CoE 3SK3 Computer Aided Engineering
Midterm Test
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1. This test is nominally 120 minutes long.
2. A cheat-sheet (letter-size, both sides) and the standard McMaster calculator are allowed.
3. **Return this question sheet with your solutions.**

Problem 1 (15 Points)

Briefly answer the questions below. No justification is required.

(a) Given $x>0$ and $\mathrm{fl}(1+x)=1$, what is the range of x? $\mathrm{fl}(x)$ is the floating point representation of x in a computer.

(b) False-position method and bisection method are the same if the function is linear. True or false?

(c) Secant method works in the same principle as Newton method, but it does not need to compute the derivative of function $f(x)$. True or false?

(d) The 2-norm of a matrix A is the largest eigenvalue of A. True or false?

(e) If a matrix is singular, its condition number is zero. True or false?

(f) Given an eigenvalue of a matrix, the corresponding eigenvector is uniquely determined. True or false?

(g) Pivoting in Gauss elimination aims to speed up the algorithm. True or false?

Problem 2 (15 points)

Use the LU decomposition technique to compute the inverse matrix of

$$\mathbf{B} = \begin{bmatrix} 2 & 3 & -4 \\ 2 & 1 & -5 \\ 1 & -1 & -2 \end{bmatrix}$$

Problem 3 (20 points)

(a) Compute the condition number of the matrix **B** in Problem 2, using 1-norm $\|\mathbf{B}\|_1$.

(b) Compute the condition number of the following matrix **A**, **using 2-norm** $\|\mathbf{A}\|_2$ (the square root of the largest eigenvalue of $\mathbf{AA}'$).

$$\mathbf{A} = \begin{bmatrix} 2 & -2 & -1 \\ -2 & -3 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

(c) When solving the linear systems **AX**=**b** and **BX**=**b** numerically, which system is more sensitive to errors in **b**? Explain your answer.

Problem 4 (30 Points)

Given the function $f(x) = \sin x - 0.5x$ (note x is in radiant NOT degree).

(a) Write an iterative formula to estimate a root x using Newton-Raphson method. Given the initial guess as $x_0=\pi/2$ find a root x using the first 3 iterations and comment on convergence.

(b) Given the initial guess as $x_0=\pi/4$, find a root x using the first 3 iterations and comment on convergence.

(c) Compare the convergence speed and precision of the roots found in (a) and (b), comment on your observations.

(d) Is it possible to locate the root of (b) using the bisection method with the initial values x_l=-0.5 and x_u=1.5? If yes, find the minimum number of iterations required in order to guarantee an absolute error less than 0.0001? If no, why?

Problem 5 (20 Points)

Consider a computer that uses 8 bits to represent floating-point numbers, 1 bit for the sign s, 3 bits for the exponent c ($c = e +3$), and 4 bits for fractional part f. In terms of s, e, and f, the base 10 numbers are given by $x = (-1)^s 2^e (1+f)$, c is non-negative, and $0 \le f < 1$. This is the same design as IEEE floating-point representation with a much shorter machine word length.

(a) What are the smallest and largest positive (non-zero) numbers that can be represented accurately (without any chopping/rounding error) on this computer?

(b) How many numbers can be accurately (without any chopping/rounding error) represented on this computer?

(c) By changing c to 4 bits and f to 3 bits, what do you gain and what do you lose?