Two View Geometry
Chapters 9-12

Multiple View Geometry

Two-view geometry

Epipolar geometry

F-matrix comp.

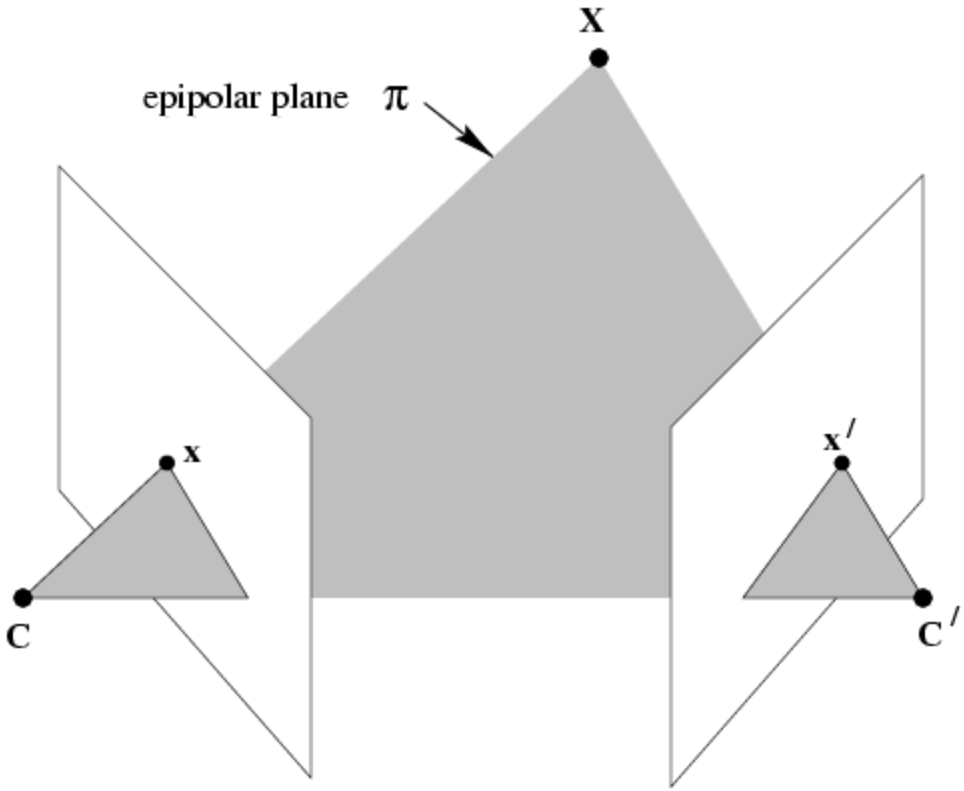
Three questions:

(i) **Correspondence geometry:** Given an image point x in the first view, how does this constrain the position of the corresponding point x' in the second image?

(ii) **Camera geometry:** Given a set of corresponding image points $\{x_i \leftrightarrow x'_i\}$, i=1,…,n, what are the cameras P and P' for the two views?

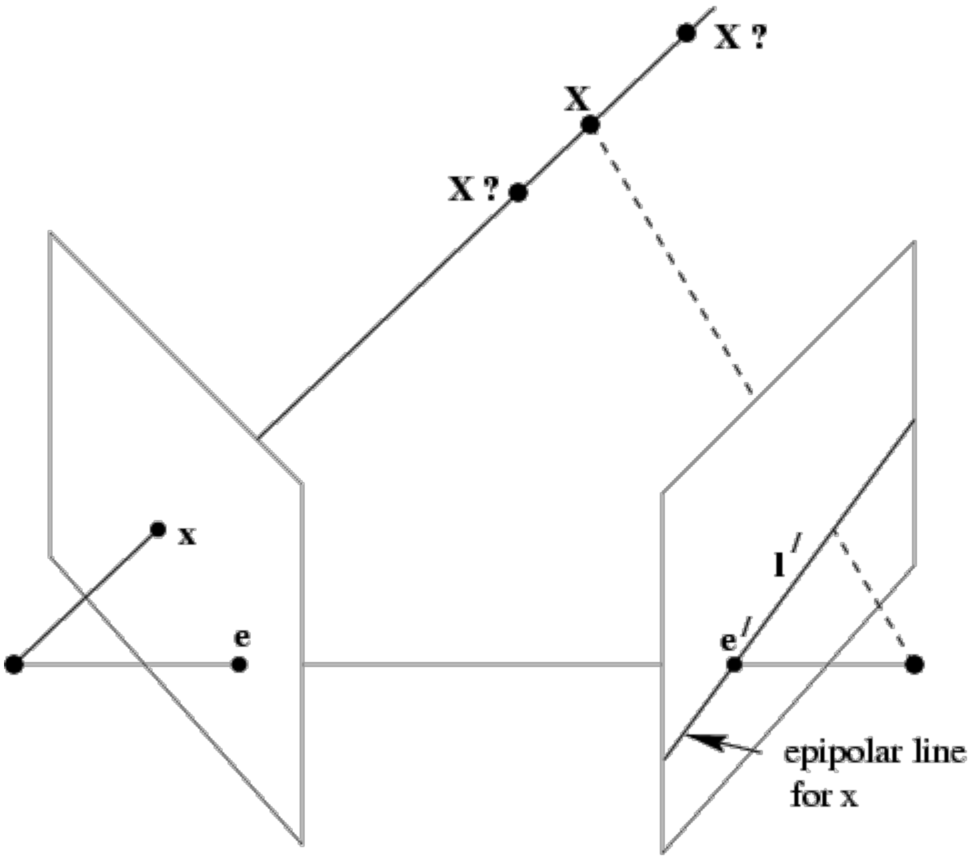
(iii) **Scene geometry (structure):** Given corresponding image points $x_i \leftrightarrow x'_i$ and cameras P, P', what is the position of (their pre-image) X in space?

The epipolar geometry



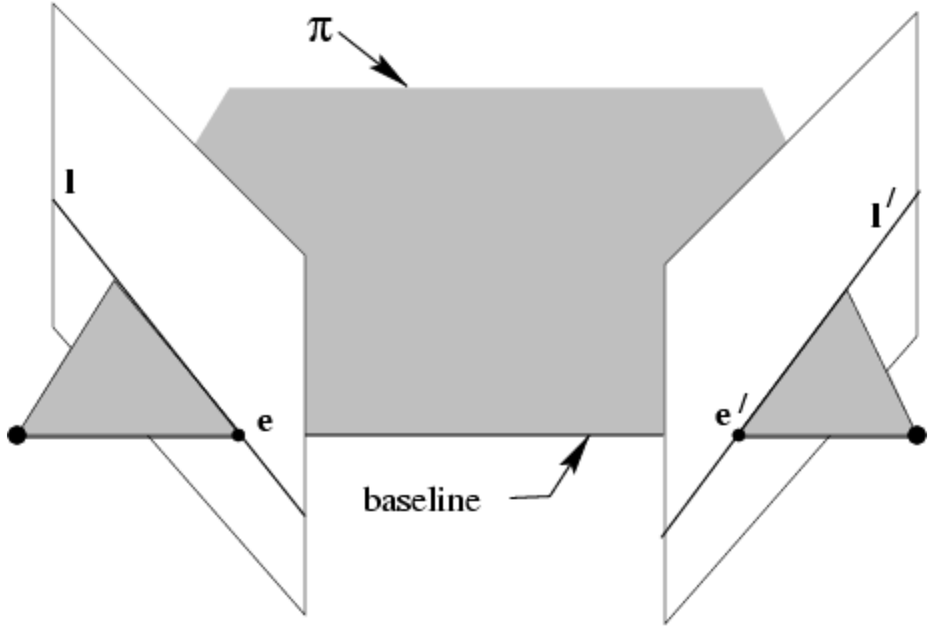
C,C',x,x' and X are coplanar

The epipolar geometry



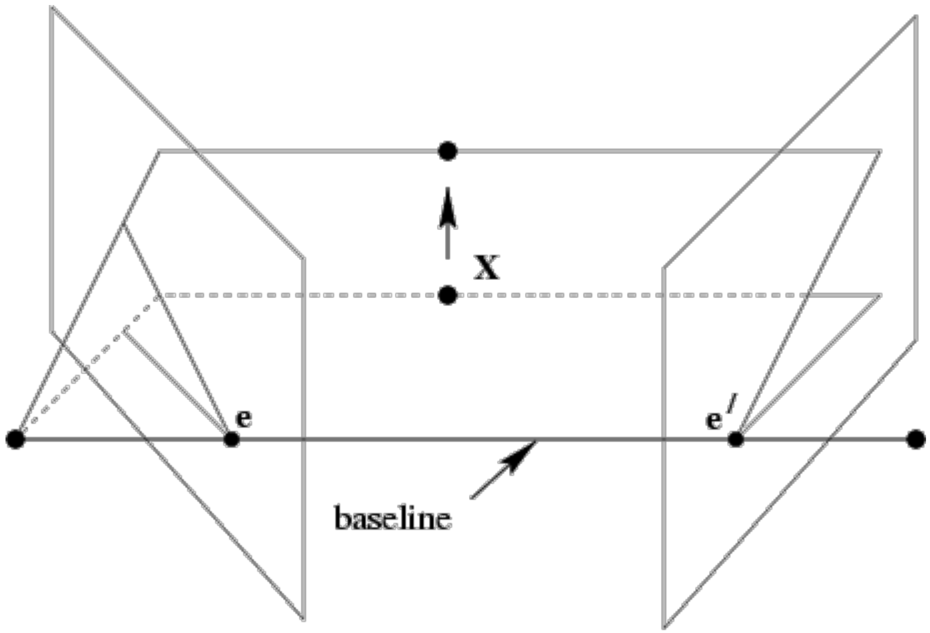
What if only C,C' ,x are known?

The epipolar geometry



All points on **π** project on l and l'

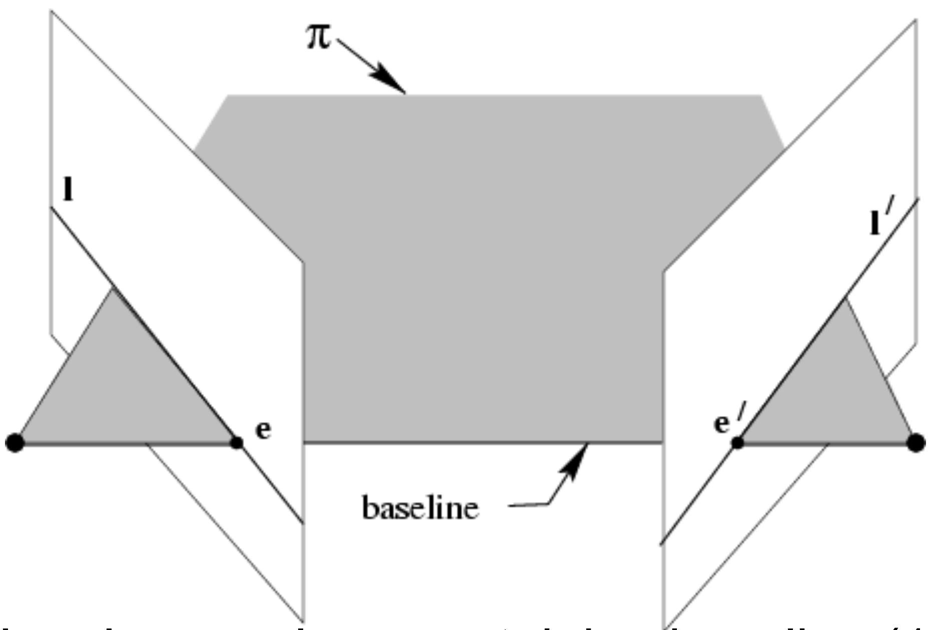
The epipolar geometry



Family of planes **π** and lines l and l'
Intersection in e and e'

The epipolar geometry

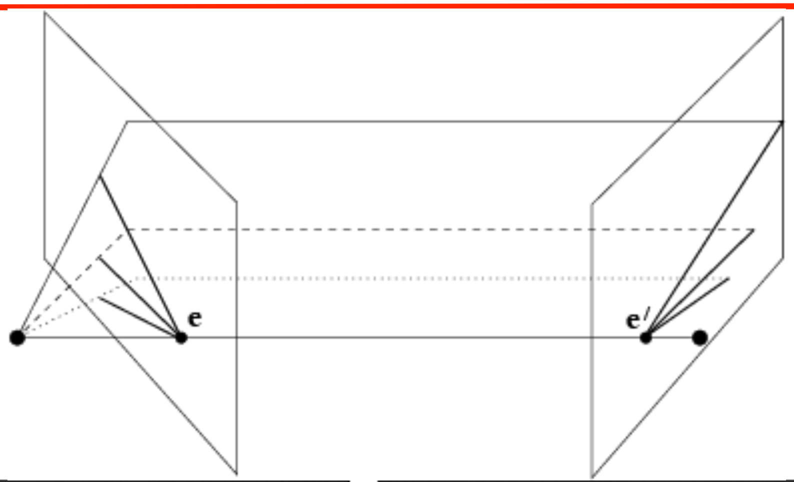
epipoles e,e'
= intersection of baseline with image plane
= projection of projection center in other image
= vanishing point of camera motion direction

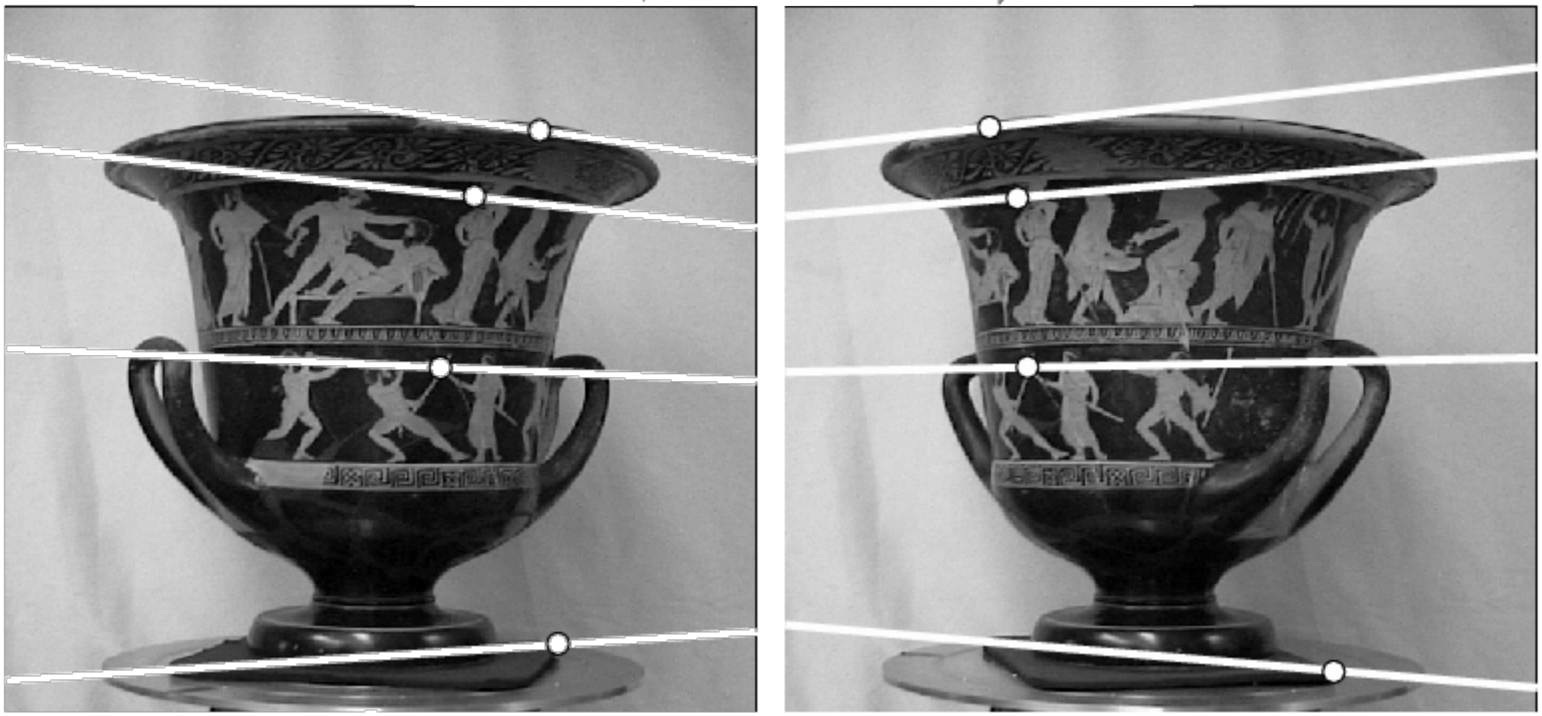


an epipolar plane = plane containing baseline (1-D family)

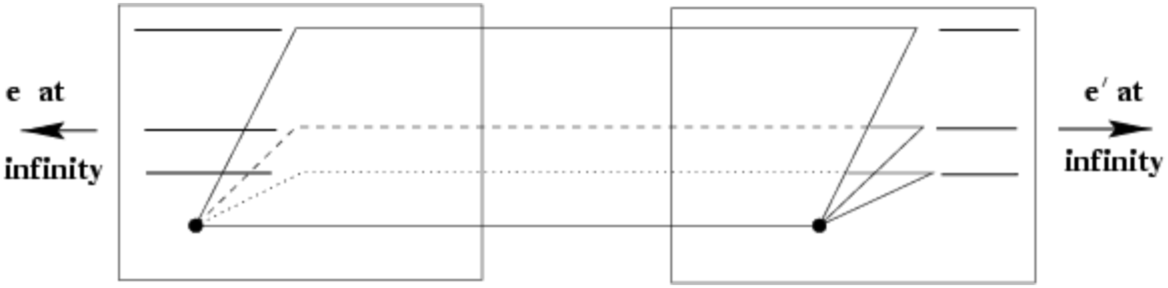
an epipolar line = intersection of epipolar plane with image
(always come in corresponding pairs)

Example: converging cameras





Example: parallel image plane



The fundamental matrix F

Algebraic representation of epipolar geometry

$$x \mapsto l'$$

we will see that mapping is (singular) correlation
(i.e. projective mapping from points to lines)
represented by the fundamental matrix F

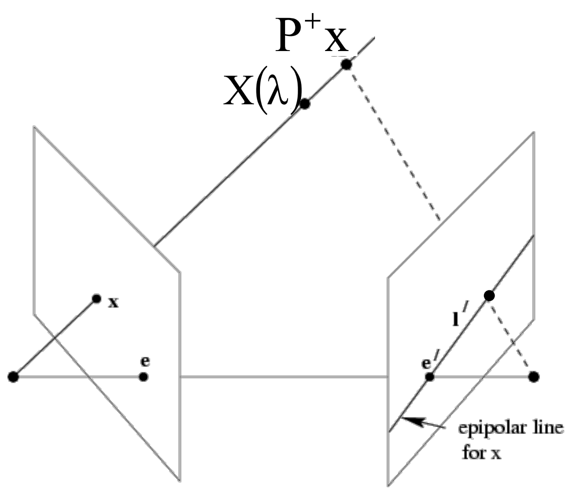
The fundamental matrix F

algebraic derivation

$$X(\lambda) = P^{+}x + \lambda C \qquad \left(P^{+}P = I\right)$$

$$l = P'C \times P'P^{+}x$$

$$F = [e']_{\times} P'P^{+}$$



The fundamental matrix F

correspondence condition

The fundamental matrix satisfies the condition that for any pair of corresponding points $x \leftrightarrow x'$ in the two images

$$x'^T F x = 0$$

$$\left(x'^T l' = 0\right)$$

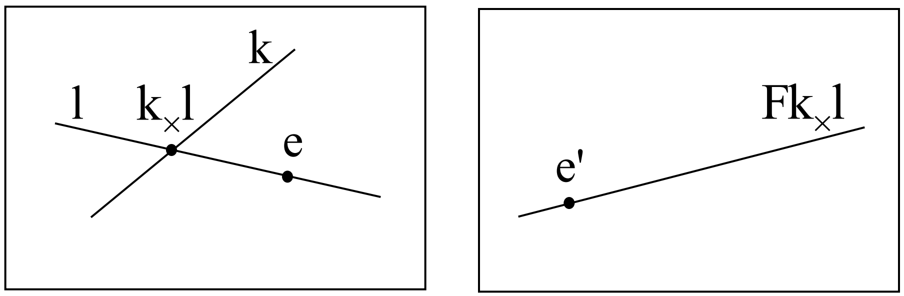
The fundamental matrix F

F is the unique 3x3 rank 2 matrix that satisfies $x'^T F x = 0$ for all $x \leftrightarrow x'$

- **(i) Transpose:** if F is fundamental matrix for (P,P'), then F^T is fundamental matrix for (P',P)
- **(ii) Epipolar lines:** $l' = Fx$ & $l = F^T x'$
- **(iii) Epipoles:** on all epipolar lines, thus $e'^T F x = 0$, for all x thus: $e'^T F = 0$, similarly $Fe = 0$
- **(iv) F** has 7 d.o.f. , i.e. 3x3-1(homogeneous)-1(rank2)
- **(v) F** is a correlation, projective mapping from a point x to a line $l' = Fx$

The epipolar line geometry

l,l' epipolar lines, k line not through e
then $l' = F[k]_x l$ and symmetrically $l = F^T[k']_x l'$



(pick k=e, since $e^T e \neq 0$)

$$l' = F[e]_\times l \qquad l = F^T[e']_\times l'$$

Computing F

Epipolar geometry: basic equation

$$\mathrm{x'}^{\mathrm{T}}\mathrm{Fx} = 0$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

separate known from unknown

$$[x'x, x'y, x', y'x, y'y, y', x, y, 1][f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}]^{\mathrm{T}} = 0$$

(data) (unknowns)

(linear)

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \mathrm{f} = 0$$

$$\mathrm{Af} = 0$$

the singularity constraint

$$\mathbf{e'}^{T}\mathbf{F} = 0 \qquad \mathbf{F}\mathbf{e} = 0 \qquad \det\mathbf{F} = 0 \qquad \operatorname{rank}\mathbf{F} = 2$$

SVD from linearly computed F matrix (rank 3)

$$\mathbf{F} = \mathbf{U}\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix}\mathbf{V}^{T} = \mathbf{U}_1\sigma_1\mathbf{V}_1^{T} + \mathbf{U}_2\sigma_2\mathbf{V}_2^{T} + \mathbf{U}_3\sigma_3\mathbf{V}_3^{T}$$

Compute closest rank-2 approximation $\min \left\| \mathbf{F} - \mathbf{F}' \right\|_F$

$$\mathbf{F}' = \mathbf{U}\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix}\mathbf{V}^{T} = \mathbf{U}_1\sigma_1\mathbf{V}_1^{T} + \mathbf{U}_2\sigma_2\mathbf{V}_2^{T}$$

the minimum case – 7 point correspondences

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_7 x_7 & x'_7 y_7 & x'_7 & y'_7 x_7 & y'_7 y_7 & y'_7 & x_7 & y_7 & 1 \end{bmatrix} \mathrm{f} = 0$$

$$\mathrm{A} = \mathrm{U}_{7\mathrm{x}7} \mathrm{diag}(\sigma_1, \ldots, \sigma_7, 0, 0) \mathrm{V}_{9\mathrm{x}9}{}^{\mathrm{T}}$$

$$\Rightarrow \mathrm{A}[\mathrm{V}_8 \mathrm{V}_9] = 0_{9\mathrm{x}2} \qquad \left(\mathrm{e.g.}\, \mathrm{V}^{\mathrm{T}} \mathrm{V}_8 = [000000010\,]^{\mathrm{T}}\right)$$

$$\mathrm{x}_i{}^{\mathrm{T}} (\mathrm{F}_1 + \lambda \mathrm{F}_2) \mathrm{x}_i = 0, \forall i = 1 \ldots 7$$

one parameter family of solutions

but F_1+lF_2 not automatically rank 2

$$\det(\mathrm{F}_1 + \lambda \mathrm{F}_2) = a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 = 0 \quad \text{(cubic equation)}$$

the NOT normalized 8-point algorithm

$$\begin{bmatrix} x_1x_1' & y_1x_1' & x_1' & x_1y_1' & y_1y_1' & y_1' & x_1 & y_1 & 1 \\ x_2x_2' & y_2x_2' & x_2' & x_2y_2' & y_2y_2' & y_2' & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx_n' & y_nx_n' & x_n' & x_ny_n' & y_ny_n' & y_n' & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

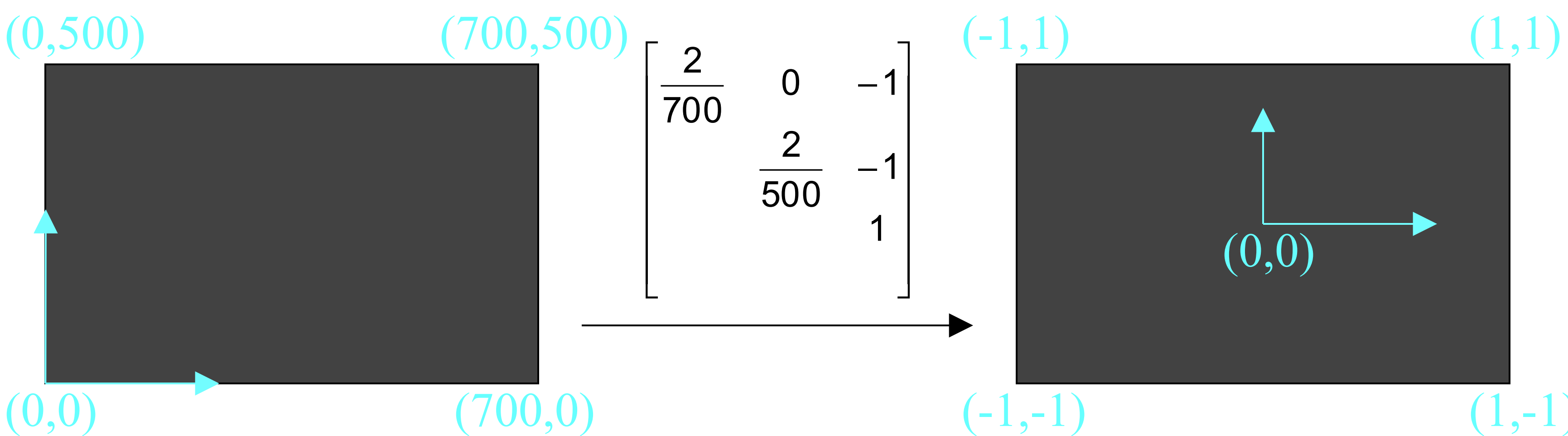
~10000 ~10000 ~100 ~10000 ~10000 ~100 ~100 ~100 1

!

Orders of magnitude difference
Between column of data matrix
Therefore, least-squares yields poor results

the normalized 8-point algorithm

Transform image to ~[-1,1]x[-1,1]



Least squares yields good results (Hartley, PAMI´97)

algebraic minimization

possible to iteratively minimize algebraic distance subject to det F=0 (see book if interested)

Gold standard

Maximum Likelihood Estimation (= least-squares for Gaussian noise)

$$\sum_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i)^2 + d(\mathrm{x}'_i, \hat{\mathrm{x}}'_i)^2 \quad \text{subject to} \quad \hat{\mathrm{x}}'^{\mathrm{T}} \mathrm{F} \hat{\mathrm{x}} = 0$$

Initialize: normalized 8-point, (P,P‘) from F, reconstruct X_i

Parameterize:

$$\mathrm{P} = [\mathrm{I} \mid 0], \mathrm{P}' = [\mathrm{M} \mid \mathrm{t}], \mathrm{X}_i$$

$$\hat{\mathrm{x}}_i = \mathrm{P}\mathrm{X}_i, \hat{\mathrm{x}}_i = \mathrm{P}'\mathrm{X}_i$$

(overparametrized)

Minimize cost using Levenberg-Marquardt

Automatic computation of F

(i) Interest points
(ii) Putative correspondences
(iii) RANSAC
(iv) Non-linear re-estimation of F

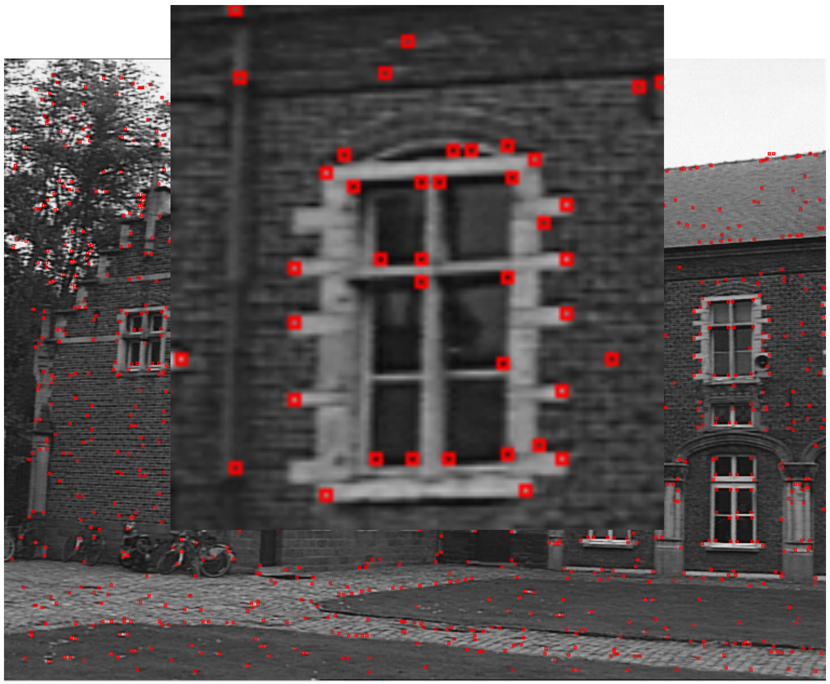
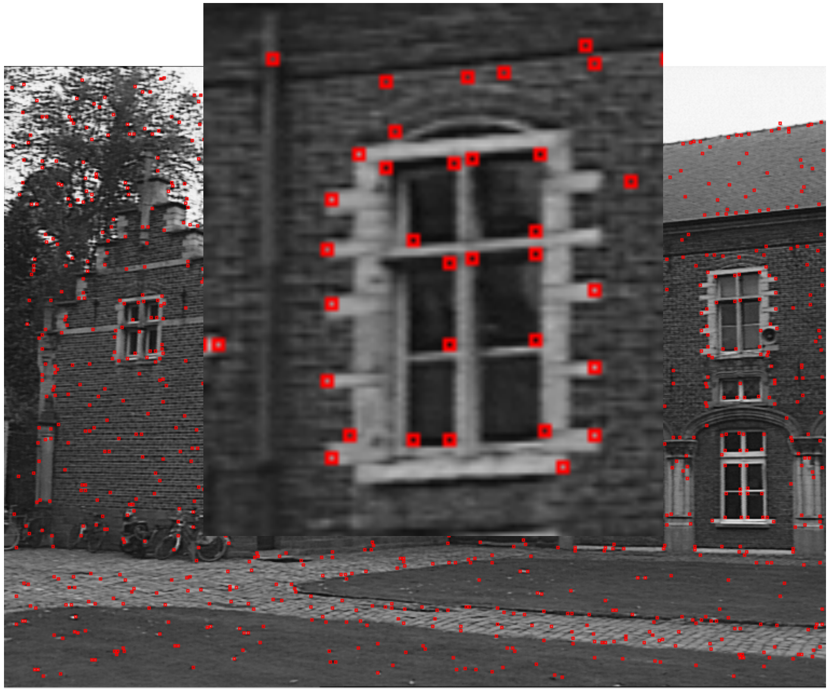
Feature points

- Extract feature points to relate images
- Required properties:
  - Well-defined

    (i.e. neigboring points should all be different)
  - Stable across views

    (i.e. same 3D point should be extracted as feature for neighboring viewpoints)

Feature points

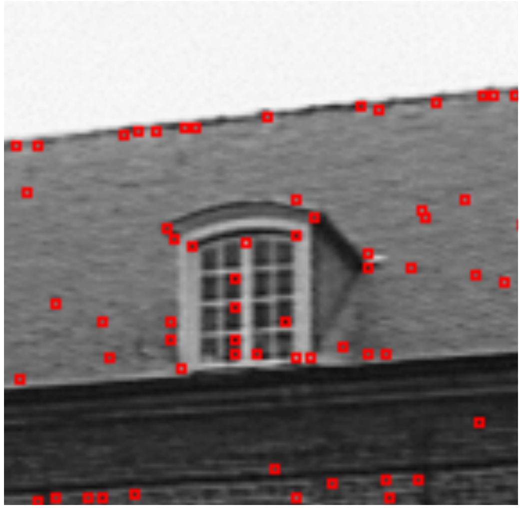
 

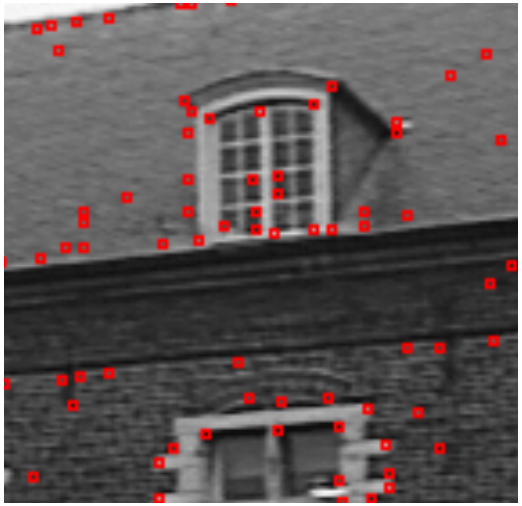
Select strongest features **(e.g. 1000/image)**

Feature matching

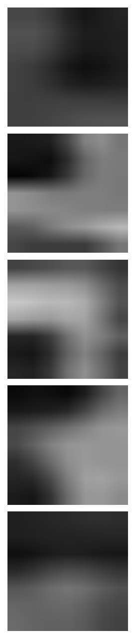
Evaluate NCC for all features with similar coordinates

$$\text{e.g.}\ (x', y') \in \left[x - \tfrac{w}{10}, x + \tfrac{w}{10}\right] \times \left[y - \tfrac{h}{10}, y + \tfrac{h}{10}\right]$$





Feature example



0.96	-0.40	-0.16	-0.39	0.19
-0.05	**0.75**	-0.47	0.51	0.72
-0.18	-0.39	**0.73**	0.15	-0.75
-0.27	0.49	0.16	**0.79**	0.21
0.08	0.50	-0.45	0.28	**0.99**

Gives satisfying results
for small image motions

RANSAC

Step 1. Extract features

Step 2. Compute a set of potential matches

Step 3. do

- Step 3.1 select minimal sample (i.e. 7 matches)
- Step 3.2 compute solution(s) for F

} (generate hypothesis)

- Step 3.3 determine inliers (verify hypothesis)

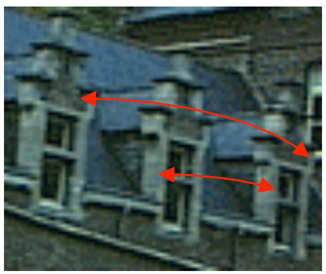
Step 4. Compute F based on all inliers

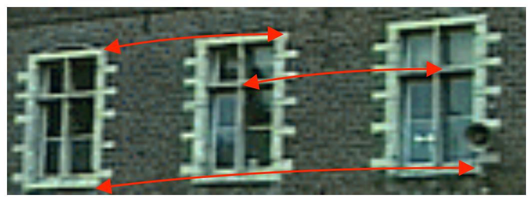
Step 5. Look for additional matches

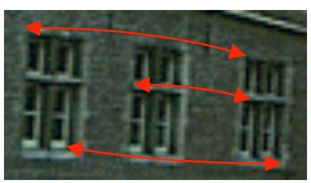
Step 6. Refine F based on all correct matches

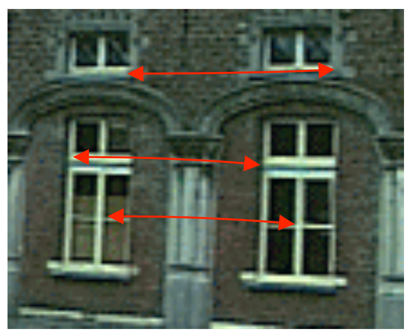
More problems:

- Absence of sufficient features (no texture)
- Repeated structure ambiguity









linear triangulation

$$\mathrm{x} = \mathrm{P}\mathrm{X} \qquad \mathrm{x}' = \mathrm{P}'\mathrm{X}$$

$$\mathrm{x} \times \mathrm{P}\mathrm{X} = 0$$

$$x\left(\mathrm{p}^{3\mathrm{T}}\mathrm{X}\right) - \left(\mathrm{p}^{1\mathrm{T}}\mathrm{X}\right) = 0$$
$$y\left(\mathrm{p}^{3\mathrm{T}}\mathrm{X}\right) - \left(\mathrm{p}^{2\mathrm{T}}\mathrm{X}\right) = 0$$
$$x\left(\mathrm{p}^{2\mathrm{T}}\mathrm{X}\right) - y\left(\mathrm{p}^{1\mathrm{T}}\mathrm{X}\right) = 0$$

$$\mathrm{A}\mathrm{X} = 0$$

$$\mathrm{A} = \begin{bmatrix} x\mathrm{p}^{3\mathrm{T}} - \mathrm{p}^{1\mathrm{T}} \\ y\mathrm{p}^{3\mathrm{T}} - \mathrm{p}^{2\mathrm{T}} \\ x'\mathrm{p}'^{3\mathrm{T}} - \mathrm{p}'^{1\mathrm{T}} \\ y'\mathrm{p}'^{3\mathrm{T}} - \mathrm{p}'^{2\mathrm{T}} \end{bmatrix}$$

homogeneous

$$\|\mathrm{X}\| = 1$$

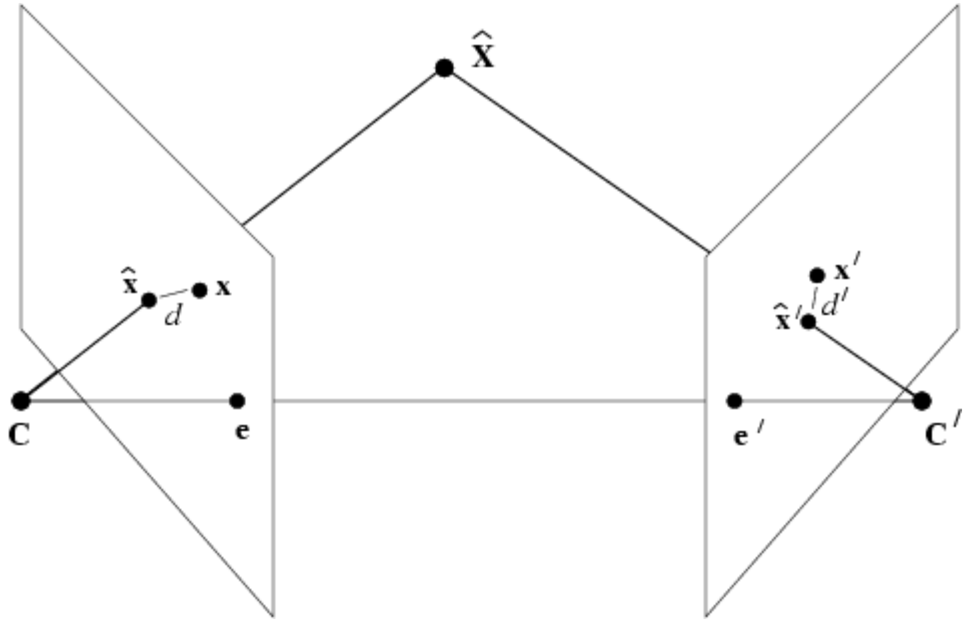
inhomogeneous

$$(X, Y, Z, 1)$$

geometric error

$$d(\mathrm{x},\hat{\mathrm{x}})^2 + d(\mathrm{x}',\hat{\mathrm{x}}')^2 \text{ subject to } \hat{\mathrm{x}}'^{\mathrm{T}}\mathrm{F}\hat{\mathrm{x}} = 0$$

$$\text{or equivalently subject to } \hat{\mathrm{x}} = \mathrm{P}\hat{\mathrm{X}} \text{ and } \hat{\mathrm{x}}' = \mathrm{P}'\hat{\mathrm{X}}$$



possibility to compute using LM (for 2 or more points)

or directly (for 2 points)