

CoE4TN4

Image Processing

Chapter 11

Image Representation & Description



Image Representation & Description

- After an image is segmented into regions, the regions are represented and described in a form suitable for computer processing (descriptors).
- Representing a region:
 1. In terms of its external characteristics (boundary)
 2. In term of its internal characteristics

Exp: A region might be represented by the length of its boundary.

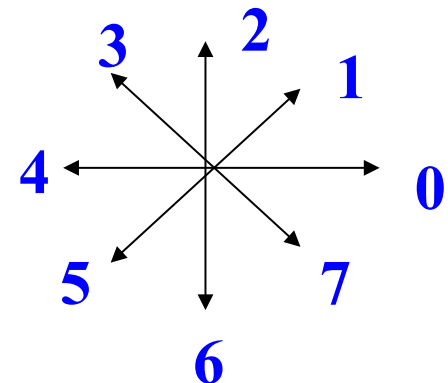
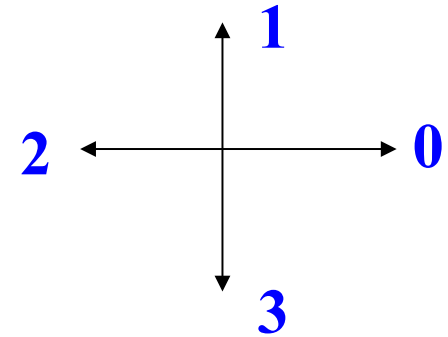
 - External representations are used when the focus is on shape of the region.
 - Internal representations are used when the focus is on color and texture.
 - Representations should be insensitive to rotation and translation.

Chain Code

- Chain Code: Used to represent a boundary by a connected sequence of straight line segments.
 - 4 or 8 connectivity is used
 - The direction of each segment is coded by a numbering scheme.

Method:

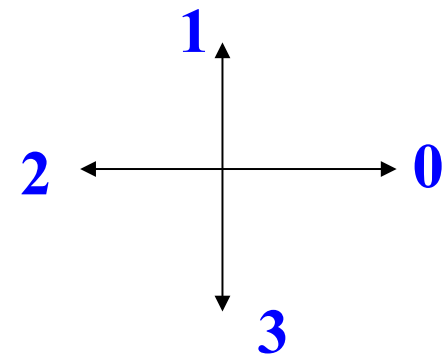
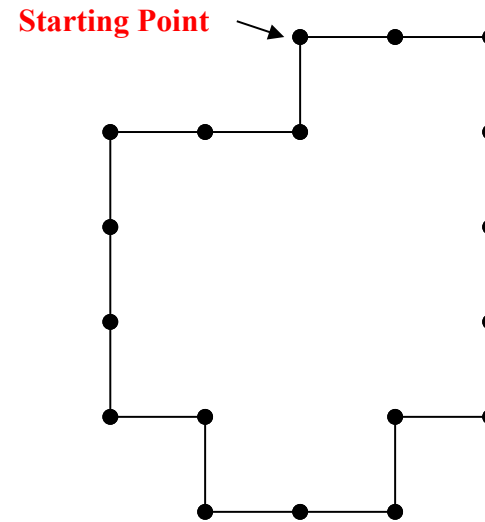
- Follow the boundary in a specific (clockwise) direction.
- Assign a direction to the segment connecting every pair of pixels.



Chain Code

Exp: 003333232212111001

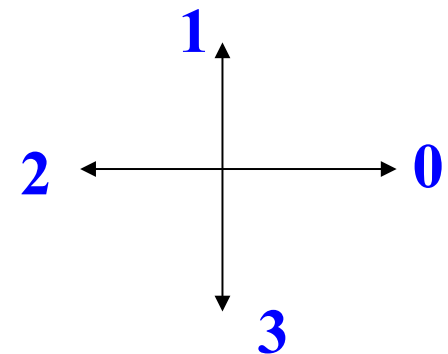
- Problems:
 - The chain code depends on the starting point.
 - It changes with rotation.



Chain Code

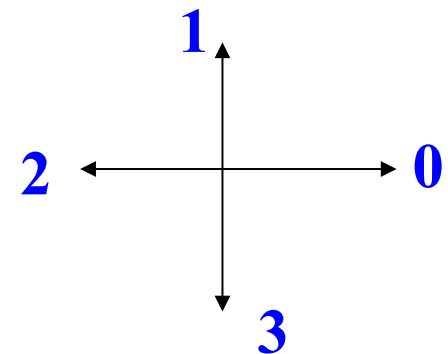
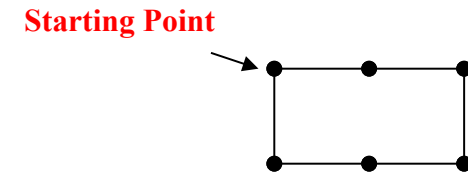
Solutions:

- Treat the chain code as a circular sequence of numbers. Circulate until the number is of minimum magnitude.
- Use the difference of chain code instead of the code itself: count counterclockwise the number of directions that separates two adjacent elements (**First difference**)
- Exp: 10103322
- First difference code: 33133030



Shape Number

- Shape number: first difference of smallest magnitude (in the chain code)
 - Exp: chain code 003221
 - First difference code: 303303
 - Shape number: 033033



Signature

- Signature: a 1-D functional representation of a boundary
- Different ways of generating signature
- Plot distance from centroid to boundary as a function of angle

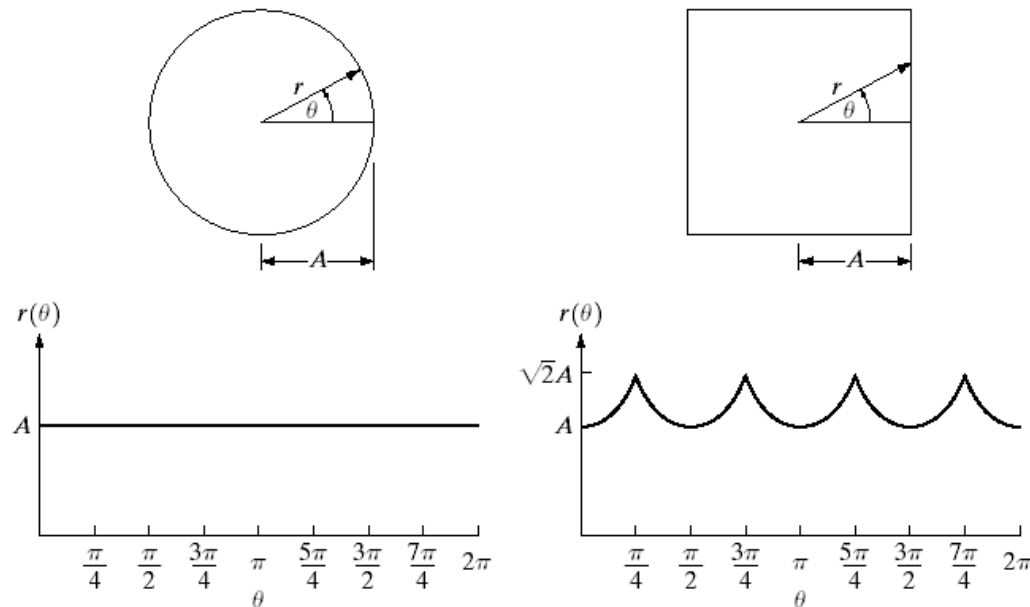
a b

FIGURE 11.5

Distance-versus-angle signatures.

In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern

$r(\theta) = A \sec \theta$ for $0 \leq \theta \leq \pi/4$ and $r(\theta) = A \csc \theta$ for $\pi/4 < \theta \leq \pi/2$.



Signature

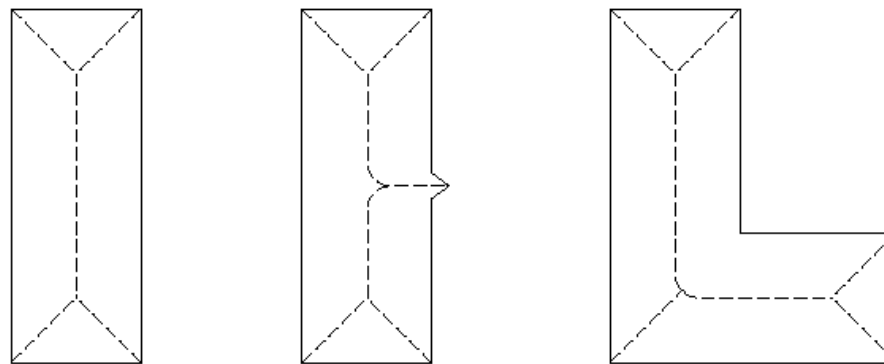
- Invariant to translation, depend on rotation and scaling
- To make it invariant to rotation we should select the same starting point regardless of the orientation
 - Select starting point farthest from centroid (if unique)
- To make it invariant to scaling we can normalize to a particular range
- Other signatures: traverse the boundary, at each point plot the angle between a line tangent to the boundary and a reference line
- Slope-density-function: histogram of tangent-angle values
- Straight segments will form the peaks of histogram

Skeletons

- An important approach to representing structural shape of a plane region is to reduce it to a graph
- This may be accomplished by obtaining the skeleton of the region via a thinning algorithm.
- Applications in automated inspection
- Definition of skeleton is based on medial axis transformation (MAT)

Skeletons

- MAT of a region R with border B: for each point p in R, find the closest neighbor in B. If p has more than one such neighbor, it belongs to medial axis (skeleton)
- MAT is based on “prairie fire concept”.
- Direct implementation of MAT is computationally expensive
- Alternative algorithms have been proposed that “thin” the boundary of a region until the skeleton is left



a b c

FIGURE 11.7
Medial axes
(dashed) of three
simple regions.

Skeletons

- An algorithm for thinning binary regions (assume region points are 1 and background points are 0)
- The algorithm has two steps which are applied to all the pixels on the contour of the region
- A contour point is any pixel with value 1 and having at least one 8-neighbor valued 0.
- In each step the boundary point that satisfy a set of conditions are flagged and then deleted

Skeletons

- Step 1 flags a contour point p_1 if the following conditions are satisfied:

- a) $2 \leq N(p_1) \leq 6$
- b) $T(p_1) = 1$
- c) $p_2.p_4.p_6 = 0$
- d) $p_4.p_6.p_8 = 0$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

- $N(p_1)$: number of nonzero neighbors of p_1
- $T(p_1)$: number of 0 to 1 transitions in ordered sequence $p_2, p_3, \dots, p_8, p_9, p_2$.

0	0	1
1	p_1	0
1	0	1

Skeletons

- After step 1 is applied to all border points those that are flagged are deleted (changed to 0)
- In step 2 conditions (a) and (b) remain the same but (c) and (d) are changed to:

$$c') \quad p2.p4.p8 = 0$$

$$d') \quad p2.p6.p8 = 0$$

- After step 2 is applied to all border points remaining after step 1, those that are flagged are deleted (changed to 0)
- This procedure is applied iteratively until no further points are deleted.

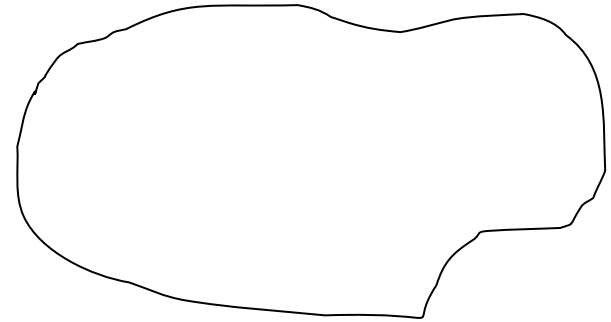
Skeletons



FIGURE 11.16
Human leg bone
and skeleton of
the region shown
superimposed.

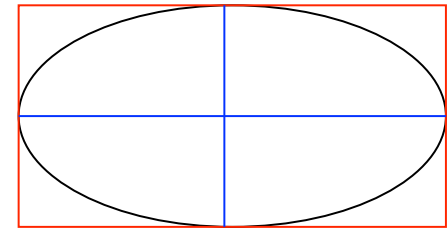
Simple boundary descriptors

- Length: number of pixels along the contour of a region
- Diameter: $\text{Dima}(B) = \max[D(p_i, p_j)]$
 - p_i, p_j are points on the boundary.
- Curvature: rate of change of slope.
- For digital images: the difference between the slopes of adjacent boundary segments.



Simple boundary descriptors

- Major axis: straight line segment joining the two points farthest from each other on the boundary
- Minor axis: Perpendicular to the major axis and of such length that a box could be formed to enclose the boundary.
- Eccentricity=Major axis/Minor axis
- Basic rectangle (bounding box): the rectangle formed by minor and major axes enclosing the boundary.



Fourier Descriptor

N point boundary

$$(x_0, y_0), (x_1, y_1), \dots, (x_{N-1}, y_{N-1})$$

$$s(k) = x_k + jy_k$$

N point DFT of $s(k)$:

$$a(u) = \frac{1}{N} \sum_{k=0}^{N-1} s(k) \exp(-j2\pi uk / N)$$

$a(u)$ are called Fourier descriptors.

Fourier Descriptor

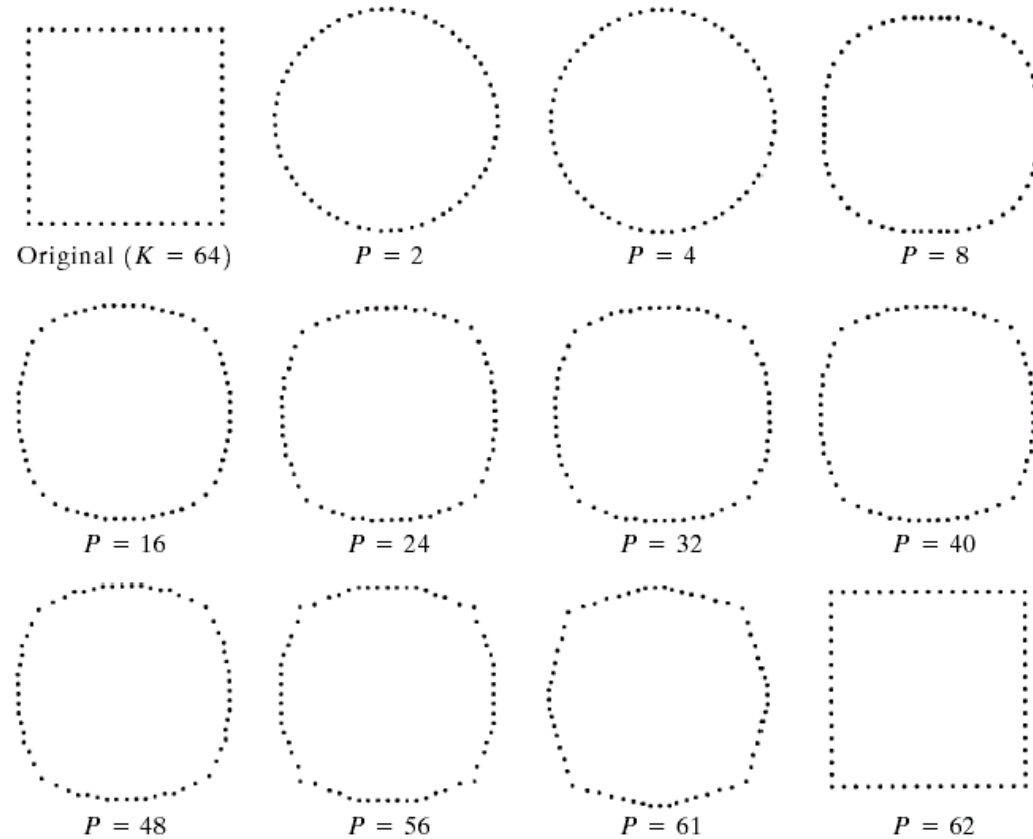
If P of the Fourier descriptors are used

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u) \exp(j2\pi uk / N)$$

$P < N \Rightarrow$ High frequency details of the boundary (e.g., corners) are removed.

- Fourier descriptors are not directly insensitive to translation, rotation and scaling.
- Magnitude of the Fourier descriptors is insensitive to rotation.

Fourier Descriptor

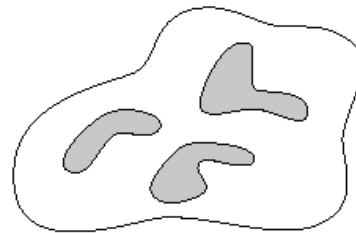
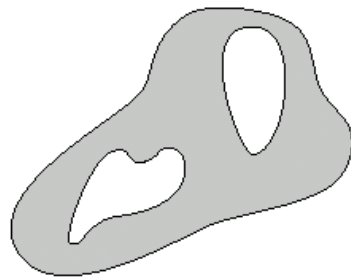


Regional Descriptors

- Area: number of pixels contained within a region
- Compactness: $(\text{perimeter})^2/\text{area}$
- Min and Max of the gray levels in the region
- Mean and median of gray levels

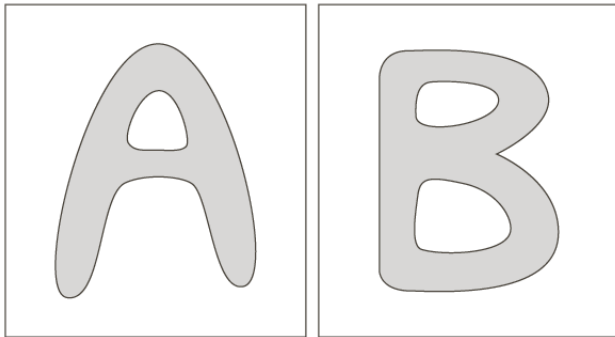
Topological Descriptors

- Topology: study of properties of a figure that are unaffected by any deformation, as long as there is no tearing or joining of the figure (rubber sheet distortions)
- Since stretching affects distance, topological properties do not depend on the notion of distance
- Number of holes in a region (H)
- Number of connected components (C)



Topological Descriptors

- Euler number: $E = C - H$
- Sometimes a region is represented by straight-line segments (polygonal network)
- V : number of vertices, Q : number of edges, F : number of faces $\Rightarrow V - Q + F = C - H = E$
- For the figure below right: $7 - 11 + 2 = 1 - 3 = -2$



a b

FIGURE 11.25
Regions with Euler number equal to 0 and respectively.

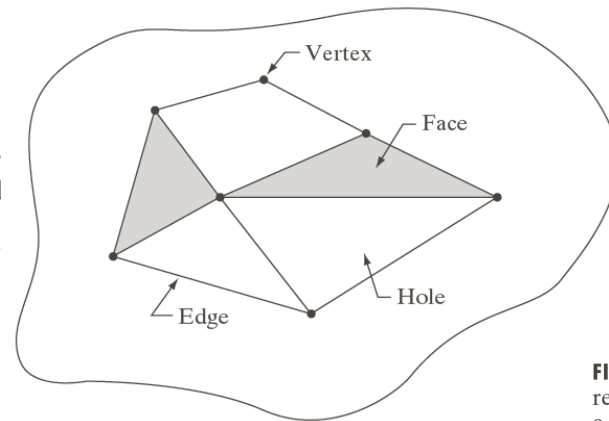
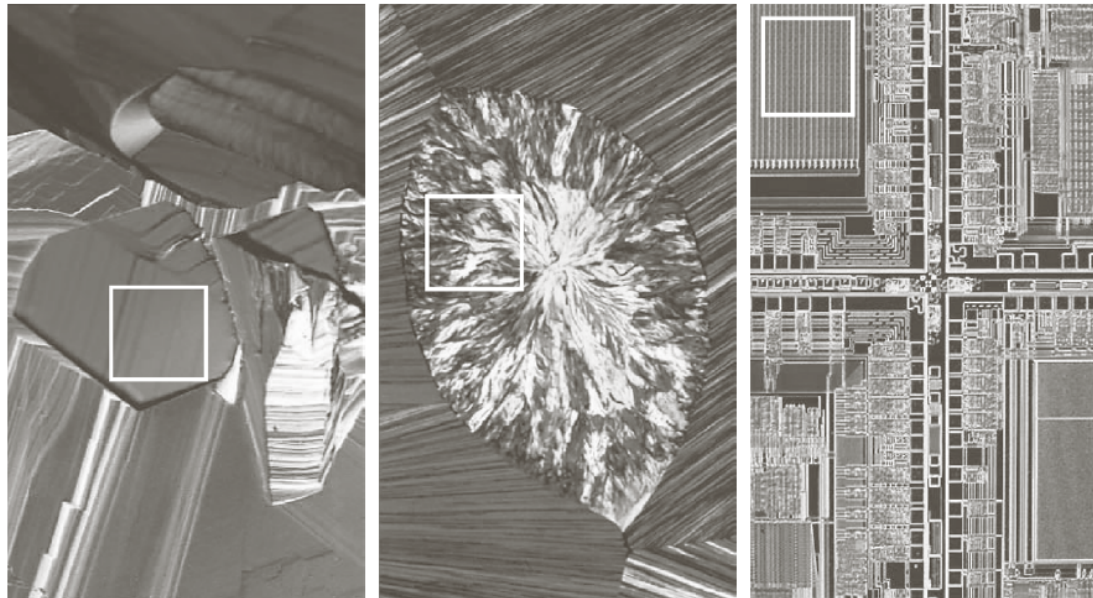


FIGURE 11.26 A region containing a polygonal network.

Texture



a b c

FIGURE 11.28

The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Texture

- An important approach to region description is to quantify its texture content
- This descriptor provides measures of smoothness, coarseness and regularity
- 3 approaches in describing the texture of a region: statistical, structural, and spectral.

Texture (statistical)

- One of the simplest approaches for describing texture is to use statistical moments of the histogram of an image or a region

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

- $\sigma^2(z) = \mu_2(z)$ is a measure of contrast

$$R = 1 - \frac{1}{1 + \sigma^2(z)}$$

Texture (statistical)

- Third moment is a measure of skewness of histogram

$$\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$$

- Measure of uniformity: U is maximum for an image in which all gray levels are equal

$$U = \sum_{i=0}^{L-1} p^2(z_i)$$

- Average entropy: a measure of variability and is zero for constant image

$$e = - \sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

Texture (statistical)

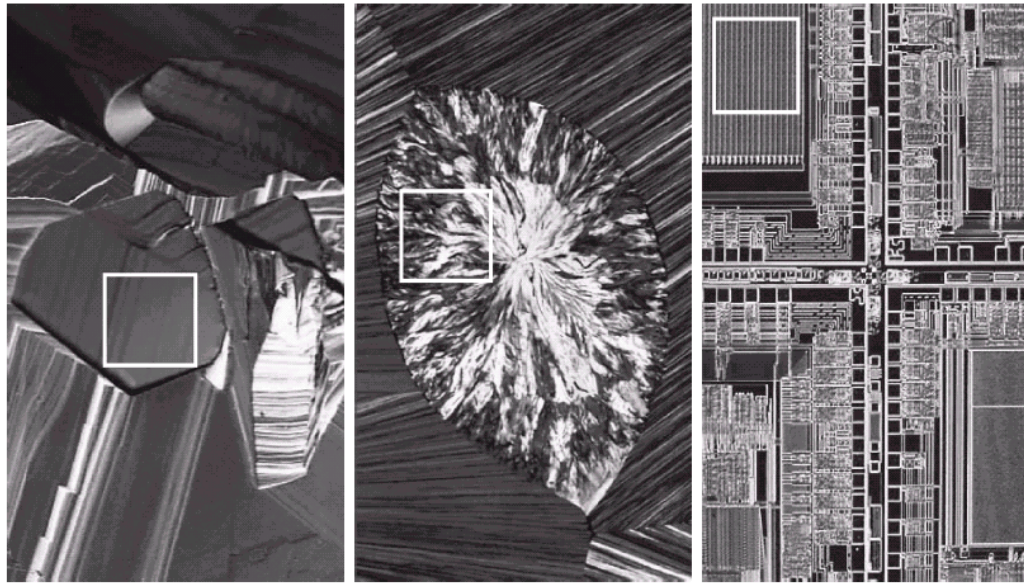


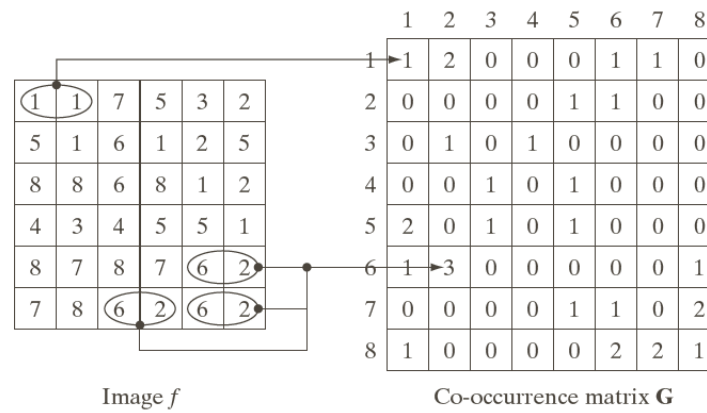
TABLE 11.2
Texture measures
for the subimages
shown in
Fig. 11.22.

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	-0.105	0.026	5.434
Coarse	143.56	74.63	0.079	-0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

Texture (statistical)

- Measures of texture computed using histogram carry no information regarding the relative position of pixels with respect to each other
- Let Q be a position operator and G a $k \times k$ matrix whose element a_{ij} is the number of times that points with gray level z_i occur (in position specified by Q) relative to points with gray level z_j .

Q: one pixel to the right



Texture (statistical)

- Let n be the total number of point pairs in the image that satisfy Q ($n=30$ in the previous example)
- $C=G/n$: gray-level co-occurrence matrix
- C depends on Q
- C is analyzed to categorize texture over which C was computed

$$\begin{aligned} & \max_{i,j}(c_{ij}) \\ & \sum_i \sum_j (i-j)^k c_{ij} \\ & \sum_i \sum_j c_{ij} / (i-j)^k \\ & \sum_i \sum_j c_{ij}^2 \\ & - \sum_i \sum_j c_{ij} \log(c_{ij}) \end{aligned}$$

Texture (structural)

- A simple texture primitive (texture elements) can be used to form more complex texture patterns by means of some rules
- Exp: $S \rightarrow aA$
 - S, A : variables (symbols or primitives)
 - a : some operation for example “put a circle to the right”
 - Rule $S \rightarrow aA$ says that starting point can be replaced by a circle to the right and a variable
- Exp:
 1. $S \rightarrow aA$
 2. $A \rightarrow bA$,
 3. $A \rightarrow cA$,
 4. $A \rightarrow a$,
 5. $A \rightarrow aA$,
 - a : circle right
 - b : circle down
 - c : circle left

155233254 is a 3x3 square of circles

Texture (spectral)

- Three features of Fourier spectrum are useful in texture description:
 1. Peaks in spectrum give the principle direction of texture pattern
 2. Location of the peaks in the frequency plane gives spatial period of the pattern
 3. Eliminating periodic components via filtering leaves non-periodic image elements that can be described by statistical techniques
- Spectrum is sometimes considered in polar coordinates:

$$S(r, \theta)$$

$$S(r) = \sum_{\theta} S(r, \theta)$$

$$S(\theta) = \sum_r S(r, \theta)$$

Texture (spectral)

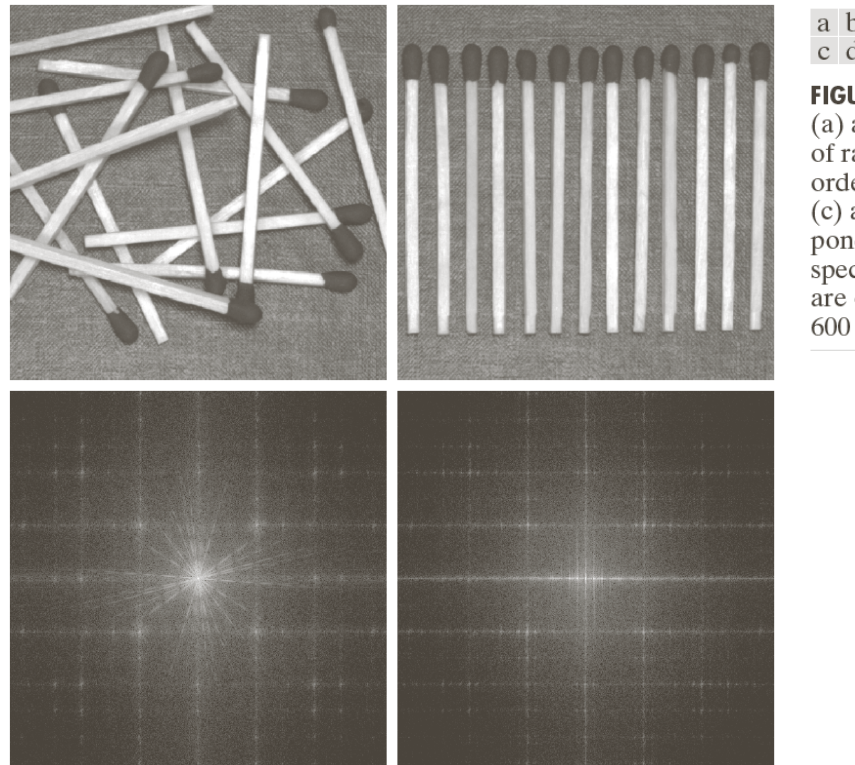
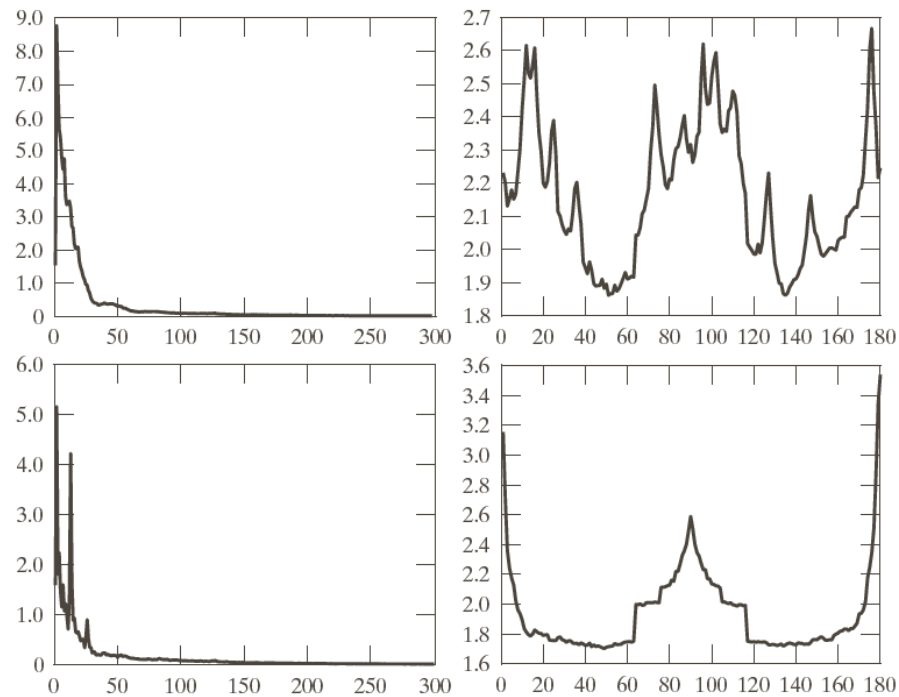


FIGURE 11.35
(a) and (b) Images
of random and
ordered objects.
(c) and (d) Corres-
ponding Fourier
spectra. All images
are of size
 600×600 pixels.

Texture (spectral)



a	b
c	d

FIGURE 11.36
Plots of (a) $S(r)$
and (b) $S(\theta)$ for
Fig. 11.35(a).
(c) and (d) are
plots of $S(r)$ and
 $S(\theta)$ for Fig.
11.35(b). All
vertical axes are
 $\times 10^5$.

Moments

$$m_{pq} = \sum_{(x,y) \in R} \sum x^p y^q f(x,y) \quad \text{Moment of order } p + q$$

$$\mu_{pq} = \sum_{(x,y) \in R} \sum (x - \bar{x})^p (y - \bar{y})^q f(x,y) \quad \text{Central Moments}$$

$$\bar{x} = \frac{m_{10}}{m_{00}} \quad \bar{y} = \frac{m_{01}}{m_{00}}$$

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \quad \text{Normalized Central Moments}$$

$$\gamma = \frac{p+q}{2} + 1$$

Principle component analysis

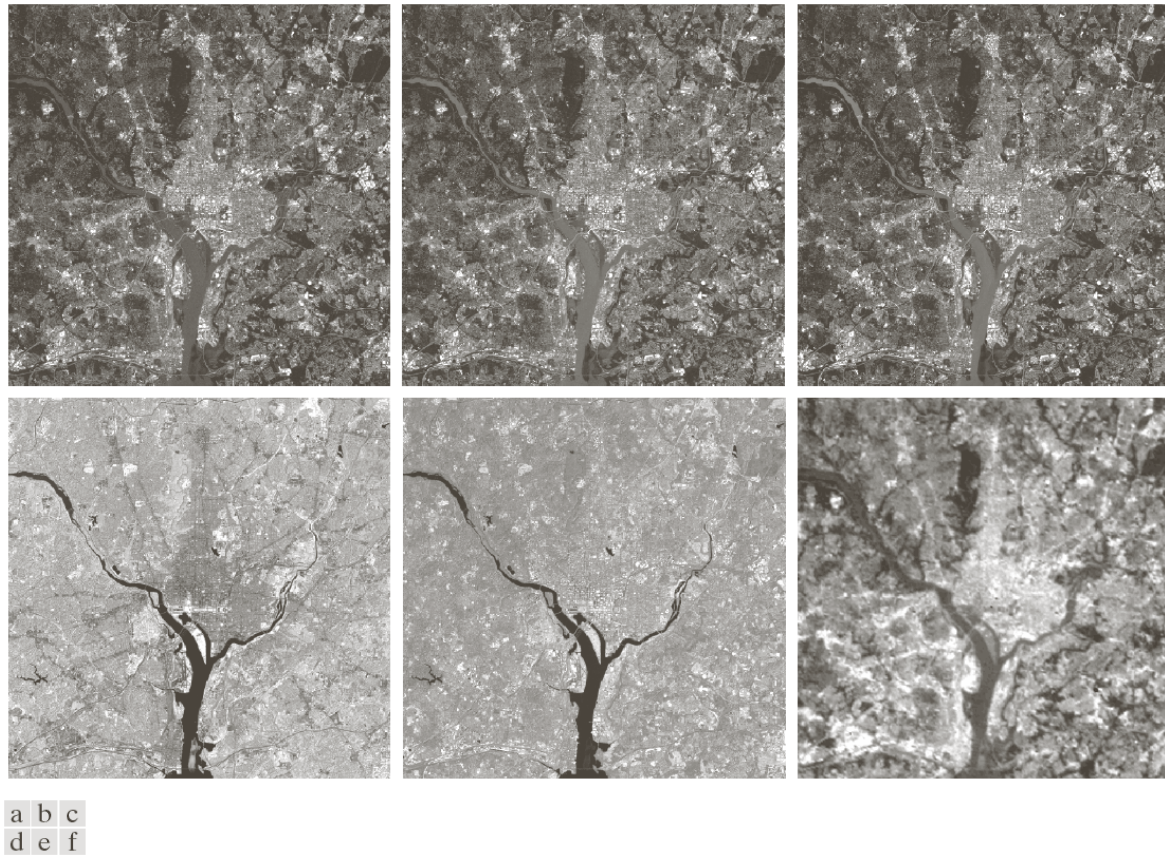
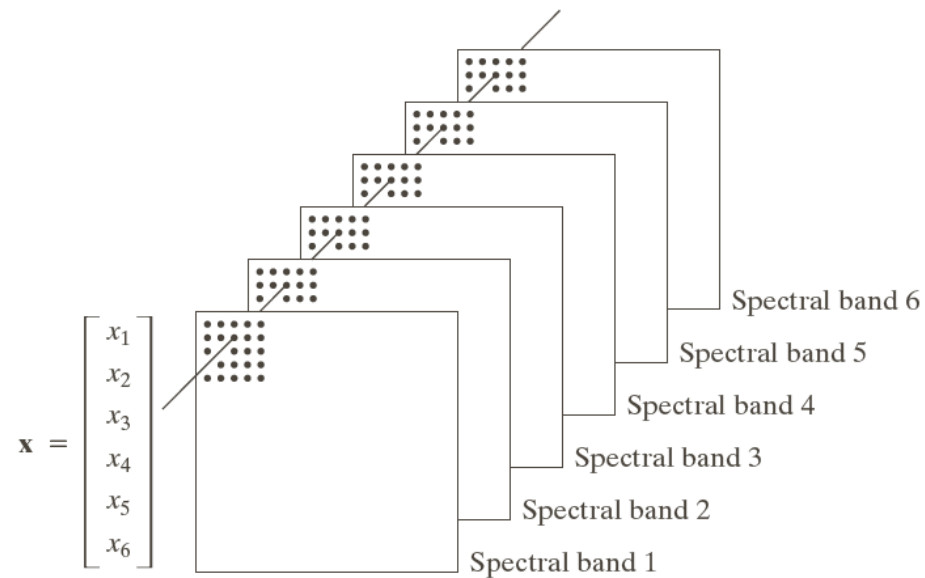


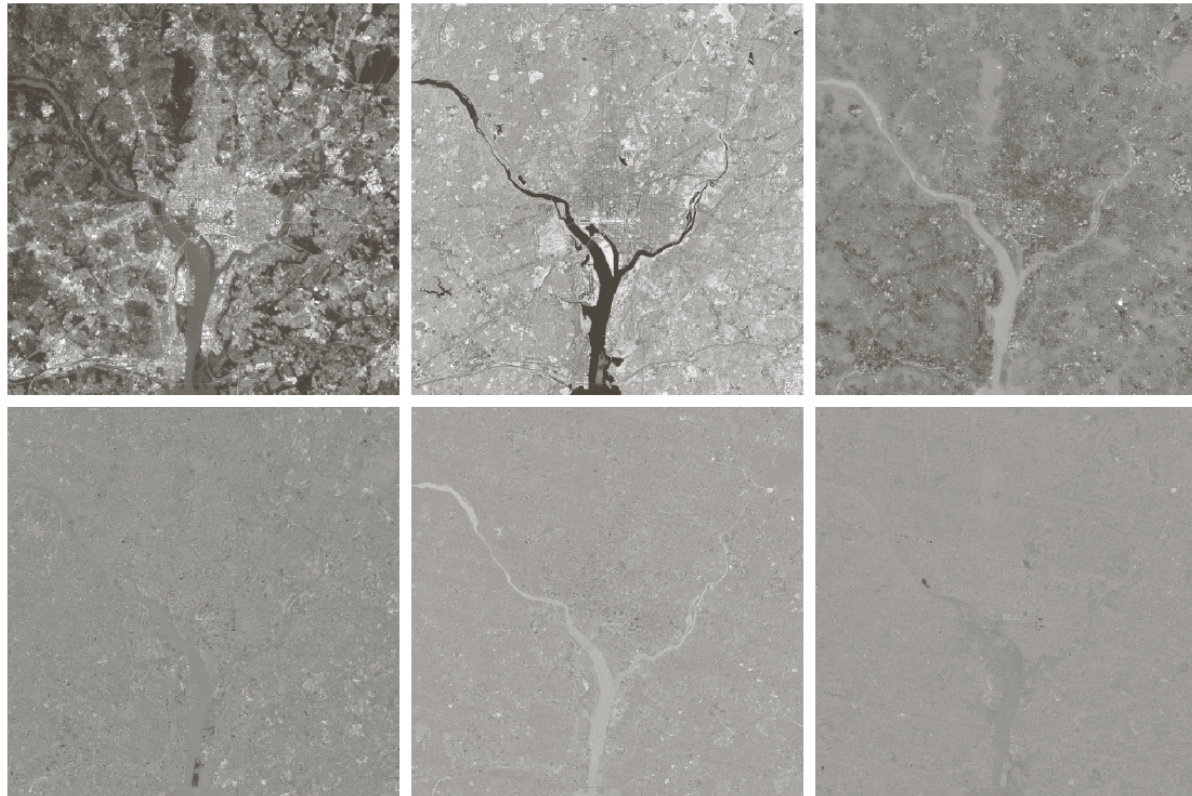
FIGURE 11.38 Multispectral images in the (a) visible blue, (b) visible green, (c) visible red, (d) near infrared, (e) middle infrared, and (f) thermal infrared bands. (Images courtesy of NASA.)

Principle component analysis

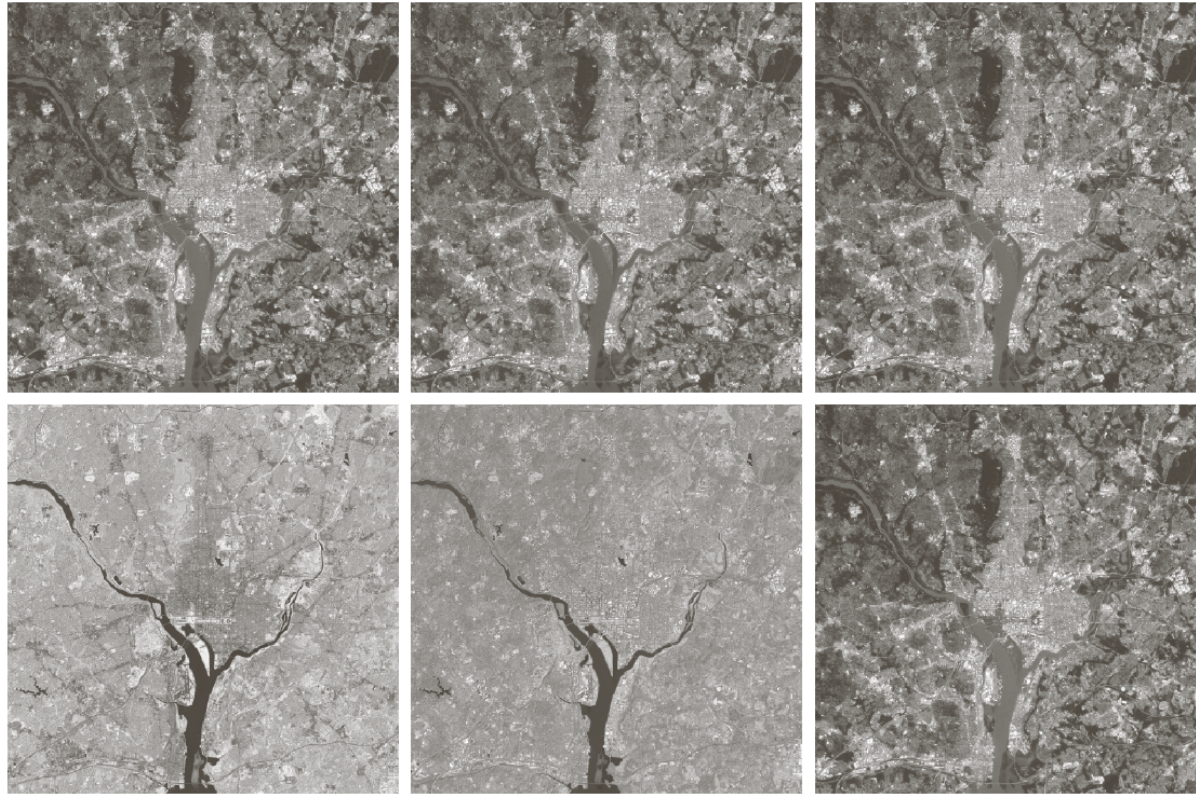


λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
10344	2966	1401	203	94	31

Principle component analysis



Principle component analysis



Principle component analysis

