1. Employ the following methods to find the maximum of $f(x) = 4x – 1.8x^2 + 1.2x^3 – 0.3x^4$:

(a) Golden-section search ($x_l = -2$, $x_u = 4$, relative error less than 1%)

(b) Quadratic interpolation ($x_0 = 1.75$, $x_1 = 2$, $x_2 = 2.5$, iterations = 4)

(c) Newton's method ($x_0 = 3$, absolute error less than 0.0001)

2. To control an illness, there are three drugs A, B, and C.

- Drugs A, B and C have effectiveness 0.8, 0.6 and 0.3 per dose, respectively;
- Drugs A, B and C have side effect 0.15, 0.05 and 0.1 per dose, respectively;
- Drugs A, B and C cost \$100, \$65 and \$20 per dose, respectively.

Suppose that the effectiveness and side effect of these drugs are additive. Formulate the linear programming problem to compute dosage x of drug A, dosage y of drug B, dosage z of drug C such that

- The total effectiveness is maximized;
- Total side effect is less than 0.35;
- Total cost is less than \$500.

3. Perform one iteration of the steepest ascent method to locate the maximum of

$$f(x, y) = 4x+2y + x^2 -2x^4+ 2xy – 3y^2$$

using initial guesses $x = 0$ and $y = 0$. Employ bisection method to find the optimal step size in the gradient search direction.

4. Derive the three-piece cubic spline function to interpolate four data points or knots (-1, 0.5), (0, -0.5), (1, 2) and (2, 1), assuming that the cubic spline $f(x)$ has equal first and second derivatives at the knots $x = 0$ and $x = 1$, and $f(x)$ has zero second derivative at $x = -1$ and $x = 2$.

5. Consider a computer that uses 10 bits to represent floating-point numbers, 1 bit for s, 5 bits for c ($c = e$ +15), and 4 bits for f. In terms of s, e, and f, the base 10 numbers are given by $x = (-1)^s 2^e (1 + f)$, c is non-negative, and $0 \le f \le 1$.

(a) If $\delta = 0.5$, what is the result of $1 + \delta$ on this computer?

(b) What is the result of the following loop executed by this computer?

```
x = 1;
δ=0.005;
for i=1 to 1000 do
    x = x + δ;
```

6. Consider solving the equation **Ax** = **b** where **x** is an unknown vector, **b** is a known vector and the matrix **A** is given by $A = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$. If the vector **b** is accurate only to 10^{-5}, how accurate will be the solution **x** in terms of 1-norm?

7. Use the Taylor series to derive the centered divided difference formula for approximating the first derivative. What is the error term in O notation?

8. Integrate the following function using (a) multiple-application trapezoidal rule with n = 4; (b) Simpson's 1/3 rules; (c) Simpson's 3/8 rules, and find the truncation error respectively.

$$\int_0^2 4x \ ? \ 1.8x^2 + 1.2x^3 \ ? \ 0.3x^4 dx$$

9. Consider fitting the function $f(x) = \alpha \sin(2x) + \beta \cos(5x)$ to the data (x_i, y_i), $i = 1, 2, \ldots, n$.

(a) Using the Least-squares criterion, derive two linear equations in terms of α and β.

(b) Given the following data, find the values of α and β using the results from (a).

i	1	2	3
x_i	0	$\pi/3$	$2\pi/3$
y_i	5.02	4.91	-5.20

(c) Given f(π) = -5, estimate the approximate error in the above evaluation.