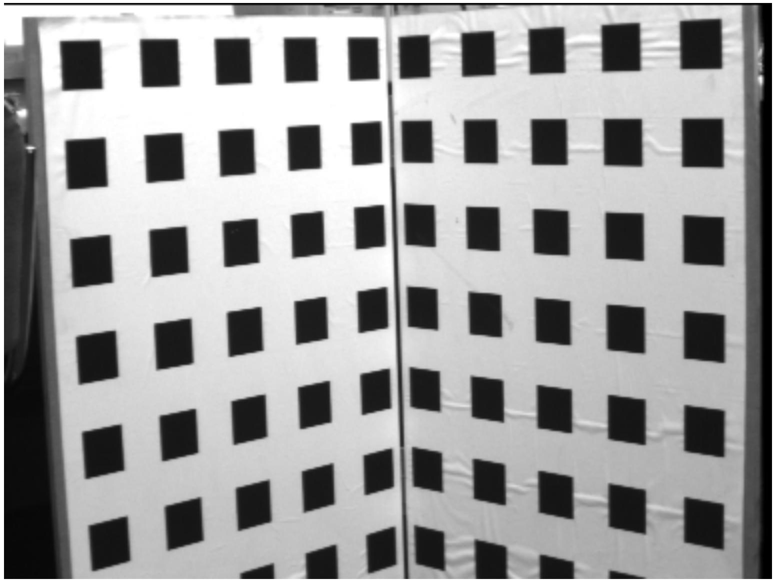
Camera Calibration
Chapter 7

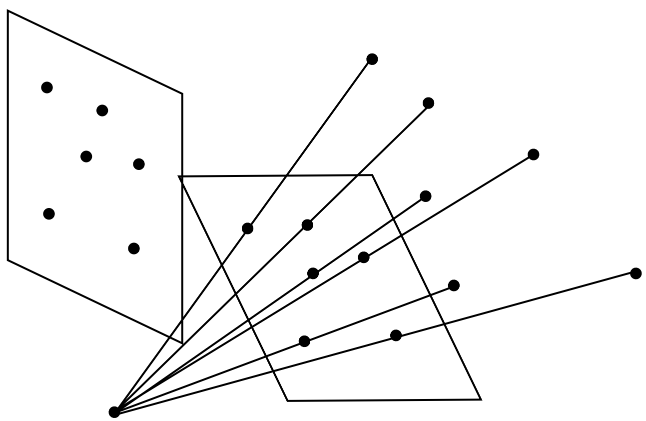
Multiple View Geometry

Camera calibration



Resectioning

$$X_i \leftrightarrow x_i \qquad P\,?$$



Basic equations

$$\mathrm{x}_i = \mathrm{P}\mathrm{X}_i$$

$$\mathbf{x_i} \times \mathbf{P}\mathbf{X_i} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \\ -y_i\mathbf{X}_i{}^\top & x_i\mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\mathrm{A}\mathrm{p} = 0$$

Basic equations

$$\mathrm{Ap} = 0$$

minimal solution

P has 11 dof, 2 independent eq./points

5½ correspondences needed (say 6)

Over-determined solution

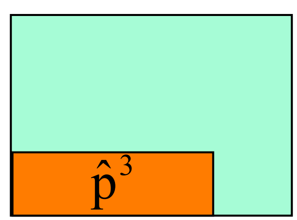
$n > 6$ points

minimize $\|\mathrm{Ap}\|$ subject to constraint

$$\|\mathrm{p}\| = 1$$

$$\|\hat{\mathrm{p}}^3\| = 1$$

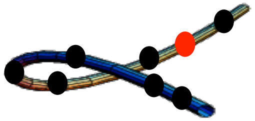
$\mathrm{P} =$



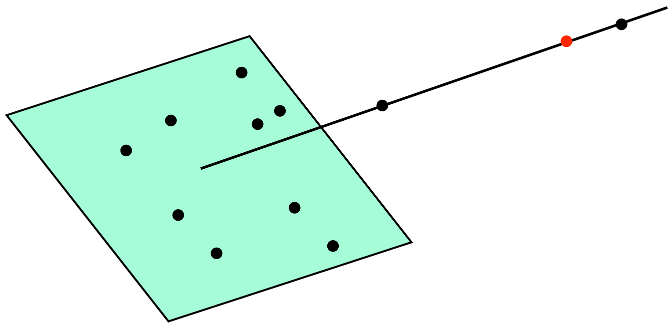
Degenerate configurations

More complicate than 2D case

(i) Camera and points on a twisted cubic



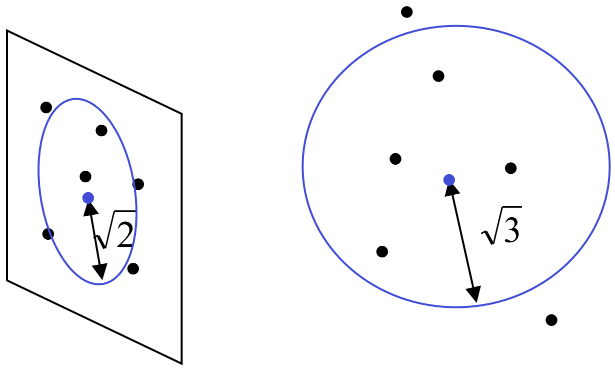
(ii) Points lie on plane or single line passing through projection center



Data normalization

Less obvious

(i) Simple, as before



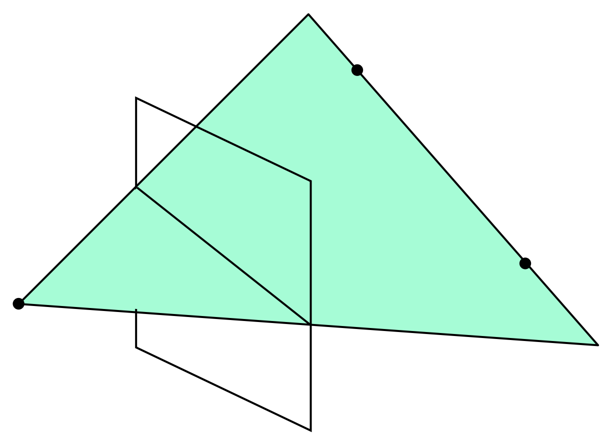
(ii) Anisotropic scaling

Line correspondences

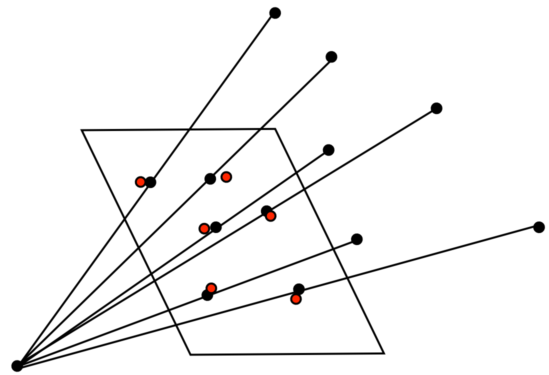
Extend Direct Linear Transformation (DLT) to lines

$$\Pi = P^{T} l_i \quad \text{(back-project line)}$$

$$l_i^{T} P X_{1i} = 0 \quad l_i^{T} P X_{2i} = 0 \quad \text{(2 independent eq.)}$$



Geometric error



$$\sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

$$\min_{\mathrm{P}} \sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2$$

Gold Standard algorithm

Objective

Given $n \geq 6$ 3D to 2D point correspondences $\{X_i \leftrightarrow x_i'\}$, determine the Maximum Likelihood Estimation of P

Algorithm

(i) Linear solution:

(a) Normalization: $\tilde{X}_i = UX_i \quad \tilde{x}_i = Tx_i$

(b) DLT:

(ii) Minimization of geometric error: using the linear estimate as a starting point minimize the geometric error:

$$\min_{\mathrm{P}} \sum_i d(\tilde{\mathbf{x}}_i, \tilde{\mathrm{P}}\tilde{\mathbf{X}}_i)^2$$

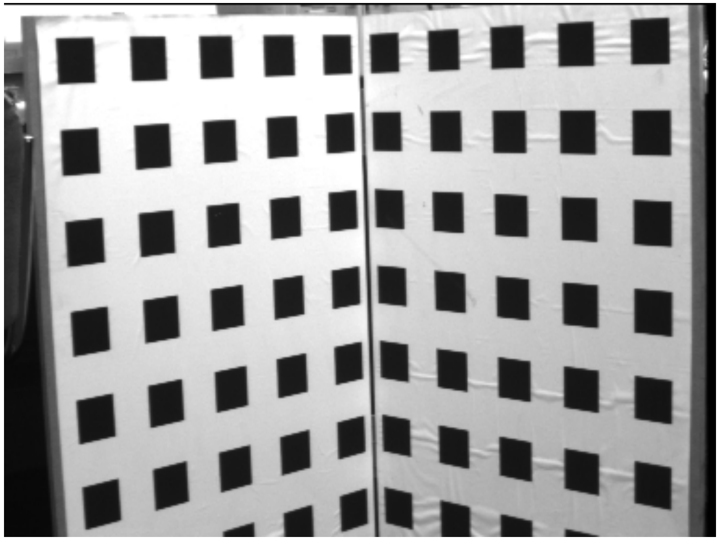
(iii) Denormalization: $P = T^{-1}\tilde{P}U$

Calibration example

(i) Canny edge detection
(ii) Straight line fitting to the detected edges
(iii) Intersecting the lines to obtain the images corners

typically precision <1/10

(HZ rule of thumb: $5n$ constraints for n unknowns



	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

Errors in the world

$$\sum_i d(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

$$\mathrm{x}_i = \mathrm{P}\widehat{\mathrm{X}}_i$$

Errors in the image and in the world

$$\sum_{i=1}^{n} d_{\mathrm{Mah}}(\mathbf{x}_i, \mathrm{P}\widehat{\mathbf{X}}_i)^2 + d_{\mathrm{Mah}}(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

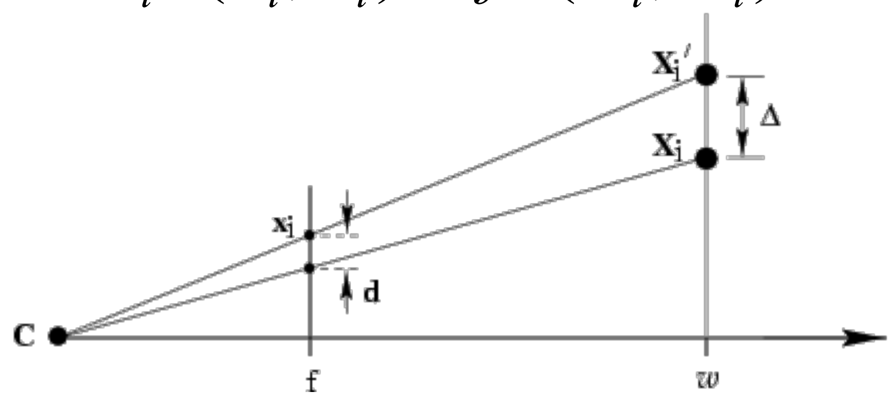
Geometric interpretation of algebraic error

$$\sum_i \left(\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i)\right)^2$$

$$\hat{w}_i\left(\hat{x}_i, \hat{y}_i, 1\right) = \mathrm{PX}_i \qquad \hat{w}_i = \pm\left\|\hat{\mathrm{p}}^3\right\| \mathrm{depth}(\mathrm{X};\mathrm{P})$$

$$\text{therefore, if } \left\|\hat{\mathrm{p}}^3\right\| = 1 \text{ then}$$

$$\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i) \sim f d(\mathrm{X}_i, \hat{\mathrm{X}}_i)$$



note invariance to 2D and 3D similarities given proper normalization

Estimation of affine camera

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$||\mathtt{A}\mathbf{p}||^2 = \sum_i \left(x_i - \mathbf{P}^{1\top}\mathbf{X}_i\right)^2 + \left(y_i - \mathbf{P}^{2\top}\mathbf{X}_i\right)^2 = \sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

note that in this case algebraic error = geometric error

Gold Standard algorithm

Objective

Given n≥4 3D to 2D point correspondences $\{X_i \leftrightarrow x_i'\}$, determine the Maximum Likelihood Estimation of P

(remember $P^{3T}=(0,0,0,1)$)

Algorithm

(i) **Normalization:** $\tilde{X}_i = UX_i \quad \tilde{x}_i = Tx_i$

(ii) For each correspondence

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$A_8 p_8 = b$$

(iii) solution is

$$p_8 = A_8{}^+ b$$

(iv) **Denormalization:** $P = T^{-1}\tilde{P}U$

Restricted camera estimation

Find best fit that satisfies

- skew s is zero
- pixels are square
- principal point is known
- complete camera matrix K is known

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

Minimize geometric error

- impose constraint through parametrization
- Select a set of parameters that characterize the camera matrix

Minimize algebraic error

- assume map from param q to P=K[R|-RC], i.e. p=g(q)
- minimize ||Ag(q)||

Reduced measurement matrix

One only has to work with 12x12 matrix, not 2nx12

$$A^\top A = (VDU^\top)(UDV^\top) = (VD)(DV^\top) = \hat{A}^\top \hat{A}$$

$$\|Ap\|^2 = p^T A^T A p = \|\hat{A} p\|^2$$

Restricted camera estimation

$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

Initialization

- Use general DLT
- Clamp values to desired values, e.g. s=0, $\alpha_x = \alpha_y$

Note: can sometimes cause big jump in error

Alternative initialization

- Use general DLT
- Impose soft constraints

$$\sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2 + ws^2 + w(\alpha_x - \alpha_y)^2$$

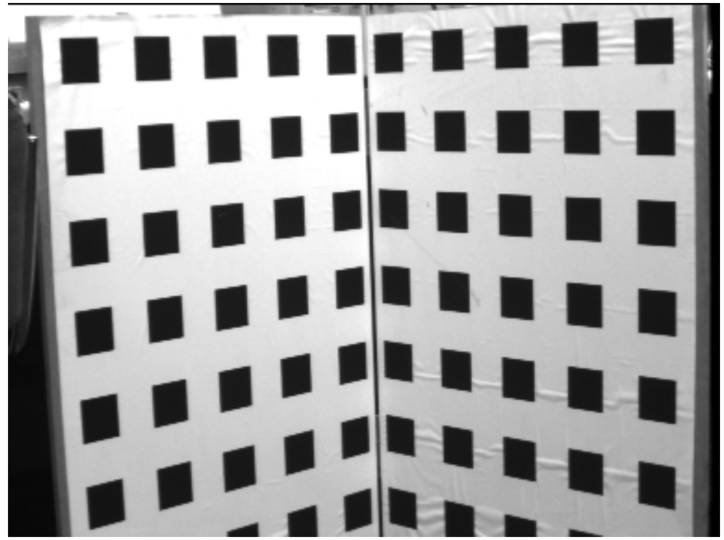
- gradually increase weights

Exterior orientation

Calibrated camera, position and orientation unkown

Pose estimation

6 dof : 3 points minimal (4 solutions in general)



	f_y	f_x/f_y	skew	x_0	y_0	residual
algebraic	1633.4	1.0	0.0	371.21	293.63	0.601
geometric	1637.2	1.0	0.0	371.32	293.69	0.601

	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

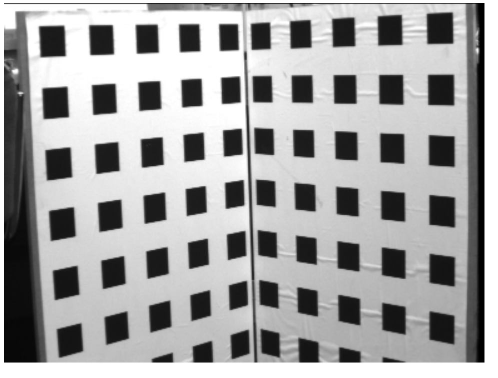
Covariance estimation

ML residual error

$$\epsilon_{\text{res}} = \sigma(1 - d/2n)^{1/2}$$

$$\epsilon_{\text{res}} \longleftrightarrow \sigma$$

Example: n=197, ϵ_{res} =0.365, σ=0.37



Covariance for estimated camera

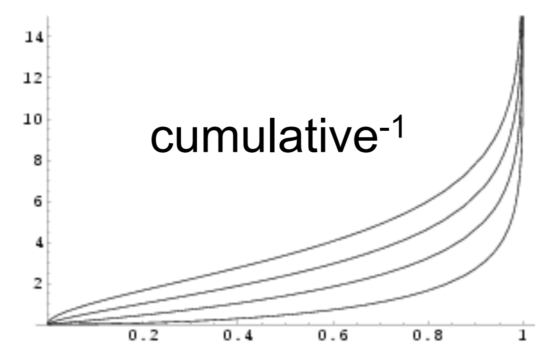
Compute Jacobian at ML solution, then

$$\Sigma_P = \left(J^T \Sigma_x^{-1} J\right)^+$$

(variance per parameter can be found on diagonal)

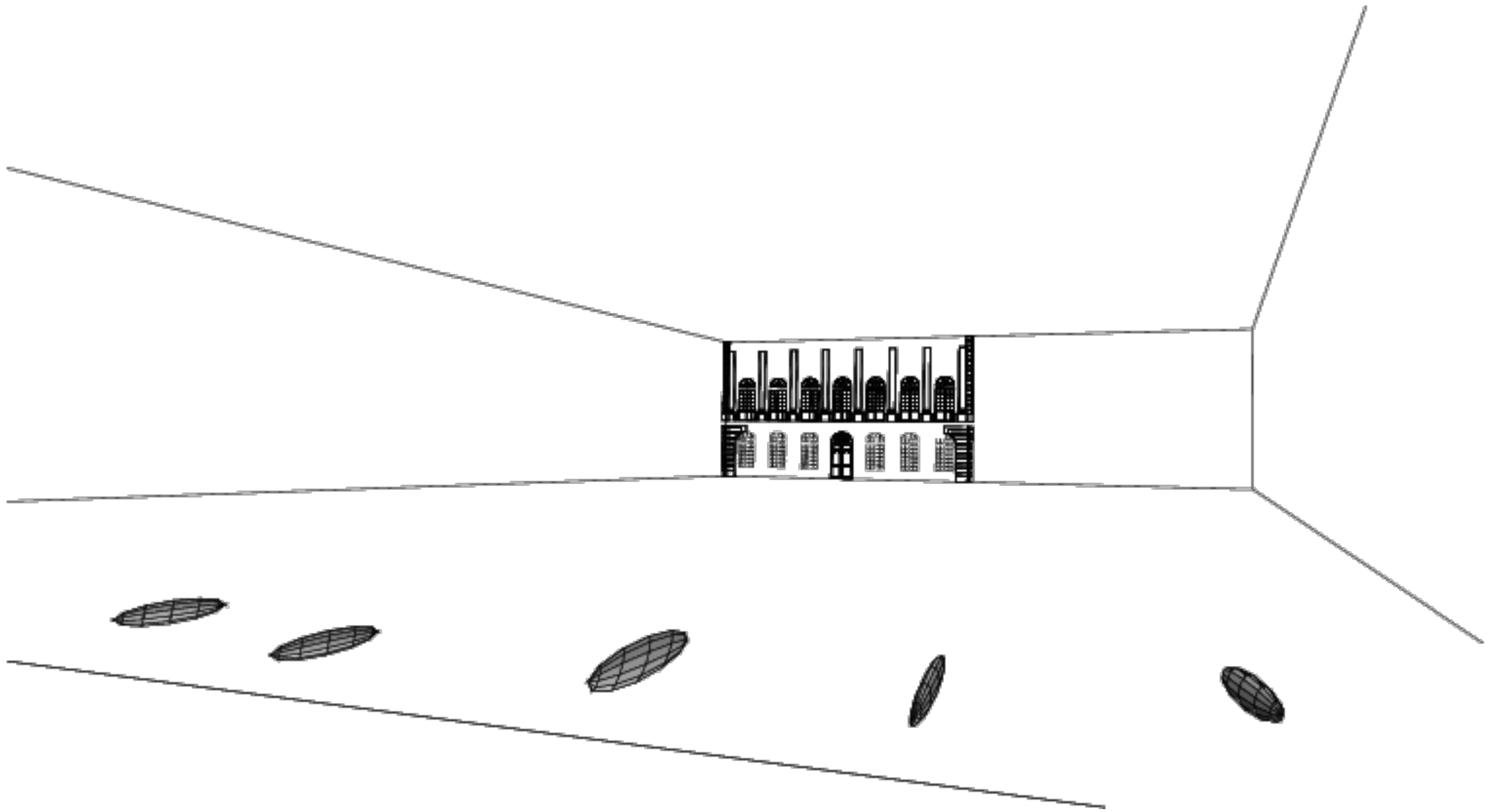
$$(\mathbf{C} - \bar{\mathbf{C}})^\top \Sigma_{\mathbf{C}}^{-1} (\mathbf{C} - \bar{\mathbf{C}}) = k^2$$

k^2



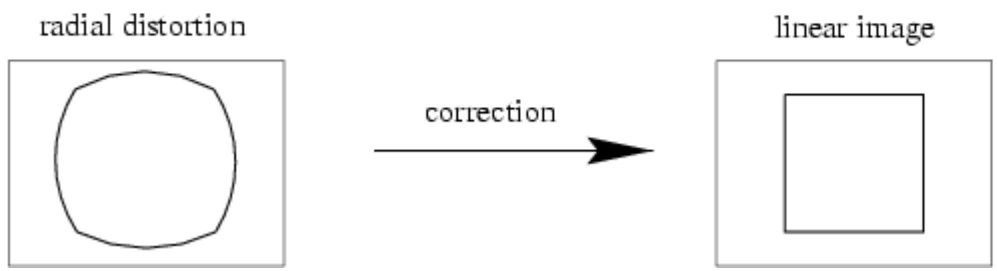
χ^2

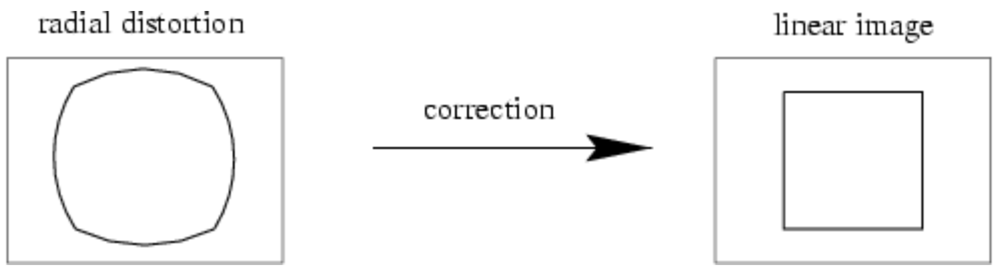
(chi-square distribution
=distribution of sum of squares)



Radial distortion

short and long focal length





$$(\tilde{x}, \tilde{y}, 1)^\top = [\mathtt{I} \mid \mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

$$\begin{pmatrix} x_d \\ y_d \end{pmatrix} = L(\tilde{r}) \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

Correction of distortion

$$\hat{x} = x_c + L(r)(x - x_c) \qquad \hat{y} = y_c + L(r)(y - y_c)$$

Choice of the distortion function and center

$$x = x_o + (x_o - c_x)(K_1 r^2 + K_2 r^4 + \ldots)$$
$$y = y_o + (y_o - c_y)(K_1 r^2 + K_2 r^4 + \ldots)$$

$$r = (x_o - c_x)^2 + (y_o - c_y)^2 \ .$$

Computing the parameters of the distortion function

(i) Minimize with additional unknowns
(ii) Straighten lines
(iii) …