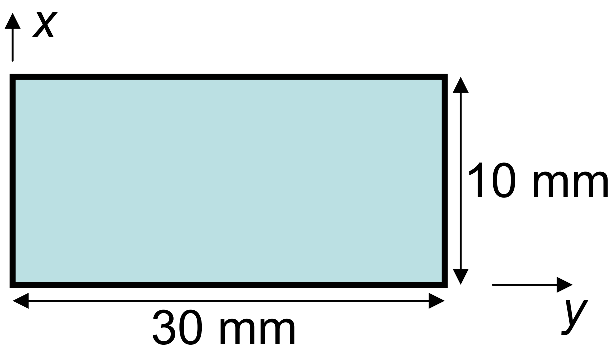
Project ECE750: FINITE-DIFFERENCE METHODS

OPTION I

Find the cut-off frequencies of a rectangular waveguide whose cross-section is 30 mm by 10 mm in a frequency band up to 40 GHz.



Some background notes

In a homogeneous medium, Maxwell's equations reduce to a second-order wave equation in the time domain (or to a Helmholtz equation in the frequency domain).

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$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad \nabla \times \nabla \times \tilde{\mathbf{E}} - k_0^2 \tilde{\mathbf{E}} = 0, \;\; k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$$

In a homogeneous, source-free medium: $\nabla \cdot \mathbf{E} = 0$

$$\nabla \times \nabla \times \mathbf{E} = \nabla\nabla \cdot \mathbf{E} - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}$$

$$\Rightarrow \nabla^2 \tilde{\mathbf{E}} + k_0^2 \tilde{\mathbf{E}} = 0$$

In rectangular coordinates, the ∇^2 operator acting on a vector conveniently decouples into three scalar operators acting on the vector's three components

$$\nabla^2 \tilde{E}_\xi + k_0^2 \tilde{E}_\xi = 0, \;\; \xi \equiv x, y, z$$

$$\frac{\partial^2 \tilde{E}_\xi}{\partial x^2} + \frac{\partial^2 \tilde{E}_\xi}{\partial y^2} + \frac{\partial^2 \tilde{E}_\xi}{\partial z^2} + k_0^2 \tilde{E}_\xi = 0, \;\; \xi \equiv x, y, z$$

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Consider a guided wave propagating in the +*z* direction.

$$\tilde{E}_\xi = \tilde{E}_{\xi\perp}(x,y)e^{-jk_z z}$$

$$\frac{\partial^2 \tilde{E}_\xi}{\partial z^2} = -k_z^2 \tilde{E}_{\xi\perp} \qquad \Rightarrow \frac{\partial^2 \tilde{E}_{\xi\perp}}{\partial x^2} + \frac{\partial^2 \tilde{E}_{\xi\perp}}{\partial y^2} + (k_0^2 - k_z^2)\tilde{E}_{\xi\perp} = 0$$

The characteristic numbers of the Helmholtz equation satisfy (remember the solution using separation of variables):

$$k_x^2 + k_y^2 + k_z^2 = k_0^2$$

In the rectangular cavity of a waveguide (a x b), the characteristic numbers k_x and k_y are easily found.

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$

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$$\Rightarrow k_z = \sqrt{k_0^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Obviously, there are frequencies, at which k_z=0. Those are the *cut-off frequencies* of the waveguide modes. A mode, for which k_z=0, does not propagate. A mode, for which k_z is an imaginary number, is evanescent, i.e., it decays exponentially along *z*, if excited. Evanescent modes have frequencies below their respective cut-off frequency.

Let us consider the case when k_z=0.

$$\frac{\partial^2 \tilde{E}_{\xi\perp}}{\partial x^2} + \frac{\partial^2 \tilde{E}_{\xi\perp}}{\partial y^2} + k_0^2 \tilde{E}_{\xi\perp} = 0 \quad \Leftrightarrow \frac{\partial^2 E_{\xi\perp}}{\partial x^2} + \frac{\partial^2 E_{\xi\perp}}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E_{\xi\perp}}{\partial t^2} = 0$$

The problem is in effect reduced to a 2-D one.

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Hints

Analyze both the TE_z and the TM_z 2-D modes.

Recommended excitation: Gaussian pulse covering frequencies well beyond the specified maximum frequency.

Your time sample must be sufficiently long! No less than 20000 time steps.

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OPTION II

Simulate the dominant TE_{z01} mode of a waveguide, whose broad side is a=30 mm. Find its wave impedance

$$Z_w^{TE_{z01}}(f) = \frac{\tilde{E}_x(f)}{\tilde{H}_y(f)}, \quad \Omega$$

Use band-limited excitation for frequencies from 5 GHz to 10 GHz

Employ ABC at both ports (Mur 1st order, bonus - Liao 3rd order)

Put your sample point halfway between the excitation plane and the further port

Use the symmetry of the mode

Make the waveguide long: at least 20 x a

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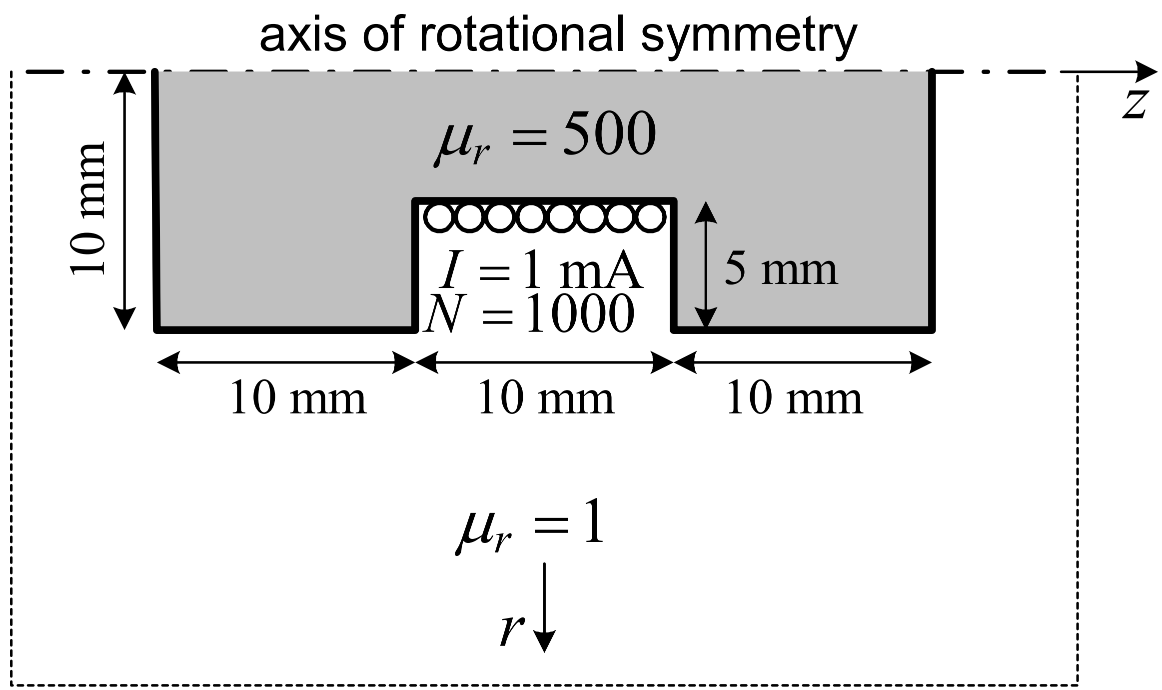
OPTION III

Calculate the inductance of the coil.



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Solve $\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} = \mathbf{J}$

Since $\mathbf{A} = A\hat{\varphi}$ and $\frac{\partial A}{\partial \varphi} = 0$

$$\frac{\partial}{\partial z}\frac{1}{\mu}\frac{\partial A}{\partial z} + \frac{\partial}{\partial r}\frac{1}{\mu}\left(\frac{A}{r} + \frac{\partial A}{\partial r}\right) = -J_{\varphi}$$

Find magnetic field $\mathbf{H} = \nabla \times \mathbf{A}$

Find magnetic energy, calculate inductance

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Outline of Projects

Objectives

Summary of theory

Discretization procedure: calculation of discretization steps in space/time, size of computational volume, numerical constants, etc.

Source code (ample comments are recommended)

Numerical results

Comparison with analytical results (if applicable)

Animations/plots of field distributions in MATLAB is a bonus