Lossless/Near-lossless Compression of Still and Moving Images

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Part 1. Basics

What is data compression?

- Data compression is the art and science of representing information in a compact form.
- Data is a sequence of symbols taken from a discrete alphabet.
- We focus here on still image data, that is a collection of arrays (one for each color plane) of values representing intensity (color) of the point in corresponding spatial location (pixel).

Why do we need Data Compression?

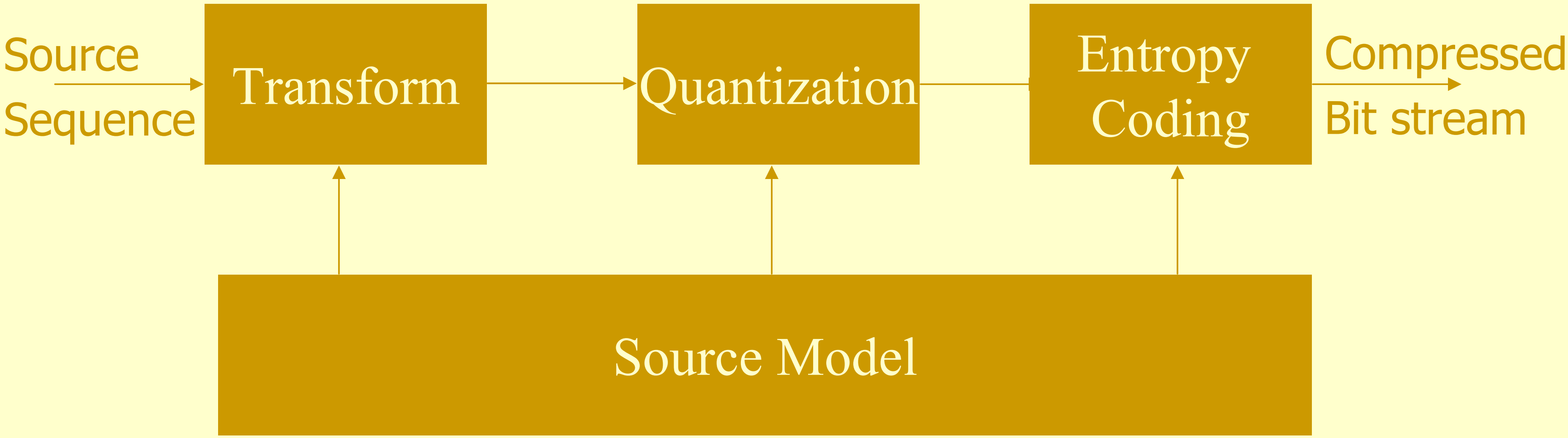
- Still Image
  - 8.5 x 11 page at 600 dpi is > 100 MB.
  - 20 1K x 1K images in digital camera generate 60 MB.
  - Scanned 3 x 7 photograph at 300 dpi is 30 MB.
- Digital Cinema
  - 4K x 2K x 3 x 12 bits/pel = 48 MB/frame, or 1.15 GB/sec, 69GB/min!

More than just storage, how about burdens on transmission bandwidth, I/O throughput?

What makes compression possible?

- Statistical redundancy
  - Spatial correlation -
    - Local - Pixels at neighboring locations have similar intensities.
    - Global - Reoccurring patterns.
  - Spectral correlation – between color planes.
  - Temporal correlation – between consecutive frames.
- Tolerance to fidelity
  - Perceptual redundancy.
  - Limitation of rendering hardware.

Elements of a compression algorithm



Measures of performance

- Compression measures
  - Compression ratio = $\frac{\text{Bits in original image}}{\text{Bits in compressed image}}$
  - Bits per symbol
- Fidelity measures
  - Mean square error (MSE) $\text{Avg}(\text{original} - \text{reconstructed})^2$
  - SNR - Signal to noise ratio $10 \log_{10}(\text{Signal Power} / \text{Noise power})$
  - PSNR - Peak signal to noise ratio
  - HVS based

Other issues

- Coder and decoder computation complexity
- Memory requirements
- Fixed rate or variable rate
- Error resilience
- Symmetric or asymmetric
- Decompress at multiple resolutions
- Decompress at various bit rates
- Standard or proprietary

What is information?

- Semantic interpretation is subjective
- Statistical interpretation - Shannon 1948
  - Self information i(A) associated with event A is

$$\log_2 \frac{1}{P(A)}$$

  - More probable events have less information and less probable events have more information.
  - If A and B are two independent events then self information i(AB) = i(A) + i(B)

Entropy of a random variable

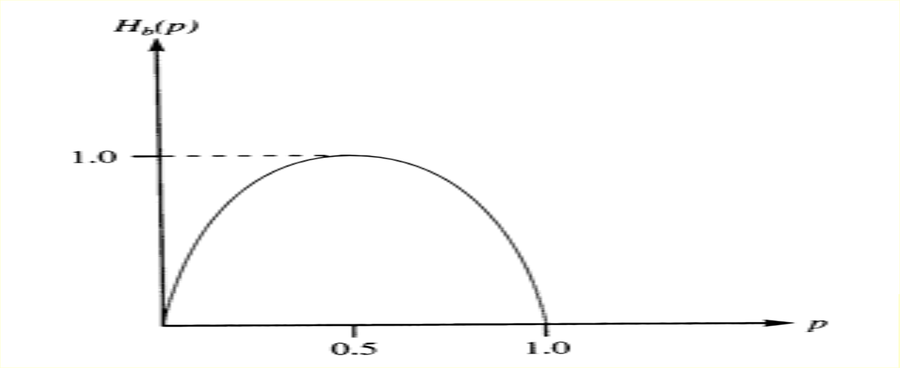
- Entropy of a random variable X from alphabet $\{X_1,\ldots,X_n\}$ is defined as

$$H(X) = -\sum p(X_i)\log p(X_i) = E\{-\log p(X_i)\}$$

- This is the average self-information of the r.v. X
- The average number of bits needed to describe an instance of X is bounded above by its entropy. Furthermore, this bound is tight. (Shannon’s noiseless source coding theorem)

Entropy of a binary valued r.v.

- Let X be a r.v. whose set of outcomes is {0,1}
- Let p(0) = p and p(1) = 1-p
- Plot H(X) = - p log p - (1-p) log (1-p)



  - H(X) is max when p = 1/2
  - H(X) is 0 if and only if either p = 0 or p = 1
  - H(X) is continuous

Properties of the entropy function

- Can also be viewed as measure of uncertainty in X
- Can be shown to be the only function that satisfies the following
  - If all events are equally likely then entropy increases with number of events
  - If X and Y are independent then $H(XY) = H(X)+H(Y)$
  - The information content of the event does not depend in the manner the event is specified
  - The information measure is continuous

Entropy of a stochastic process

- A stochastic process $S = \{X_i\}$ is an indexed sequence of r.v.'s characterized by joint pmf's
- Entropy of a stochastic process S is defined as

$$H(S) = \lim_{n->\infty} \frac{1}{n} E\{-\log_2 P(X_1 X_2 \cdots X_n)\}$$

- measure of average information per symbol of S
- In practice, difficult to determine as knowledge of source statistics is not complete.

Joint Entropy and Conditional Entropy

- Joint entropy H(X,Y) is defined as

$$H(X,Y) = -\sum_x \sum_y p(x,y) \log p(x,y)$$

- The conditional entropy H(Y|X) is defined as

$$H(Y|X) = \sum_x p(x) H(Y|X=x)$$

- It is easy to show that

$$H(X,Y) = H(X) + H(Y|X)$$

- Mutual Information I(X;Y) is defined as

$$I(X;Y) = H(X) - H(X|Y)$$

General References on Data Compression

- Image and Video Compression Standards - V. Bhaskaran and K. Konstantinides. Kluwer International - Excellent reference for engineers.
- Data Compression - K. Sayood. Morgan Kauffman - Excellent introductory text.
- Elements of Information Theory - T. Cover and J. Thomas - Wiley Interscience - Excellent introduction to theoretical aspects.