ENG4BF3
Medical Image Processing

Chapter 10

Image Segmentation

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Image Segmentation

- An important step in image analysis is to segment the image.
- Segmentation: subdivides the image into its constituent parts or objects.
- Autonomous segmentation is one of the most difficult tasks in image processing.
- Segmentation algorithms for monochrome images generally are based on two basic properties of gray-level values:

  1. Discontinuity 2. Similarity

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Image Segmentation

- Discontinuity: the image is partitioned based on abrupt changes in gray level.
- Similarity: partition an image into regions that are similar
  - Main approaches are thresholding, region growing and region splitting and merging.

Detection of Discontinuities

- Line detection: If each mask is moved around an image it would respond more strongly to lines in the mentioned direction.

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

−1	−1	2
−1	2	−1
2	−1	−1

+45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

2	−1	−1
−1	2	−1
−1	−1	2

−45°

FIGURE 10.3 Line masks.

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Detection of Discontinuities

- There are 3 basic types of discontinuities in digital images:

  1. Point
  2. Line
  3. Edges.

Mask: $$R = \sum_{i=1}^{9} w_i z_i$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

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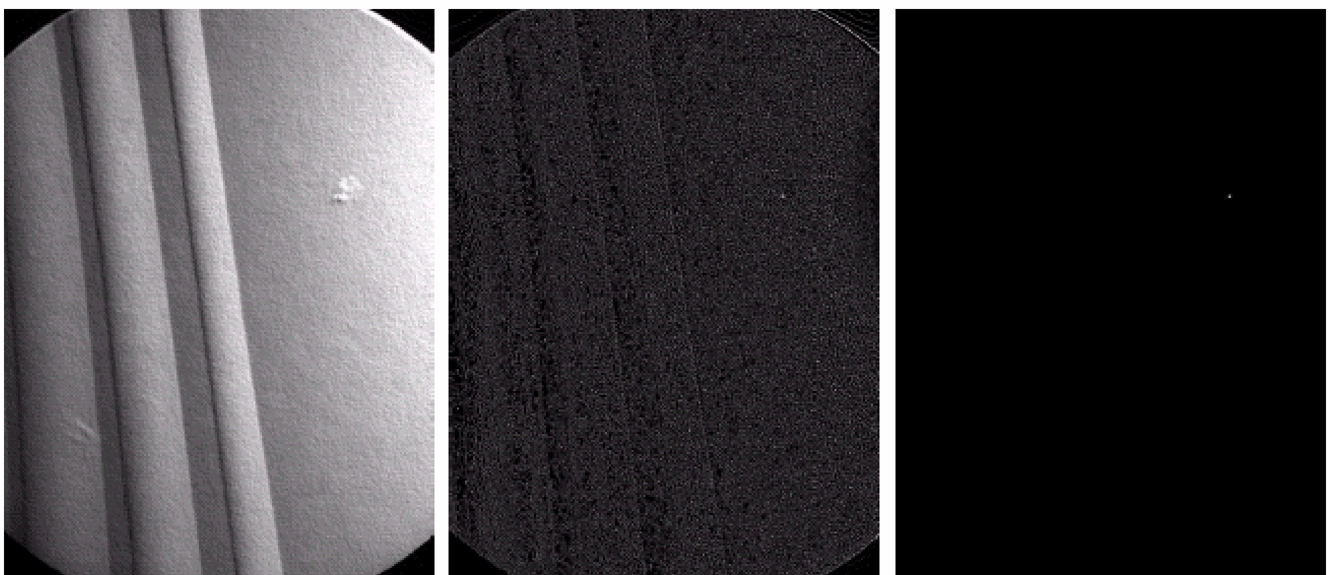
Detection of Discontinuities

- Point detection: detect an isolated point

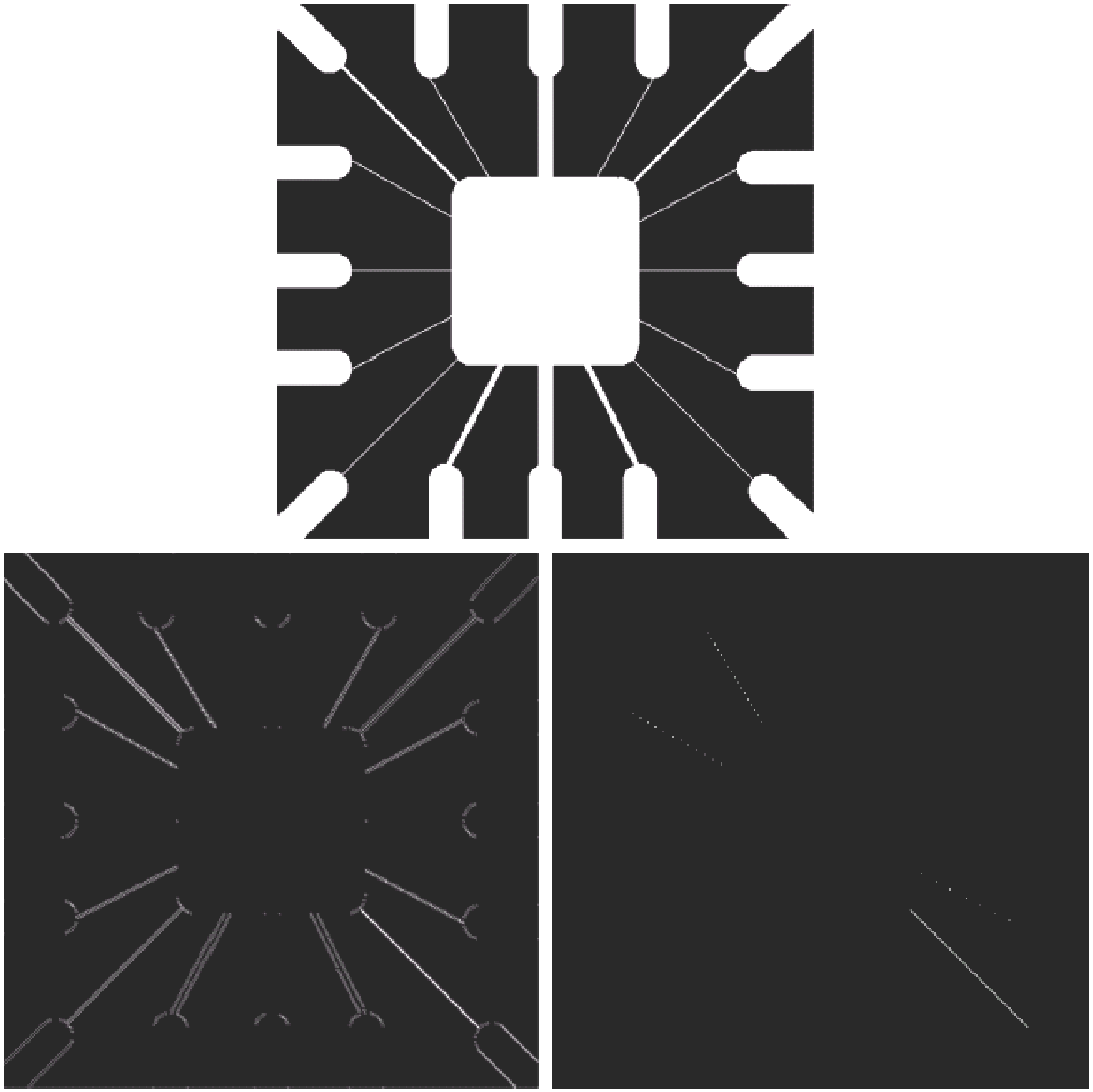
$$|R| \geq T$$

−1	−1	−1
−1	8	−1
−1	−1	−1



a
b c d

FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)



a
b c

FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with −45° line detector. (c) Result of thresholding image (b).

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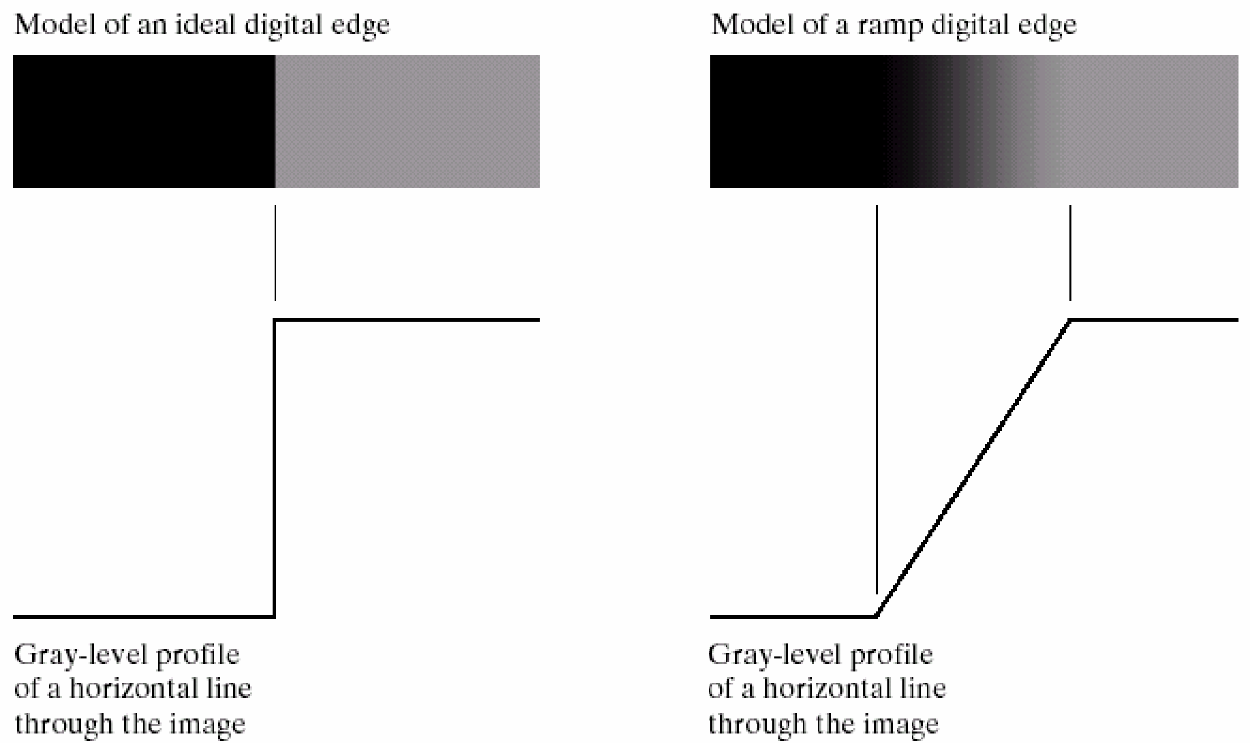
Edge detection

- Edge: boundary between two regions with relatively distinct gray levels.



a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

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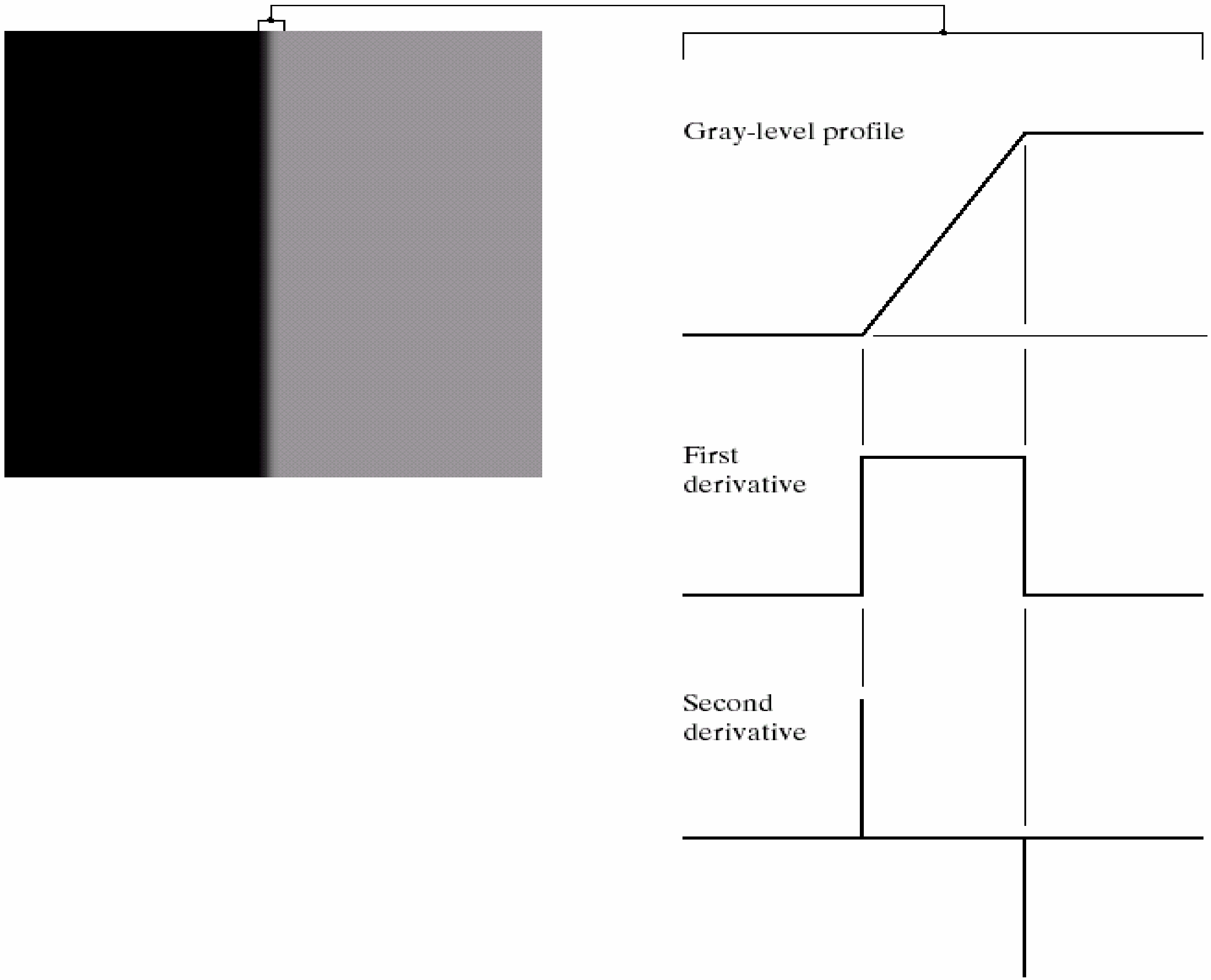
Edge detection

- Basic idea: computation of a local derivative operator.

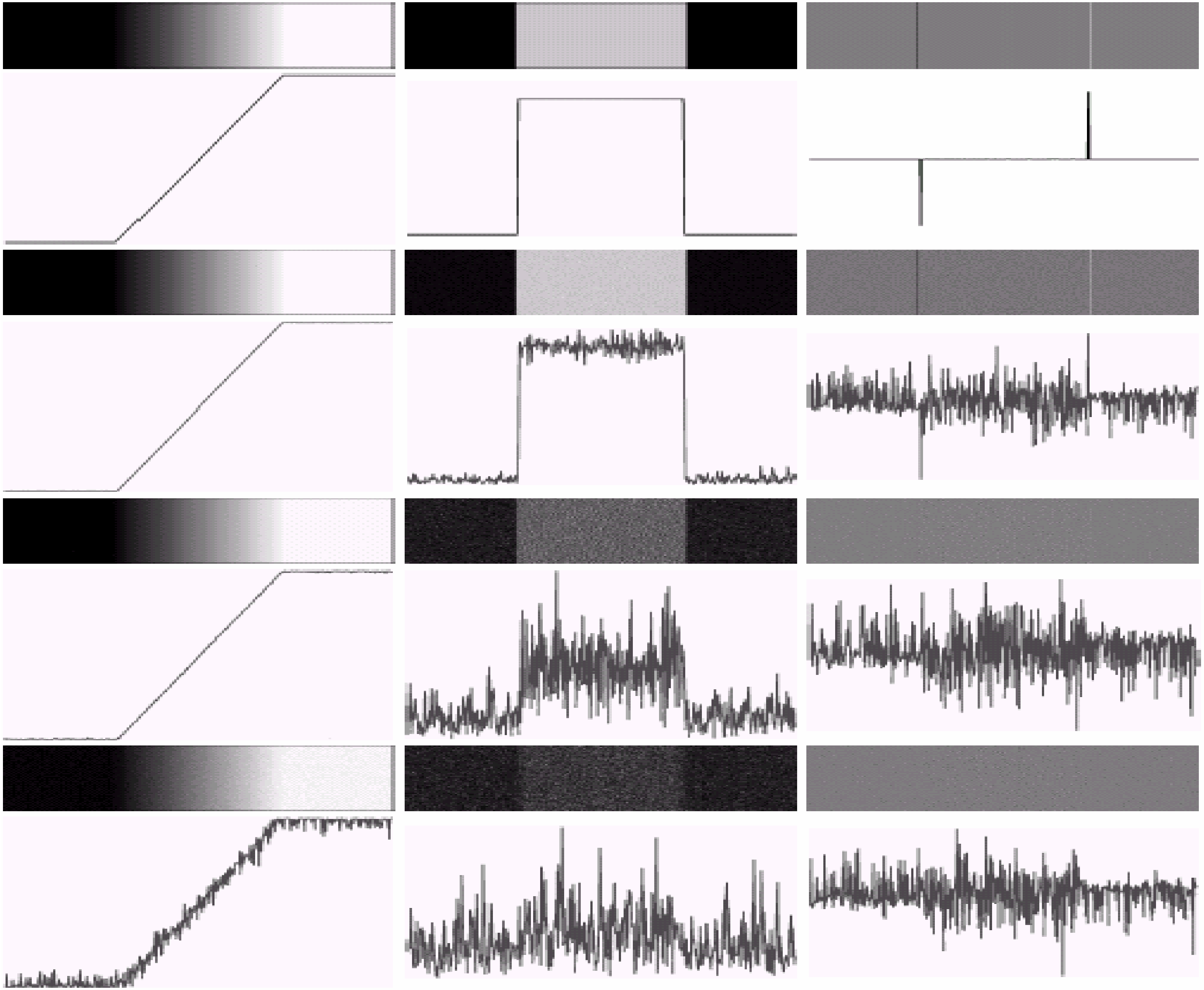
a b

FIGURE 10.6
(a) Two regions separated by a vertical edge.
(b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.



Edge detection

- The magnitude of the first derivative can be used to detect an edge
- The sign (zero crossing) of the second derivative can be used to detect an edge.
- The same idea can be extended into 2-D. 2-D derivatives should be used.
- The magnitude of the gradient and sign of the Laplacian are used.



Gradient Operator

Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Gradient magnitude

$$\nabla f = mag(\nabla \mathbf{f}) = \sqrt{\left(G_x{}^2 + G_y{}^2\right)} \qquad \nabla f \approx |G_x| + |G_y|$$

Gradient direction

$$\alpha(x, y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

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a
b c
d e
f g

FIGURE 10.8 A 3 × 3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a	b
c	d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

a b
c d

FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

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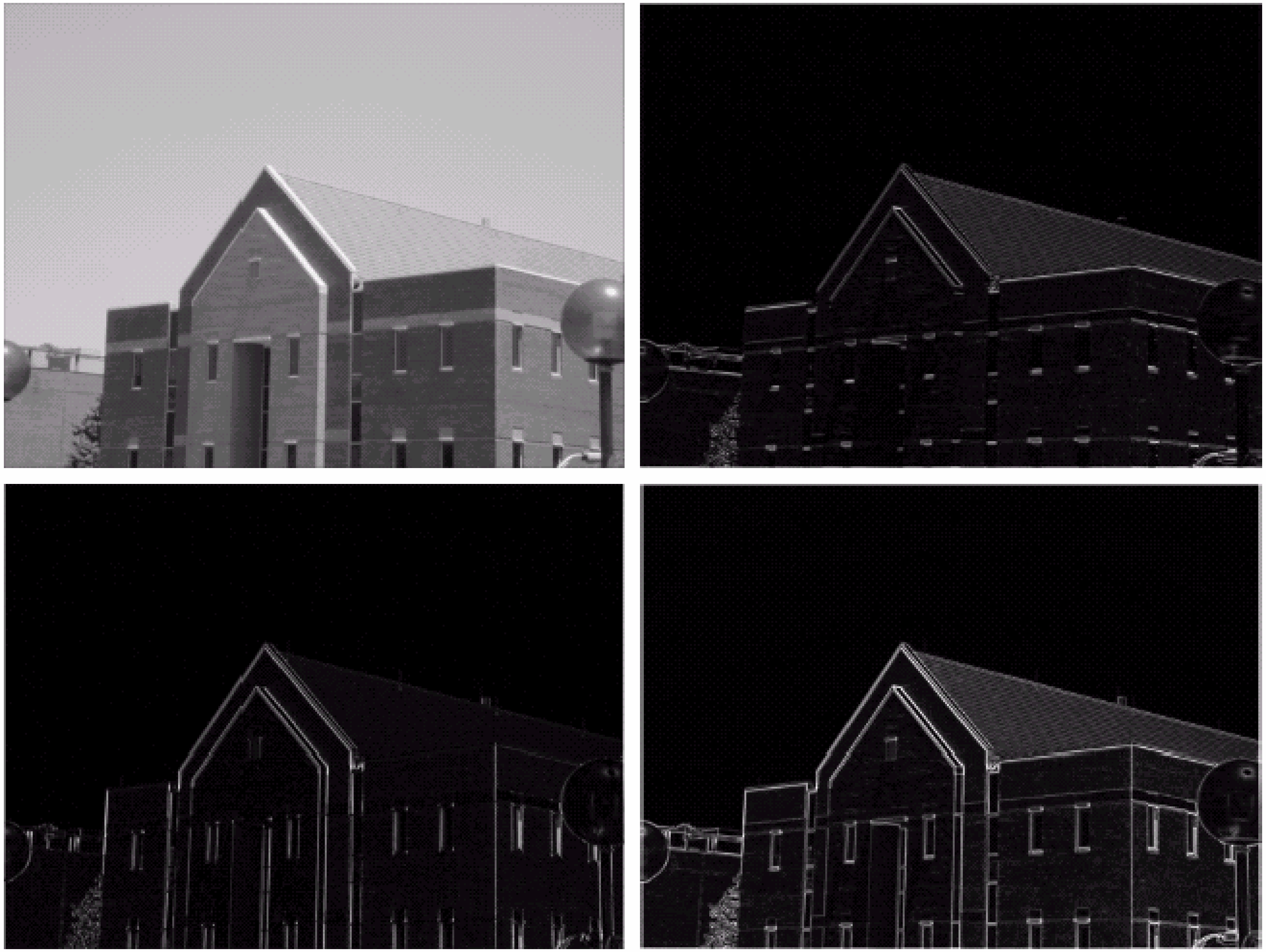

a	b
c	d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5×5 averaging filter.

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a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

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Laplacian

- Laplacian in its original form is not used for edge detection because:

  1. It is very sensitive to noise
  2. It's magnitude produces double edges
  3. Unable to detect the direction of an edge.

- To solve the first problem, the image is low-pass filtered before using the Laplacian operator.

$$f(x,y) \longrightarrow \boxed{h(x,y)} \longrightarrow \boxed{\nabla^2} \longrightarrow g(x,y)$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = 8z_5 - \sum_{i=1}^{8} z_i$$

$$\nabla^2 f = 4z_5 - (z_2 + z_4 + z_6 + z_8)$$

FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

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Laplacian

Convolution: $g(x,y) = f(x,y) * \nabla^2 h(x,y)$

Gaussian function: $h(x,y) = \exp(-\frac{x^2+y^2}{2\sigma^2})$

$r^2 = x^2 + y^2$

Second derivative: $\nabla^2 h(r) = (\frac{r^2-\sigma^2}{\sigma^4})\exp(-\frac{r^2}{2\sigma^2})$

- Cross section of $\nabla^2 h$ has a Mexican hat shape.
- The average $\nabla^2 h$ is zero. The average of image convolved with $\nabla^2 h$ is also zero.
- We will have negative pixel values in the result.

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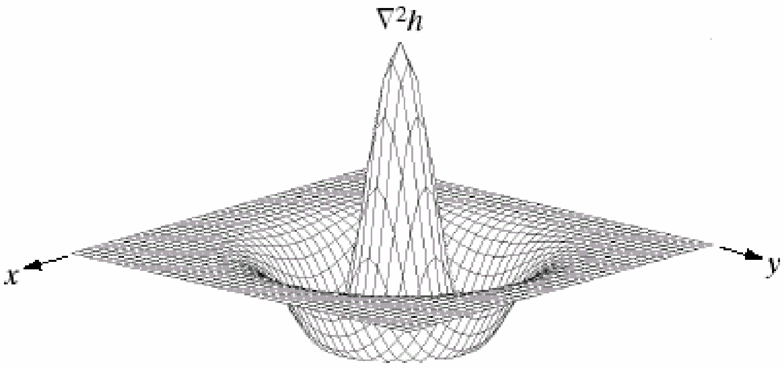
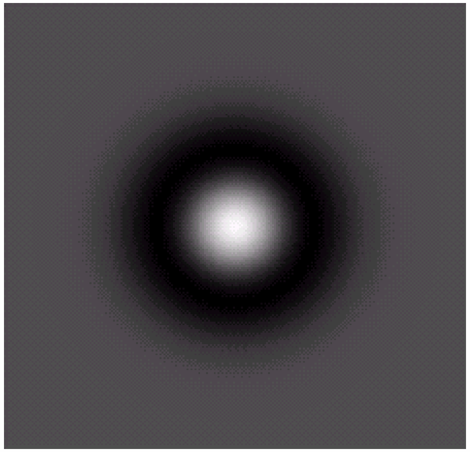
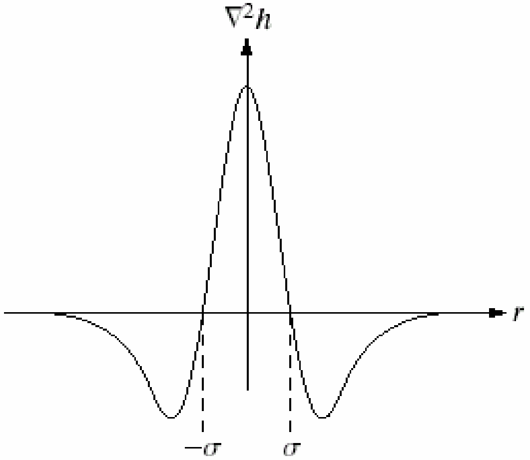
Laplacian

- To solve the problem of double edges, zero crossing of the output of Laplacian operator is used.



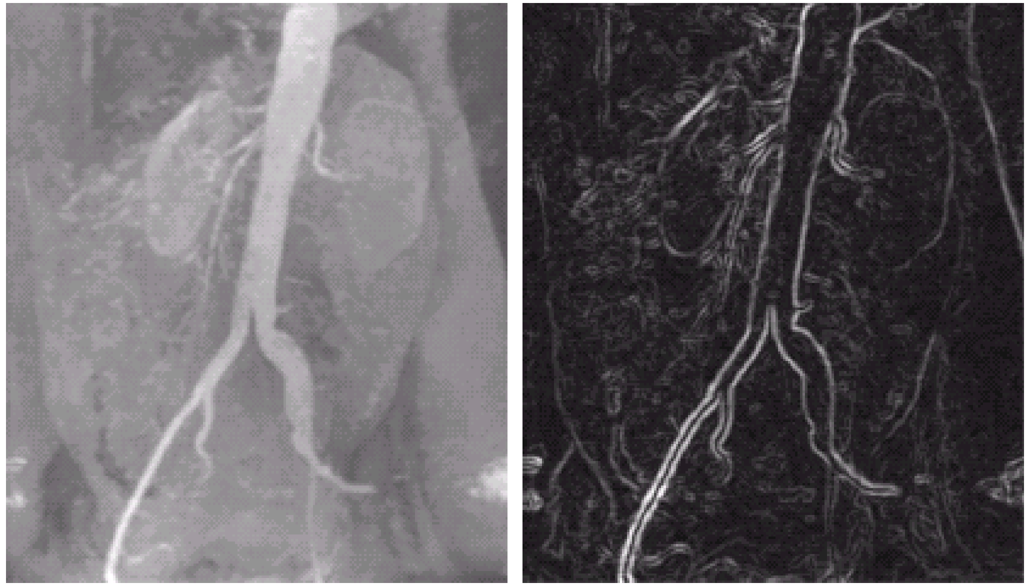




0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

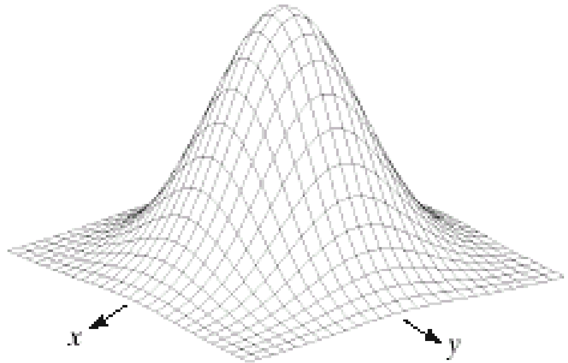
a b
c d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

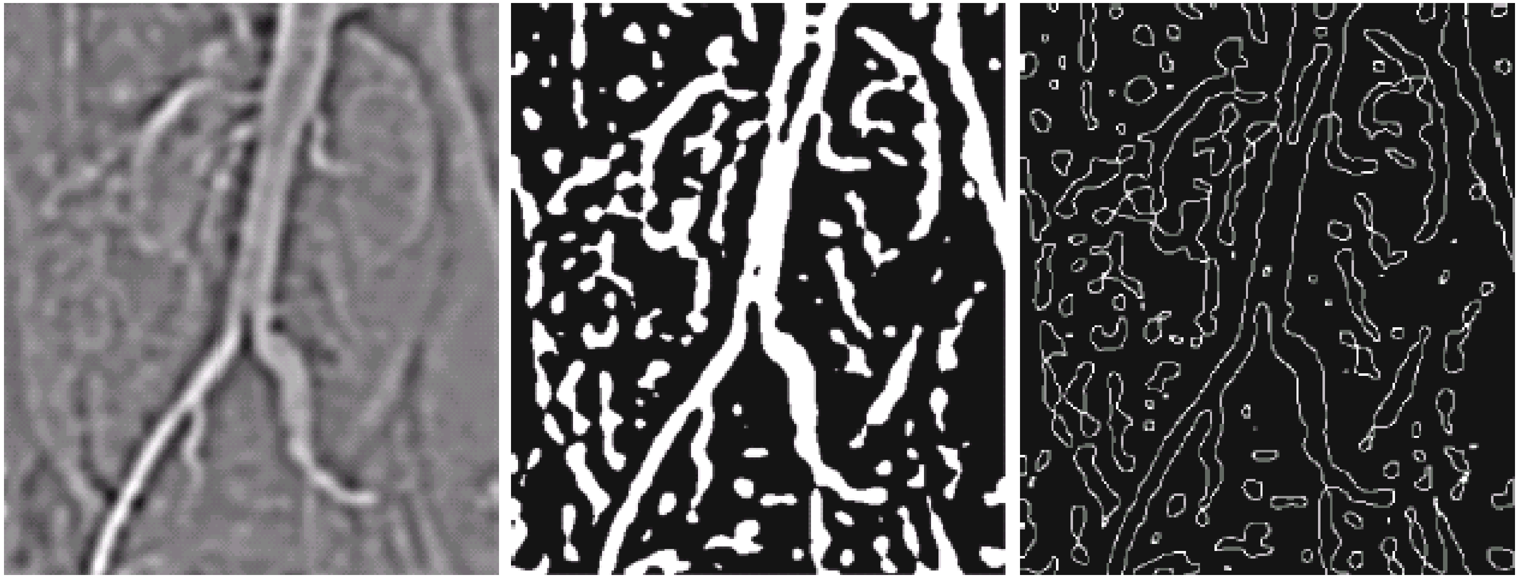


a b
c d
e f g

FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



–1	–1	–1
–1	8	–1
–1	–1	–1



a b
c d
e f g

FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Edge linking and Boundary Detection

- The output of discontinuity detection stage seldom characterizes a boundary completely.
- This is because of noise, breaks in the boundary and other effects.
- Edge detection algorithms typically are followed by linking and boundary detection procedures.
- These procedures assemble edge pixels into meaningful boundaries.

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Edge linking and Boundary Detection

- All the points that are similar are linked forming a boundary of pixels.
- Two principle properties used for establishing similarity of edge pixels in this kind of analysis are:
  1. The strength of the response of the gradient operator used to produce the edge pixels
  2. The direction of the gradient

$$\left\| \left| \nabla f(x,y) \right| - \left| \nabla f(x_0 + y_0) \right| \right\| \leq E$$

$$\left| \alpha(x,y) - \alpha(x_0, y_0) \right| < A$$

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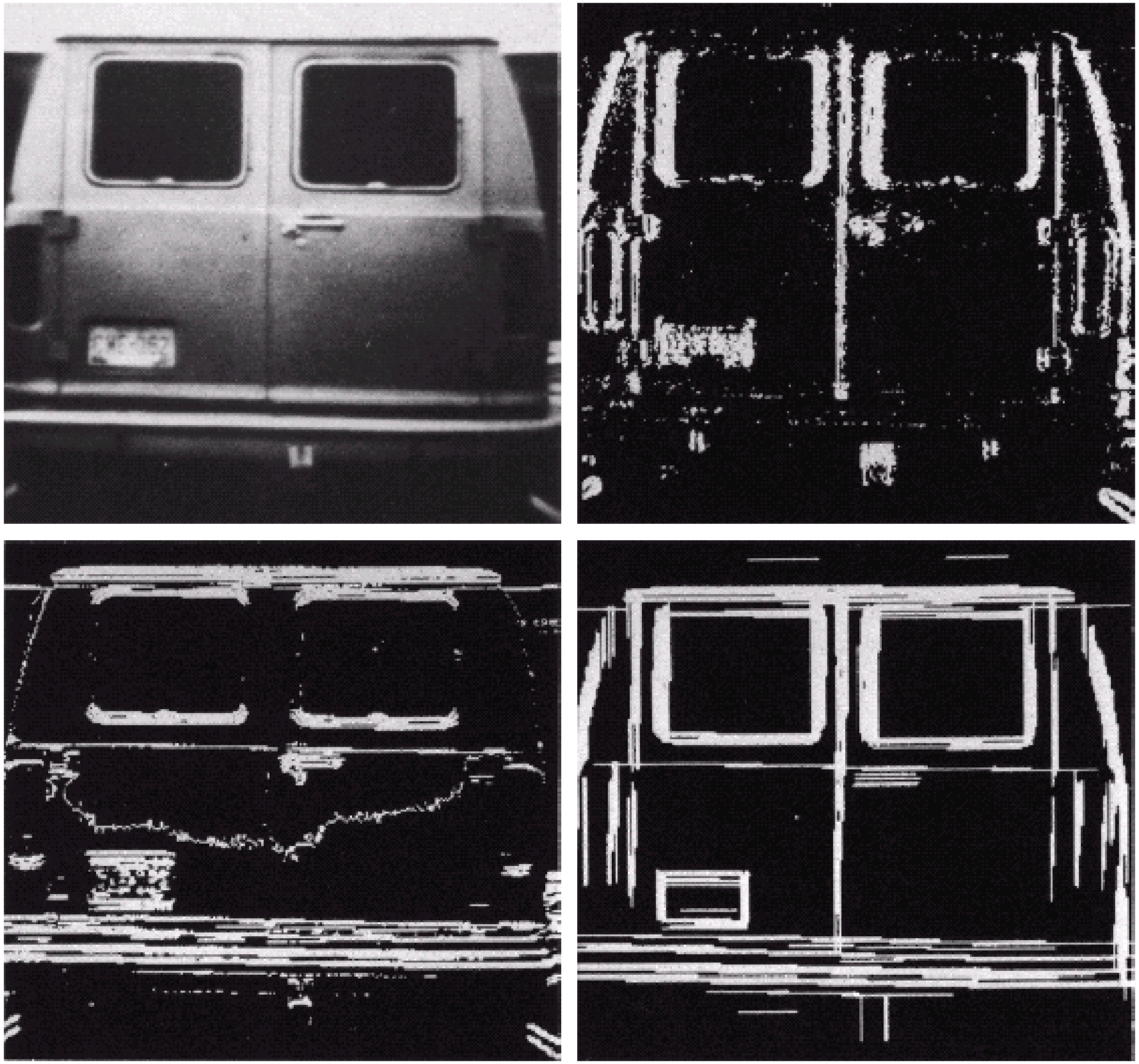
a b
c d

FIGURE 10.16
(a) Input image.
(b) G_y component of the gradient.
(c) G_x component of the gradient.
(d) Result of edge linking. (Courtesy of Perceptics Corporation.)



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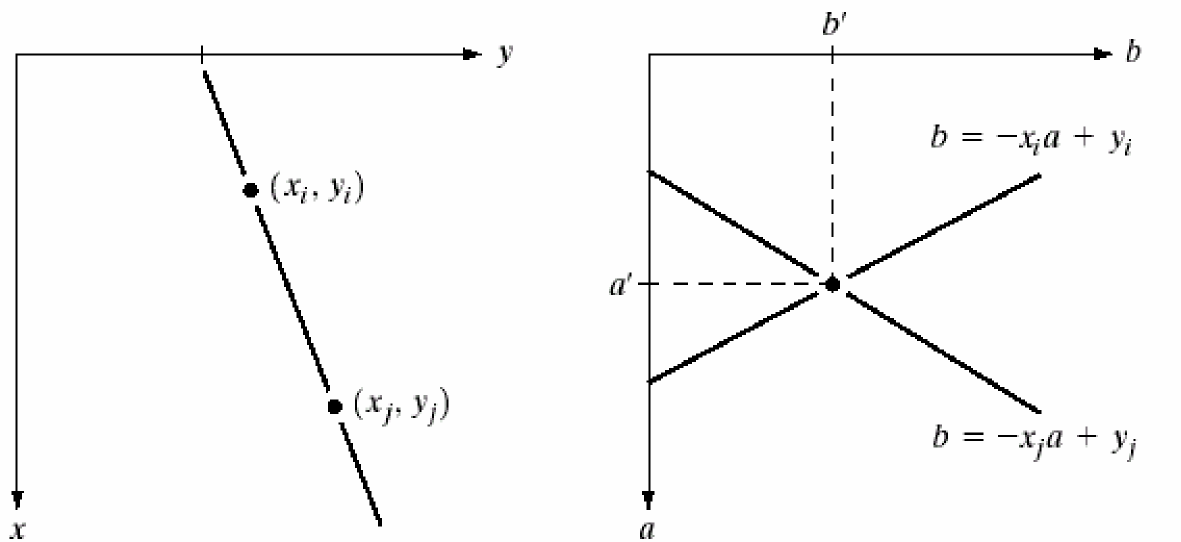
Hough Transform

- (x_i, y_i): all the lines passing this point $y_i = ax_i + b$
- $b = y_i - ax_i$: point (x_i, y_i) maps to a single line in *ab* plane.
- Another point (x_j, y_j) also has a single line in *ab* plane

$$b = y_j - ax_j$$

- a' and b' are the slope and intercept of the line containing both (x_i, y_i) and (x_j, y_j).



a b

FIGURE 10.17
(a) *xy*-plane.
(b) Parameter space.

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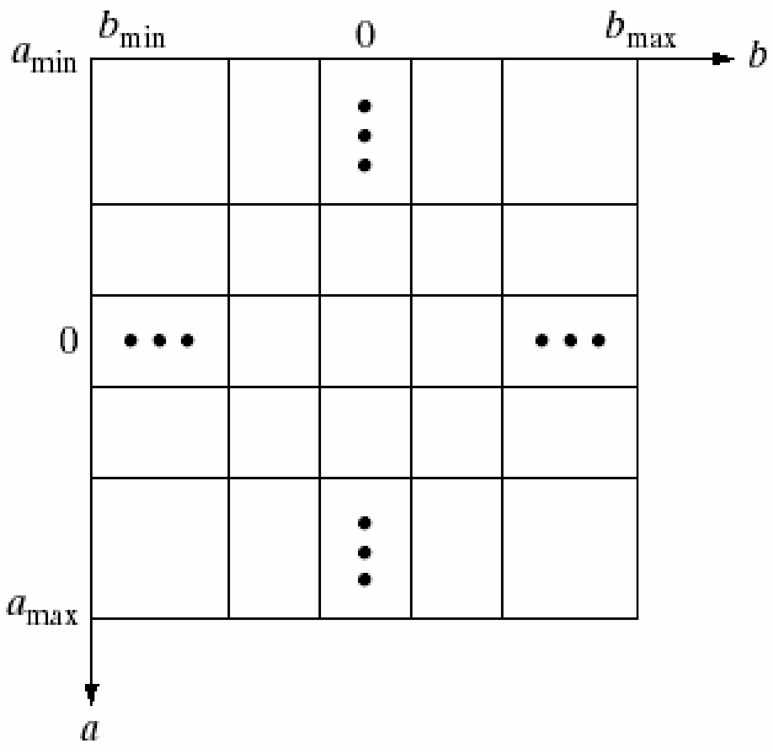
Hough transform

- (a_{min}, a_{max}): expected range of slope
- (b_{min}, b_{max}): expected range of intercepts
- $A(i,j)$: the number of points in the cell at coordinates (i,j) in the ab plane.

For every point in the image plane, we let the value of *a* equal each of the allowed subdivisions and find the corresponding *b* from $b = y_i - ax_i$ If for a_p we get b_q the $A(p,q)$ is incremented.



Hough transform

- The cell with the largest value shows the parameters of the line that contains the maximum number of points.
- Problem with this method: *a* approaches infinity as the line gets perpendicular to the *x* axis.
- Solution: use the representation of the line as:

$$x\cos\theta + y\sin\theta = \rho$$

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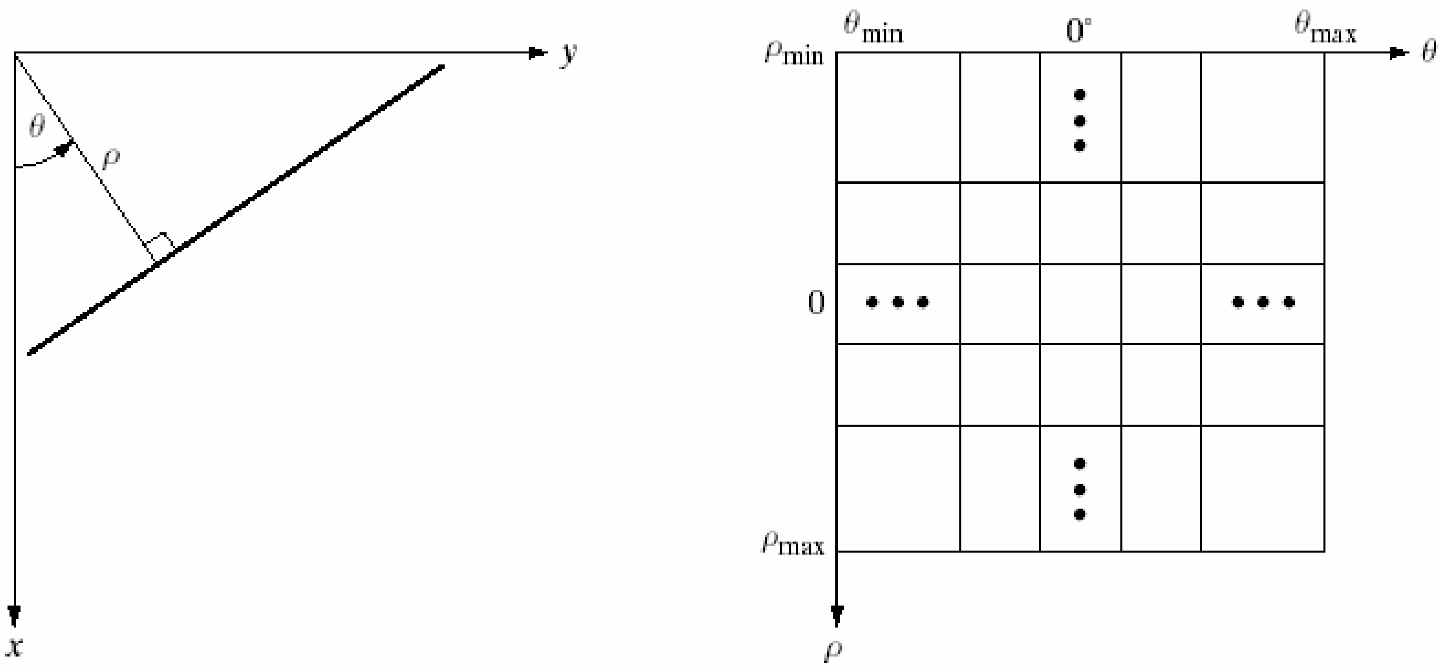
Hough transform

- A point in *xy* plane is mapped into a sinusoidal curve in $\rho\theta$ plane.

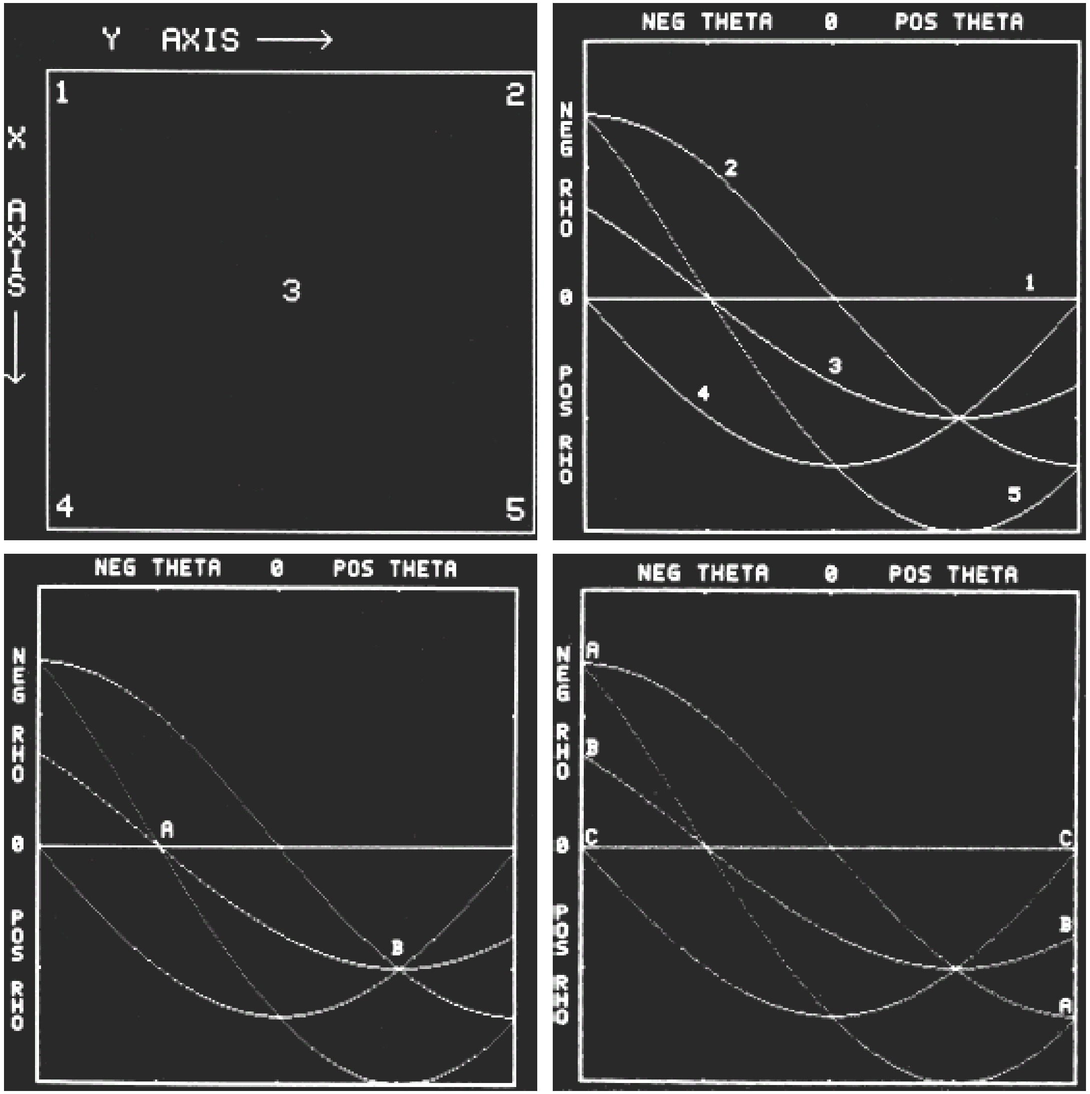


a b

FIGURE 10.19 (a) Normal representation of a line. (b) Subdivision of the $\rho\theta$-plane into cells.

a b
c d

FIGURE 10.20 Illustration of the Hough transform. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)



Hough transform

- Hough transform is applicable to any function of the form g(**v**,**c**)=0, where **v** is the vector of coordinates and **c** is the vector of coefficients.
- Exp: $(x-c_1)^2+(y-c_2)^2=c_3{}^2$
- 3 parameters (c1,c2,c3), 3-D parameter space, cube like cells, accumulators of the form A(i,j,k).
- Procedure:

1. Increment c1 and c2
2. Solve for c3
3. Update the accumulator associated with (c1,c2,c3)

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Thresholding

- Thresholding is one of the most important approaches to image segmentation.
- Gray level histogram of an image f(x,y) composed of a light object on a dark background.
- To extract the object: select a threshold T that separates the gray levels of the background and the object.

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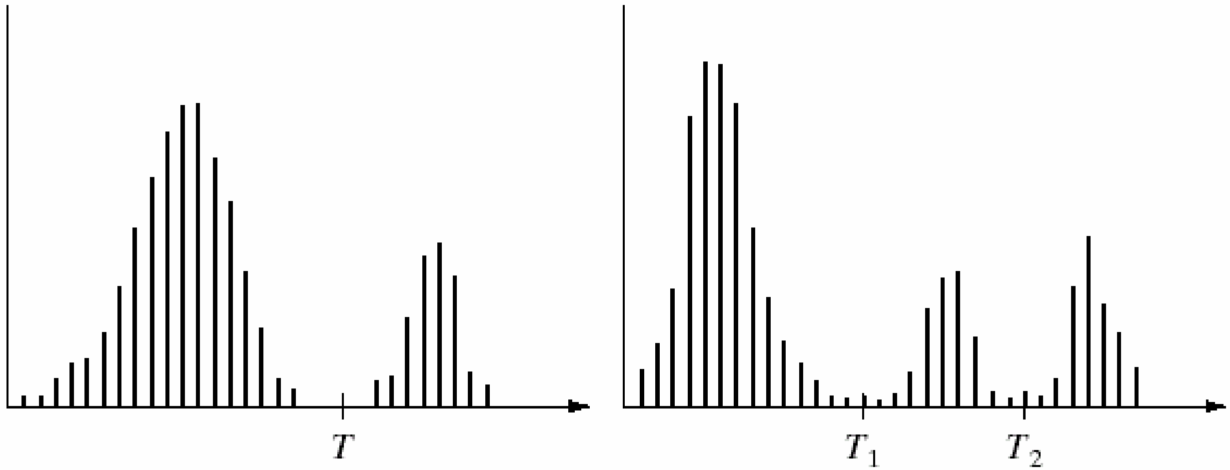
Thresholding

- Single threshold: points with f(x,y)>T belong to object; other points belong to background.
- Multiple thresholds: points with f(x,y)>T_2 belong to object; points with f(x,y)<T_1 belong to bakground.



a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

Thresholding

- Threshold in general can be calculated as:

  $T=T(x,y,p(x,y),f(x,y))$

  $f(x,y)$: gray level at (x,y)

  $p(x,y)$: some local property of the point (x,y) (e.g., the average gray level of a neighborhood centered on (x,y).

  T depends only on $f(x,y)$: global threshold

  T depends on $f(x,y)$ and $p(x,y)$: local threshold

  T depends on $f(x,y)$ and $p(x,y)$ and x,y : dynamic threshold

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Thresholding

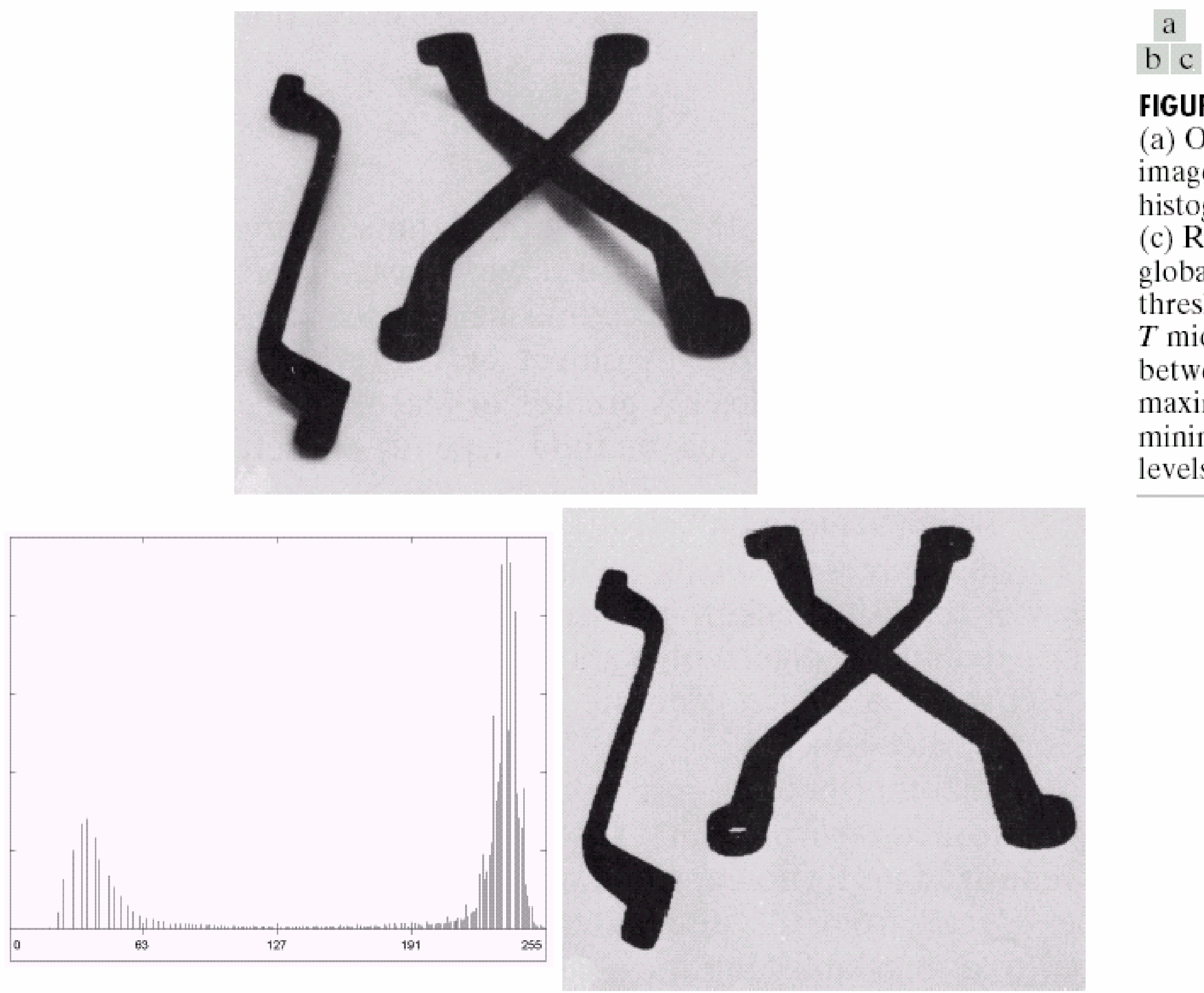
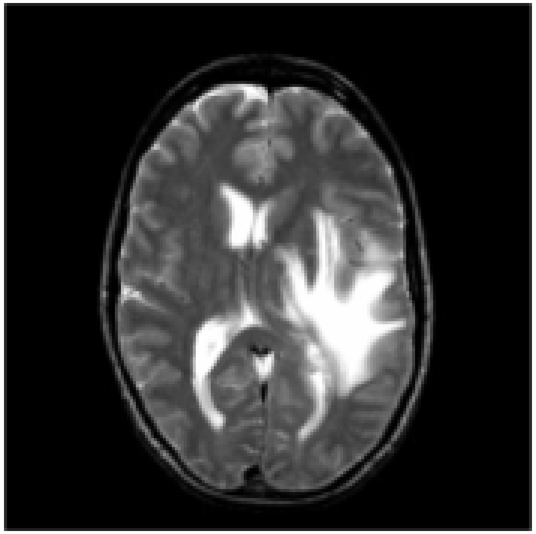
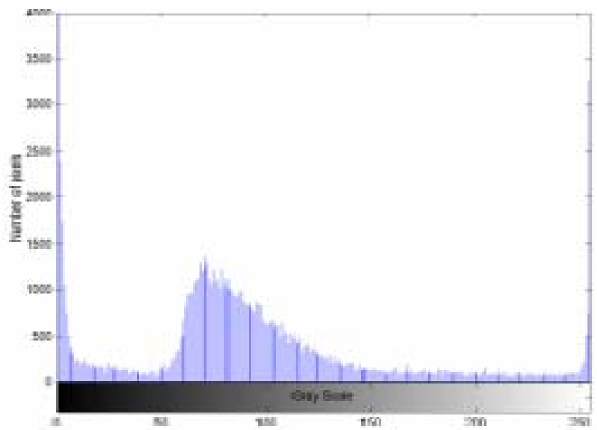


FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with *T* midway between the maximum and minimum gray levels.

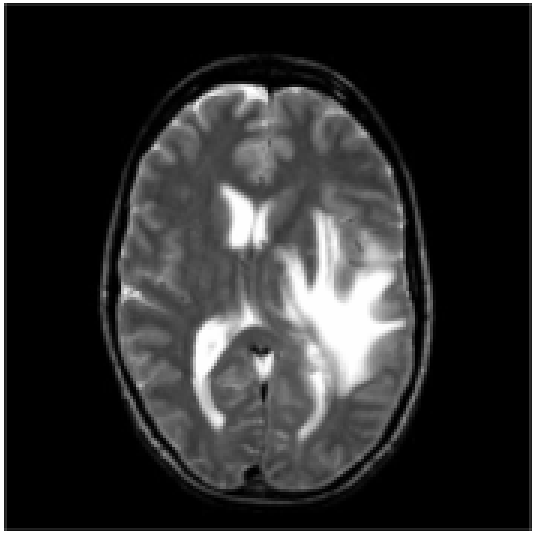
Thresholding

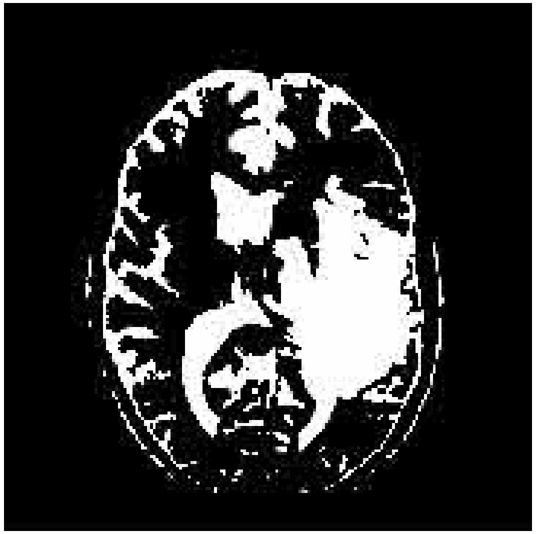




MR brain image (top left), its histogram (bottom) and the segmented image (top right) using a threshold T=12 at the first major valley point in the histogram.

Thresholding





Two segmented MR brain images using a gray value threshold T=166 (top right) and T=225 (bottom)

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Basic Global Thresholding

1. Select an initial estimate for T
2. Segment the image using T. This will produce two group of pixels G1 (with gray level greater than T) and G2 (with gray level less than T)
3. Compute average gray level values $\mu 1$ and $\mu 2$ for pixels in regions G1 and G2
4. Compute a new threshold: $T=(\mu 1+ \mu 2)/2$
5. Repeat step 2 through 4 until the difference in T in successive iterations is smaller than a predefined value

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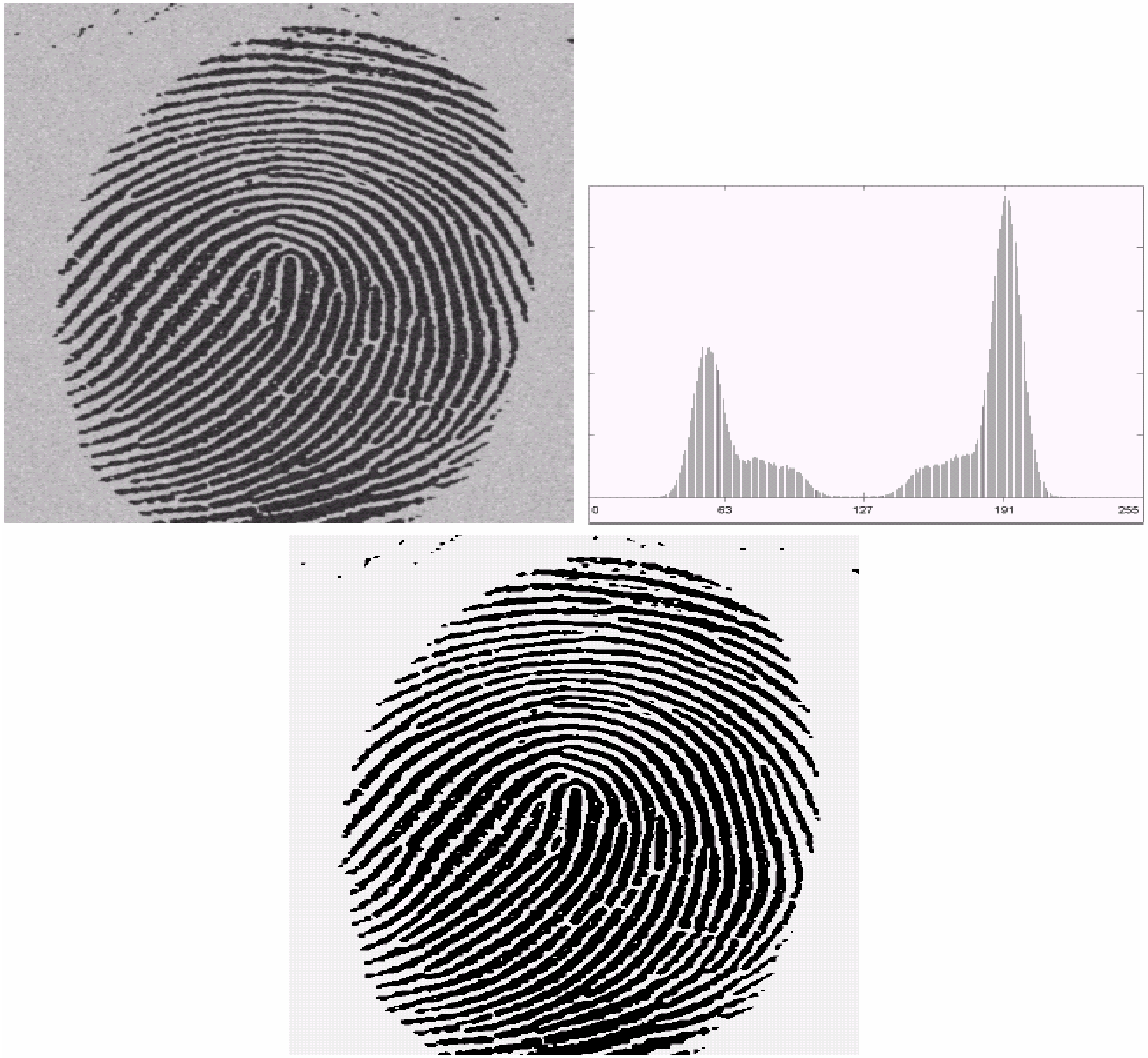
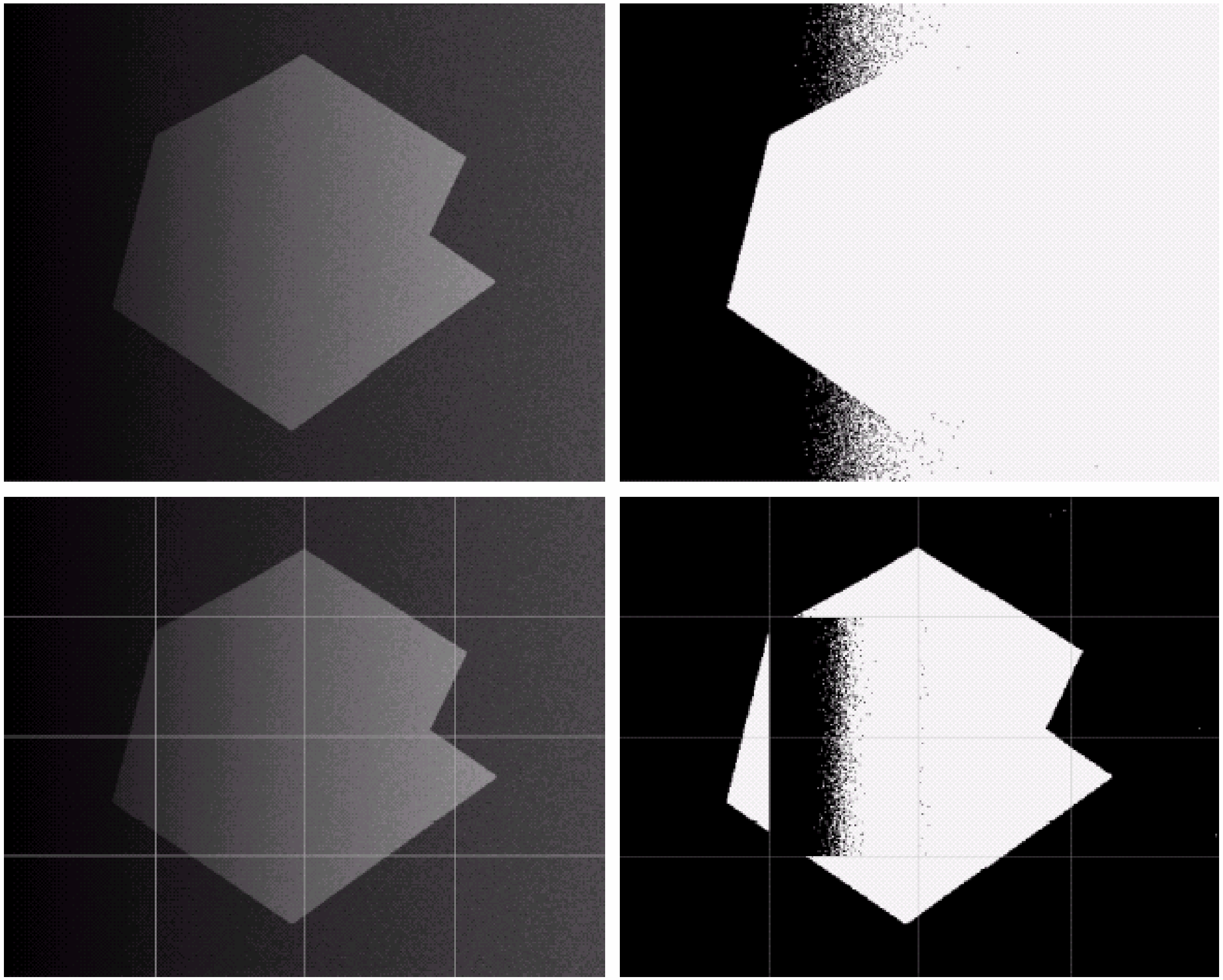
Basic Global Thresholding



a b
c

FIGURE 10.29 (a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)

Basic Adaptive (Local) Thresholding



a	b
c	d

FIGURE 10.30 (a) Original image. (b) Result of global thresholding. (c) Image subdivided into individual subimages. (d) Result of adaptive thresholding.

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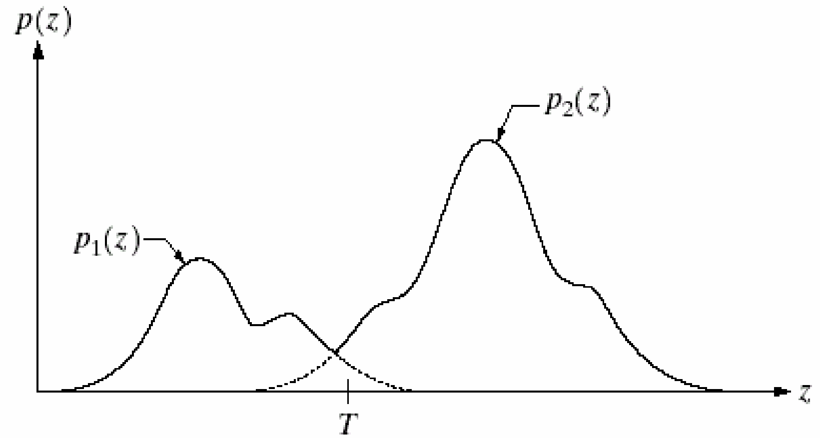
Optimal Global Thresholding

FIGURE 10.32 Gray-level probability density functions of two regions in an image.



$$p(z) = P_1 \cdot p_1(z) + P_2 \cdot p_2(z)$$

$$P_1 + P_2 = 1$$

Problem: how to optimally determine T to minimize the segmentation error?

Optimal Global Thresholding

Error 1: $$E_1(T) = \int_{-\infty}^{T} p_2(z)dz$$

Error 2: $$E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

Total error: $E(T) = P_1 E_2(T) + P_2 E_1(T)$

Goal: what's the value of T to minimize E(T)?

Result: $$P_1 p_1(T) = P_2 p_2(T)$$

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Optimal Global Thresholding: Gaussian PDFs

$$p(z) = \frac{P_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(z-\mu_1)^2}{2\sigma_1^2}\right) + \frac{P_2}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{(z-\mu_2)^2}{2\sigma_2^2}\right)$$

Solution: $$AT^2 + BT + C = 0$$

$$A = \sigma_1^2 - \sigma_2^2$$

$$B = 2(\mu_1\sigma_2^2 - \mu_2\sigma_1^2)$$

$$C = \sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2 + 2\sigma_1^2 2\sigma_2^2 \ln(\sigma_2 P_1 / \sigma_1 P_2)$$

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Region Based Segmentation

- Let R represent the entire image. Segmentation is a process that partitions R into n subregions R1,R2,…,Rn such that:

a) $\bigcup_{i=1}^{n} R_i = R$

b) R_i is a connected region.

c) $\mathrm{R}_i \cap R_j = \phi$

d) $\mathrm{P}(\mathrm{R}_i) = True$

e) $\mathrm{P}(\mathrm{R}_i \cup R_j) = False$

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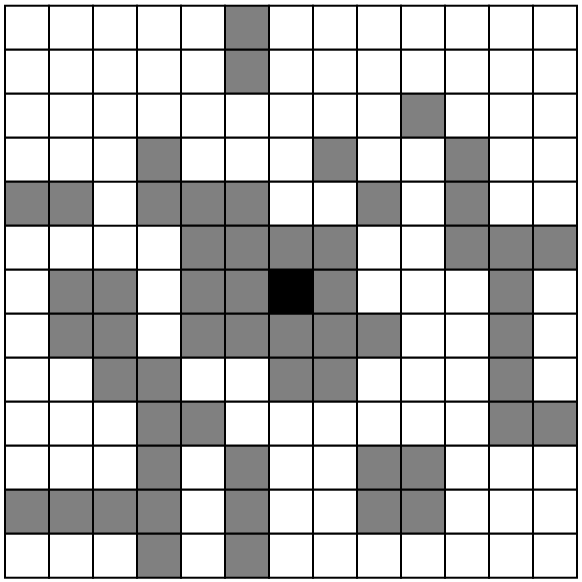
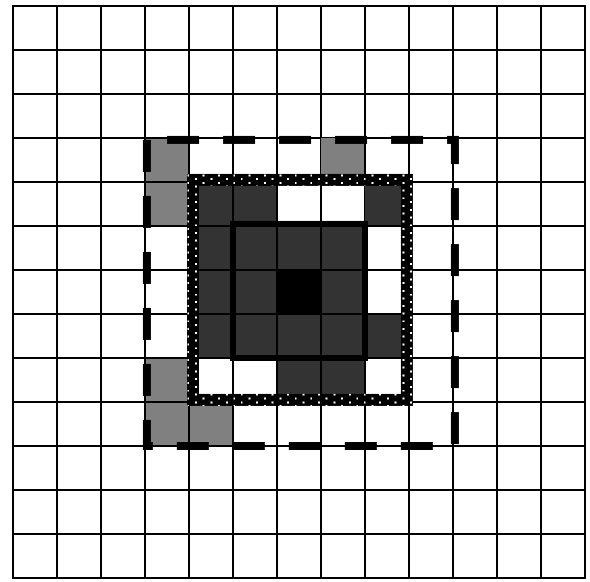
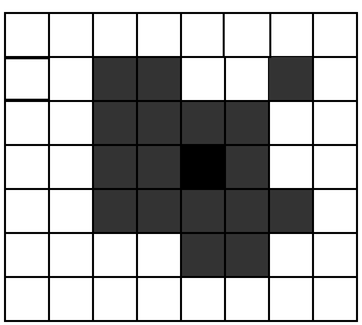
Region Growing

- Pixel aggregation: starts with a set of seed point and from these grows regions by appending to each seed point those neighboring pixels that have similar properties (e.g., gray-level, texture, color).

Region Growing







Segmented region

- Center Pixel
- Pixels satisfying the similarity criterion
- Pixels unsatisfying the criterion
- 3x3 neighborhood
- 5x5 neighborhood
- 7x7 neighborhood

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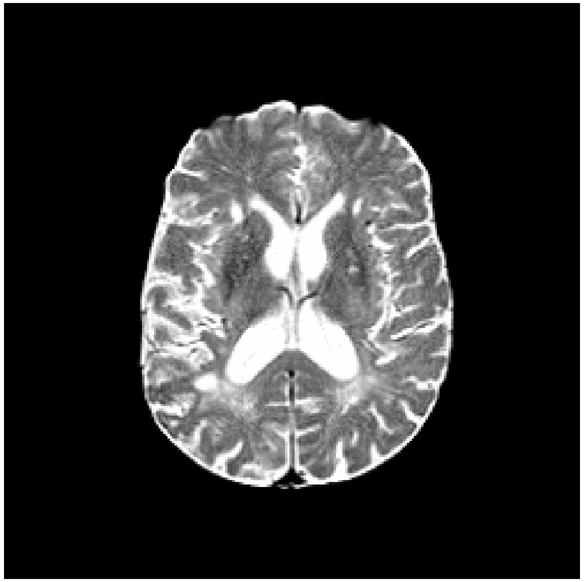
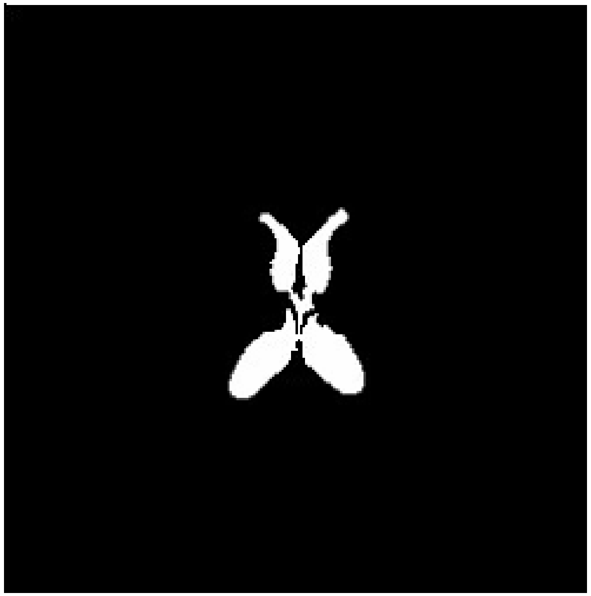
Region Growing



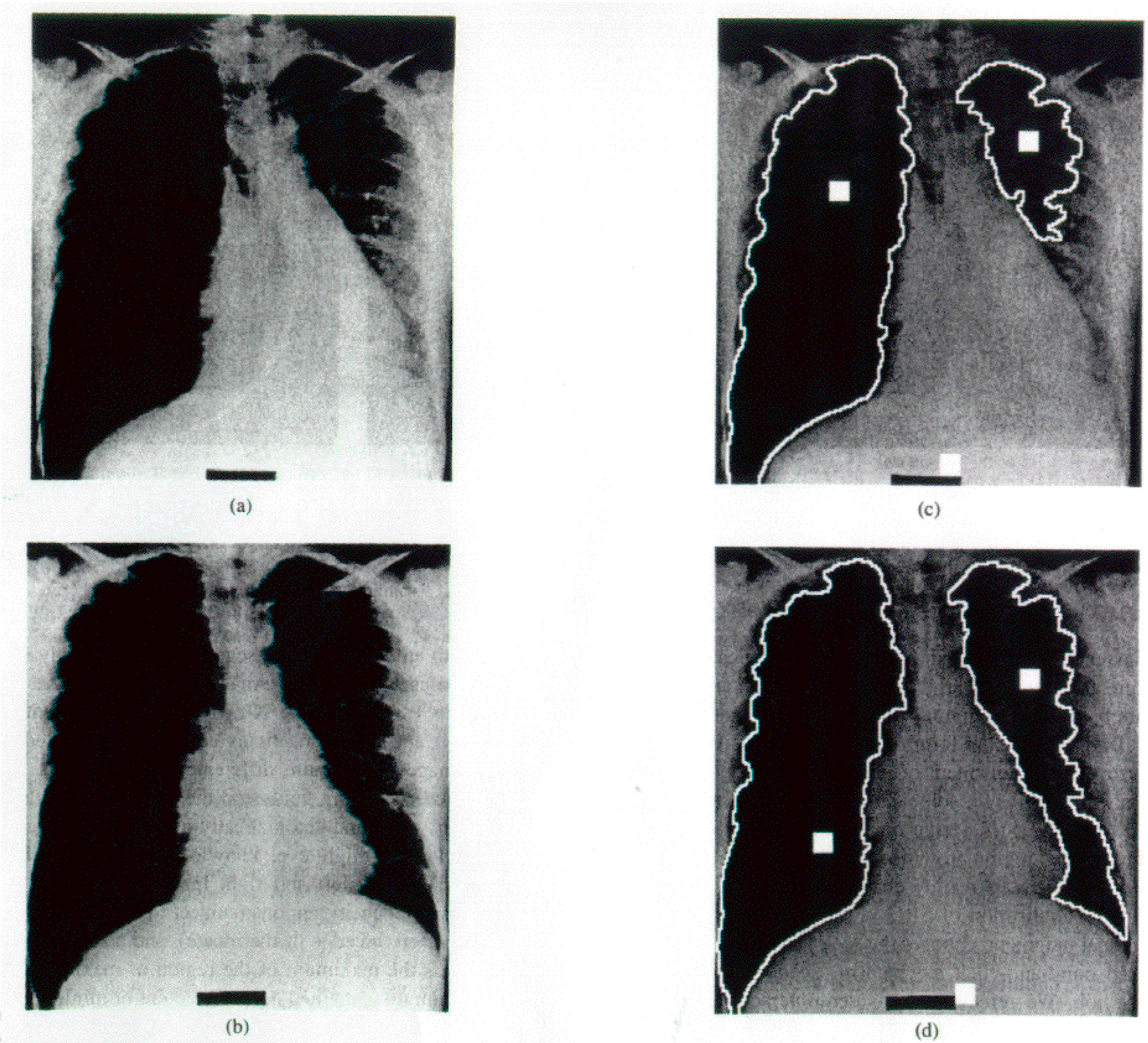


A T-2 weighted MR brain image (left) and the segmented ventricles (right) using the region-growing method.

Region growing

- Problems:
  1. Selection of initial seeds that properly represent regions of interest
  2. Selection of suitable properties
  3. Formulation of a stopping rule.

Seeded Region Growing



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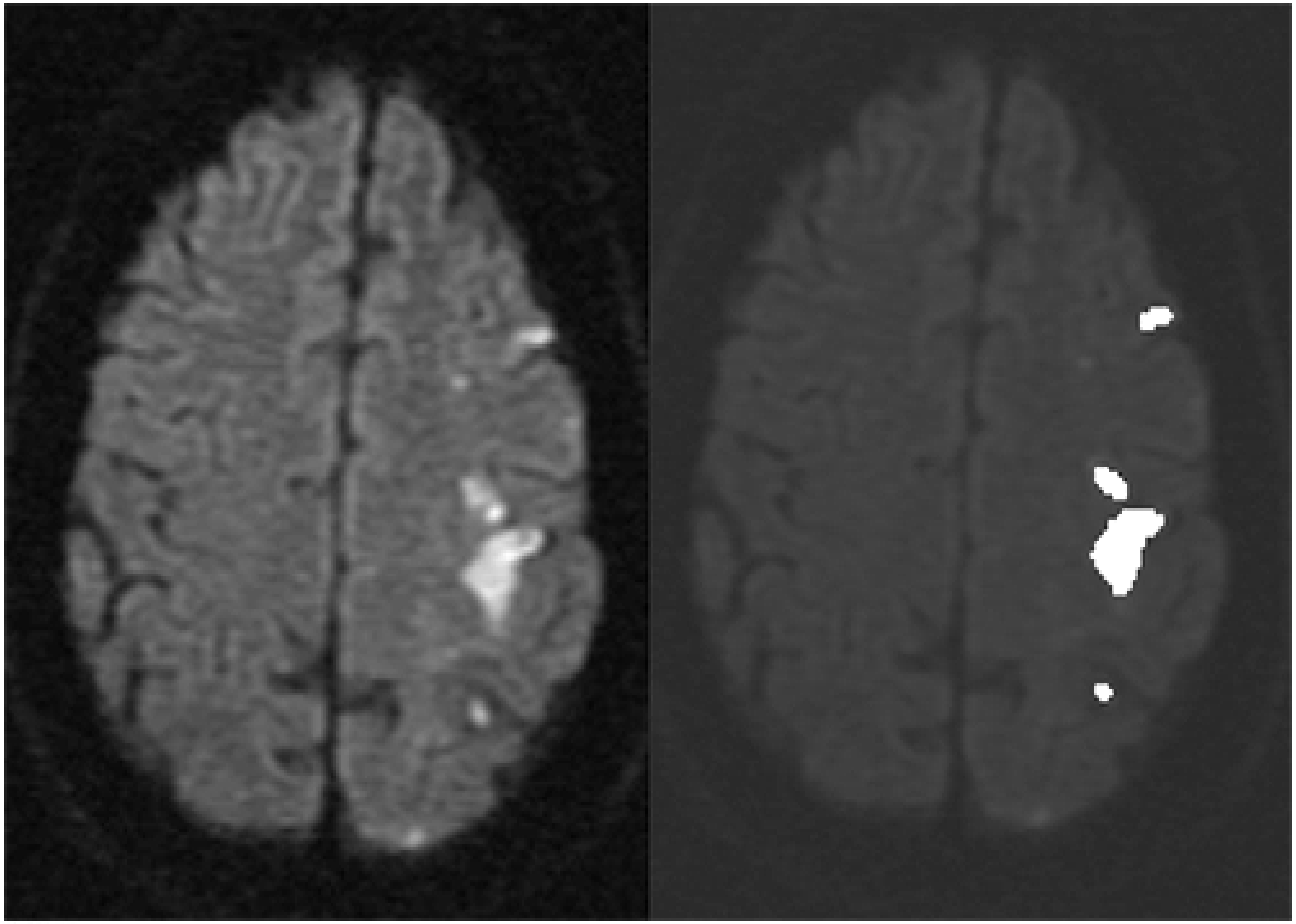
Region Growing in a Diffusion Weighted Image



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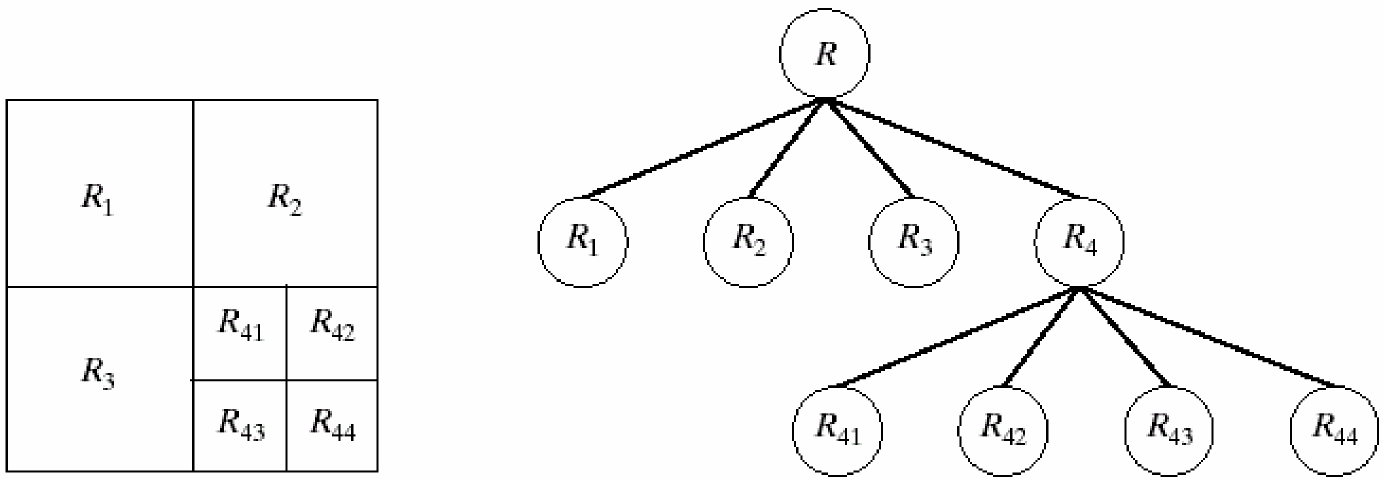
Region Splitting and Merging

- Sub-divide an image into a set of disjoint regions and then merge and/or split the regions in an attempt to satisfy the condition (P).



a b

FIGURE 10.42 (a) Partitioned image. (b) Corresponding quadtree.

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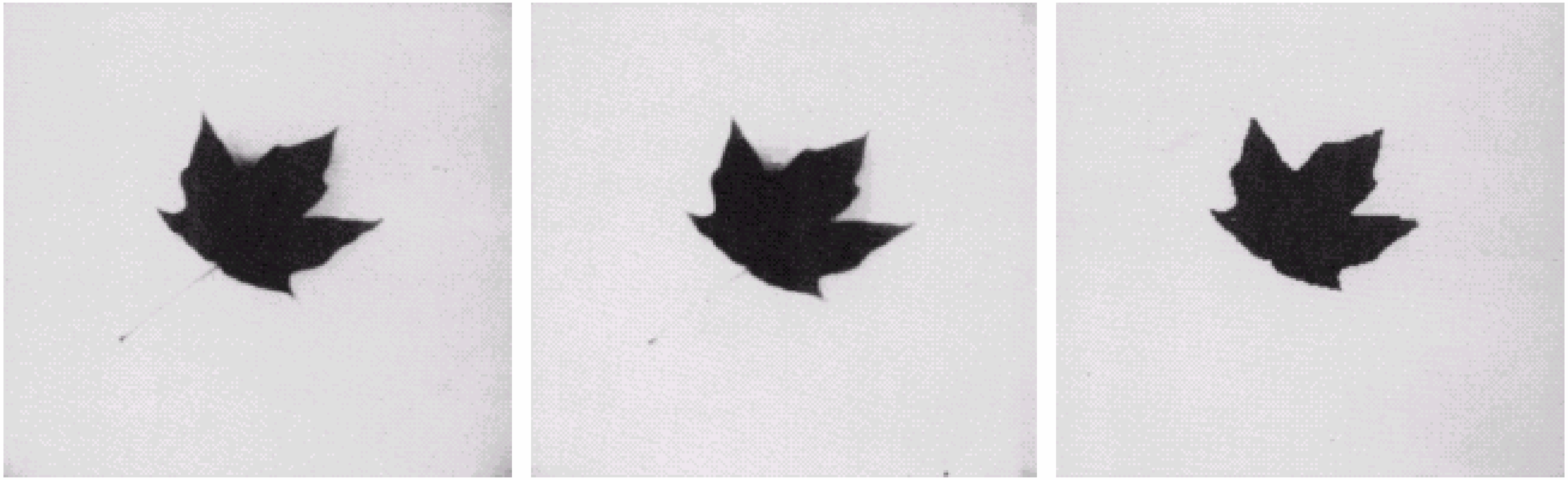
Region Splitting and Merging

- Procedure:

1. Split into 4 disjoint quadrants any region Ri where P(Ri)= False.
2. Merge any adjacent regions R_j and R_k for which $P(R_j \cup R_k)$= True.
3. Stop when no further merging or splitting is possible.

Region Splitting and Merging



a b c

FIGURE 10.43 (a) Original image. (b) Result of split and merge procedure. (c) Result of thresholding (a).

Mathematical Morphology

- Mathematical morphology involves a convolution-like process using various shaped kernels, called structuring elements
- The structuring elements are mostly symmetric: squares, rectangles, and circles
- Most common morphological operations are
  - Erosion
  - Dilation
  - Open
  - Close
- The operations can be applied iteratively in selected order to effect a powerful process

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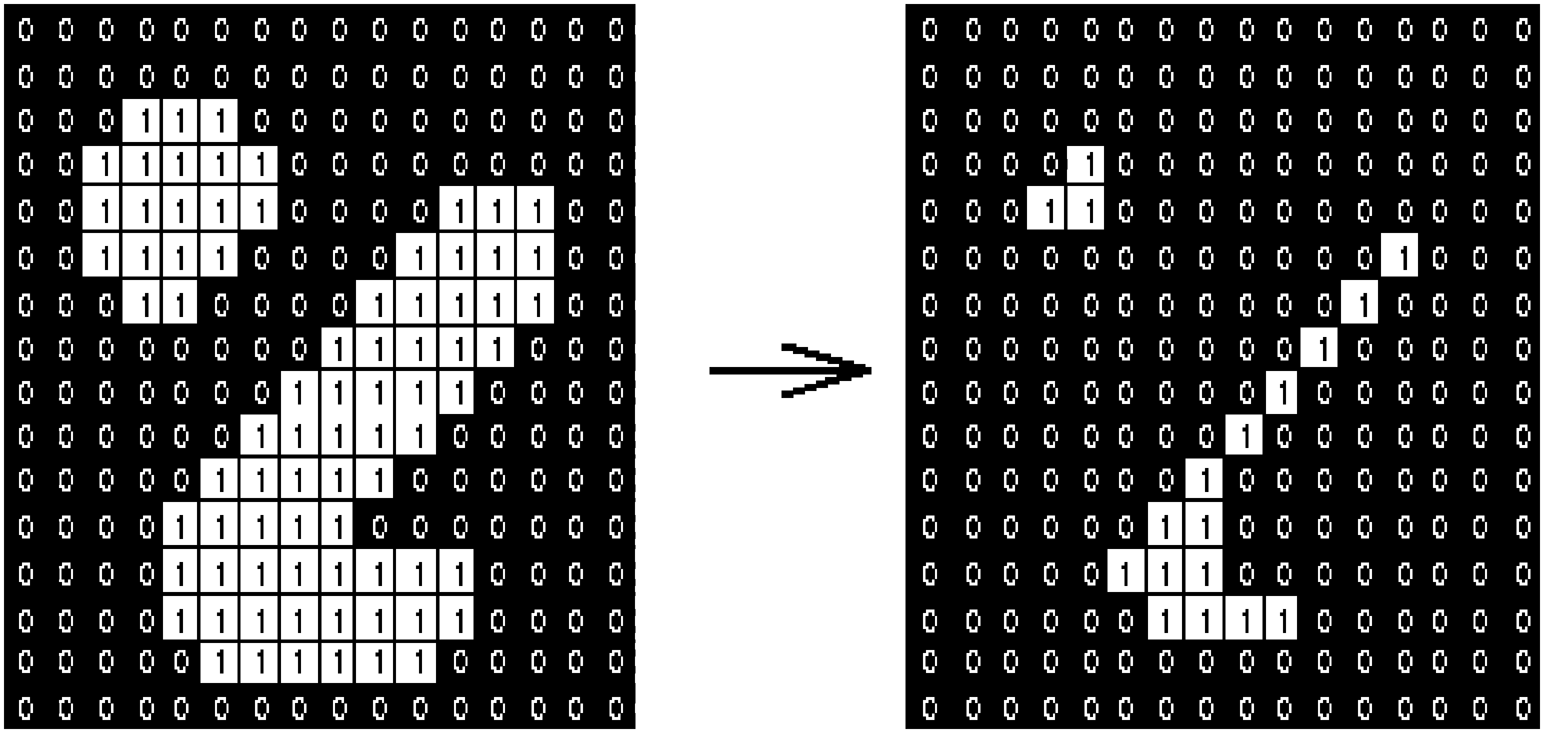
Erosion Functions

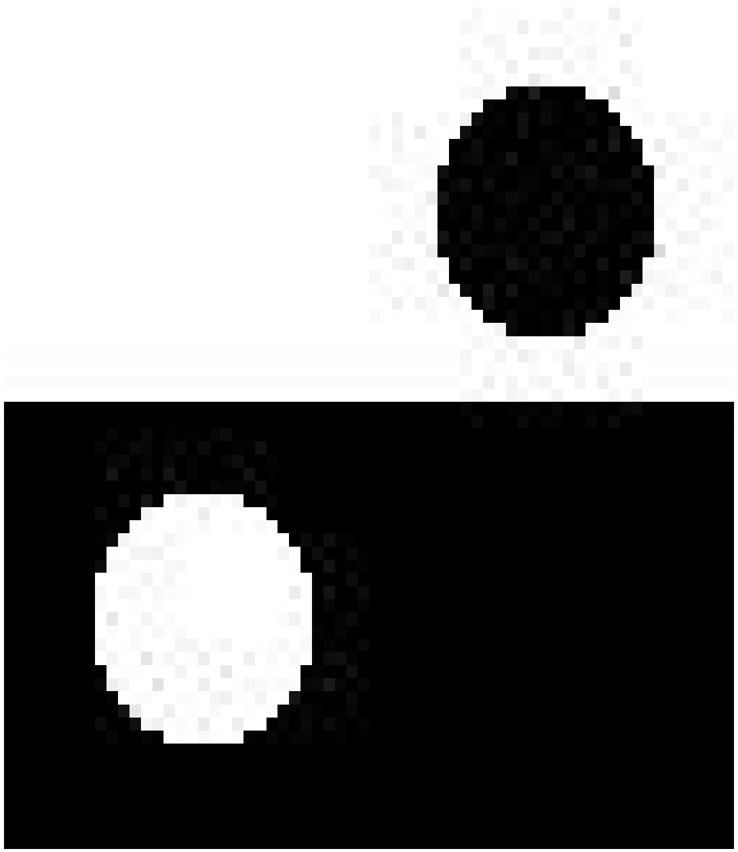
- Erosion function is a reduction operator.
- It removes noise and other small objects, breaks thin connections between objects, removes an outside layer from larger objects, and increases the size of holes within an object
- For binary images, any pixel that is 1 and has a neighbor that is 0, is set to 0
- The minimum function is the equivalence of an erosion
- The neighbors considered are defined by the structuring element

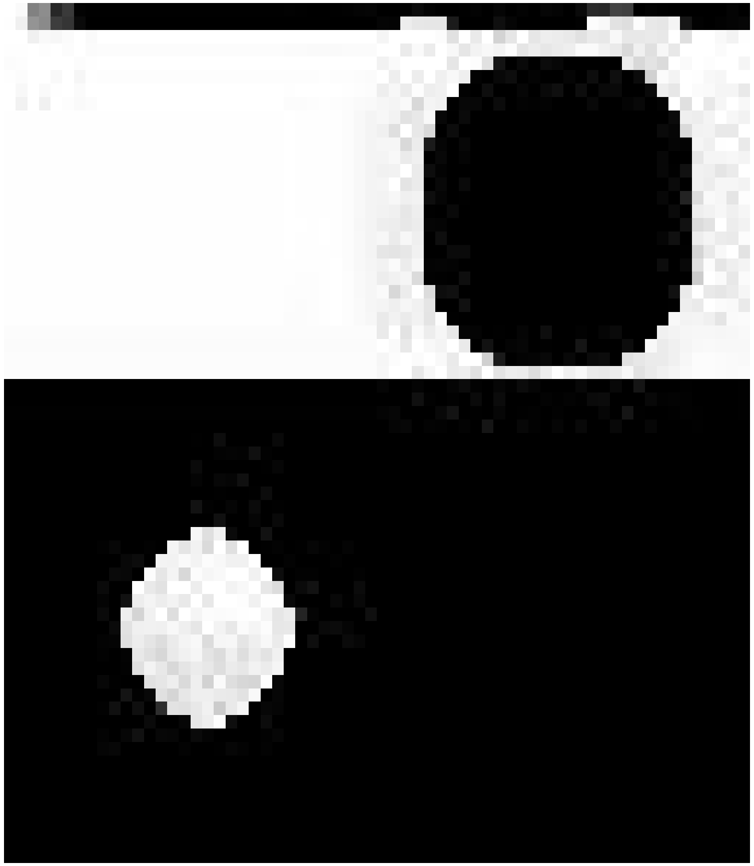
Illustration of Erosion Function



Example of Erode Function

Erosion Example





Dilation Function

- The dilation function is an enlargement operator, the reverse of erosion
- For a binary data set, any 0 pixel that has a 1 neighbor, where the neighborhood is defined by the structuring element, is set to 1
- For gray scale data, the dilation is a maximum function
- The dilation fills small holes and cracks and adds layers to objects in a binary image

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Example of Dilation

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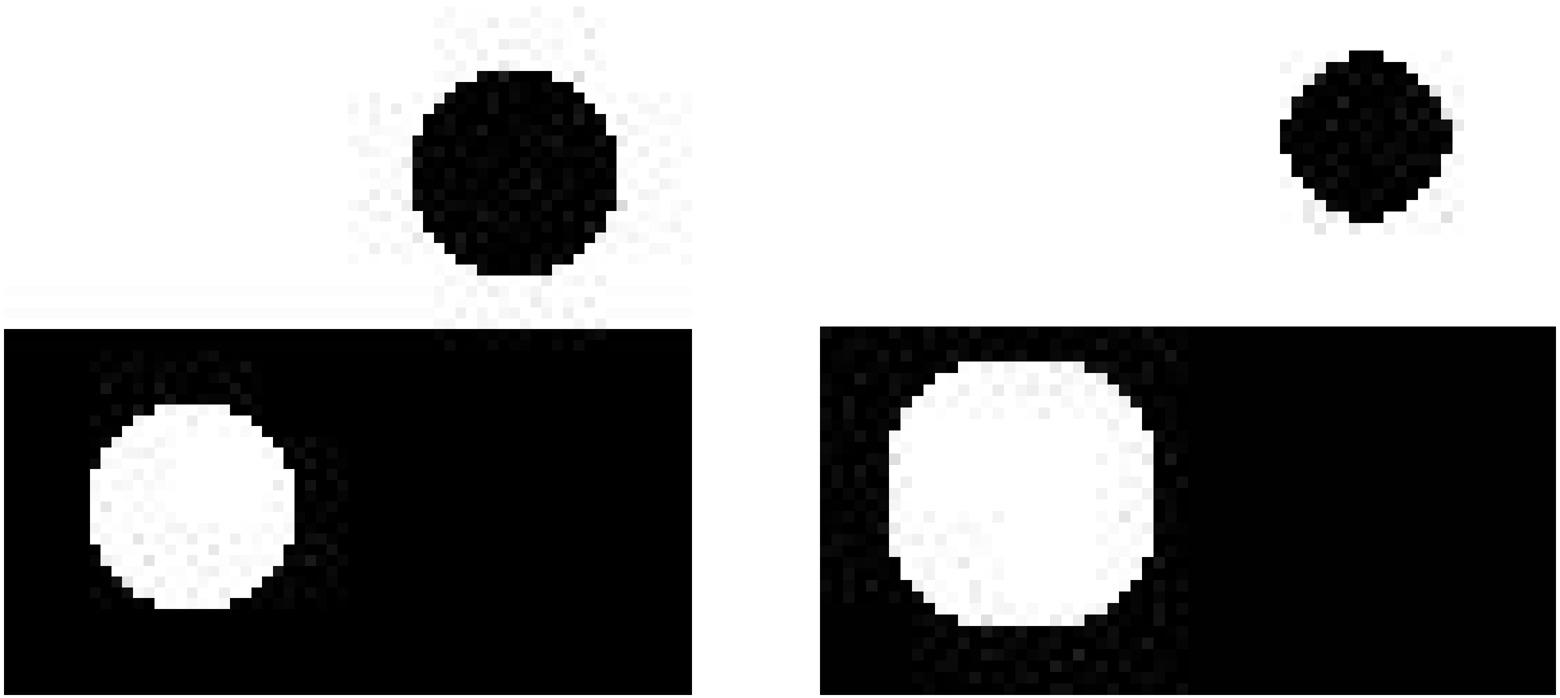
Dilated Image



Erosion – Dilation Functions

- Erosion and dilation are essentially inverse operations, they are often applied successively to an image volume
- An erosion followed by a dilation is called an **open**
- A morphological open will delete small objects and break thin connections without loss of surface layers
- A dilation followed by an erosion is called **close**
- The close operation fills small holes and cracks in an object and tends to smooth the border of an object

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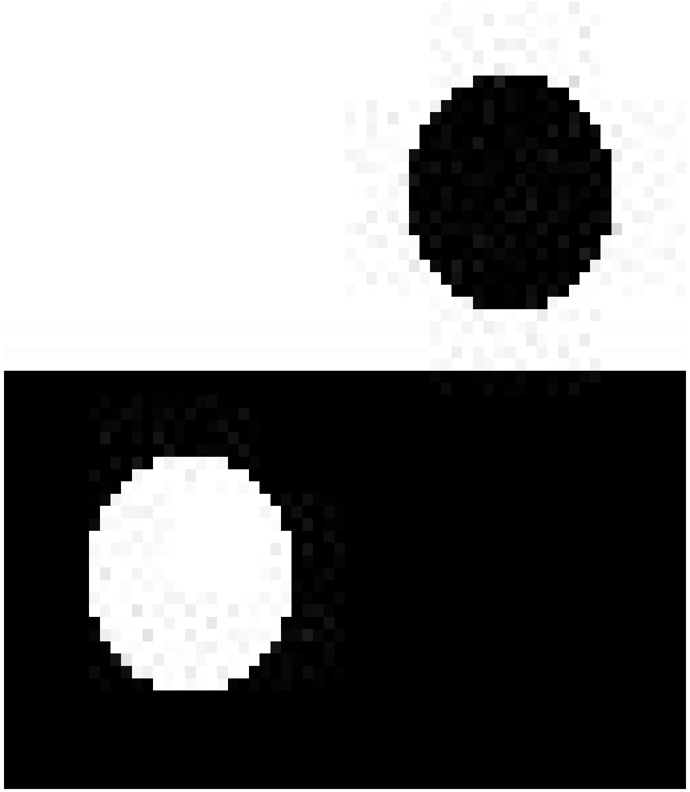
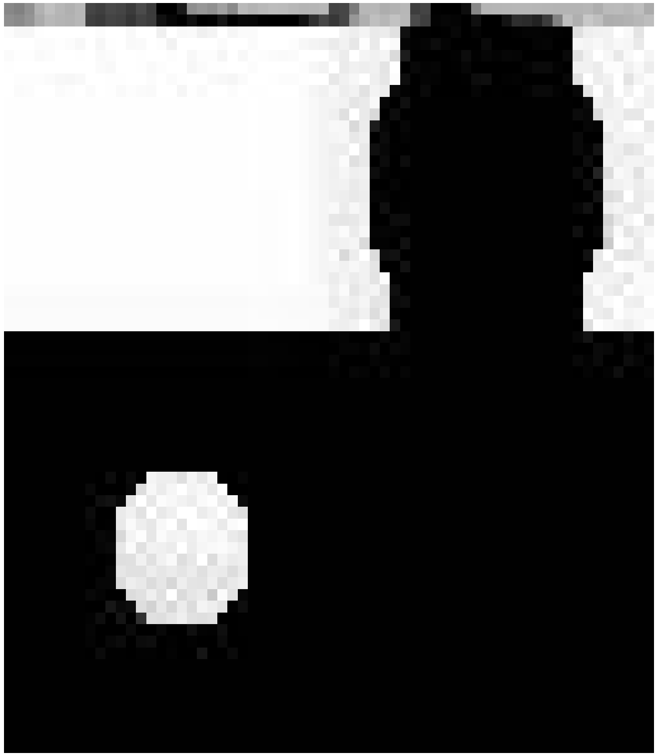
Example of Open Operation

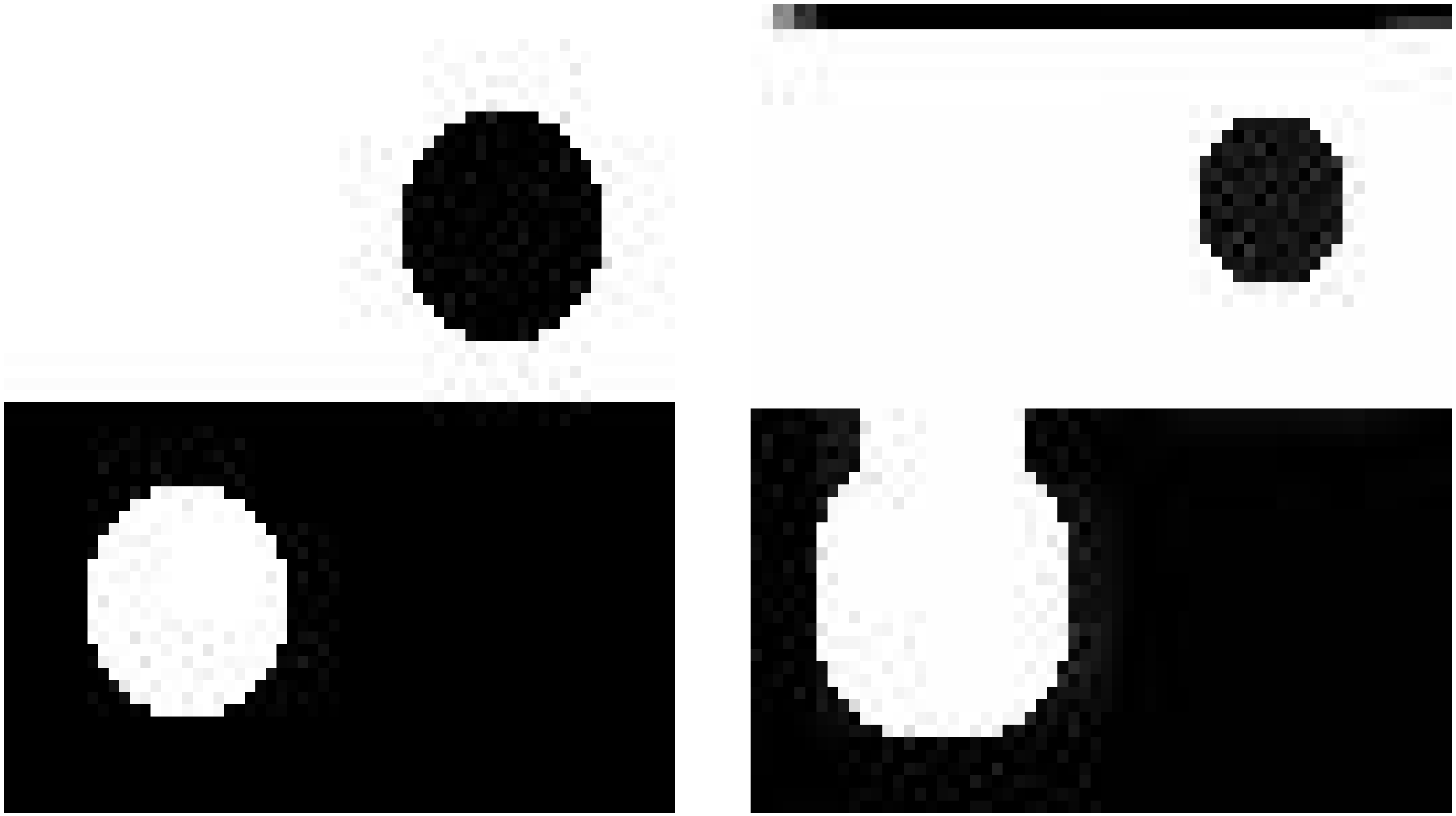
Example of Open Operation



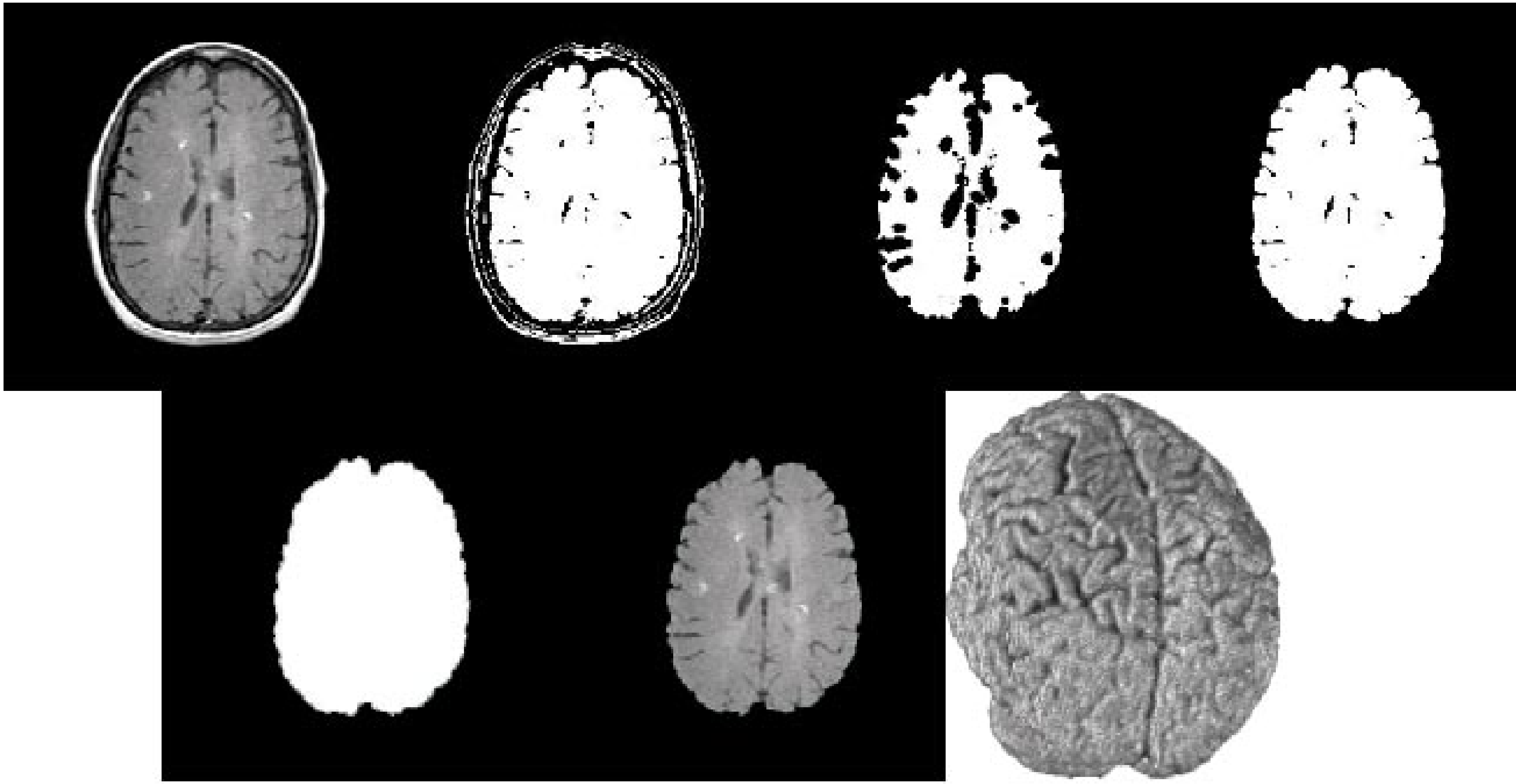


Example of Close Operation


Example of Close Operation



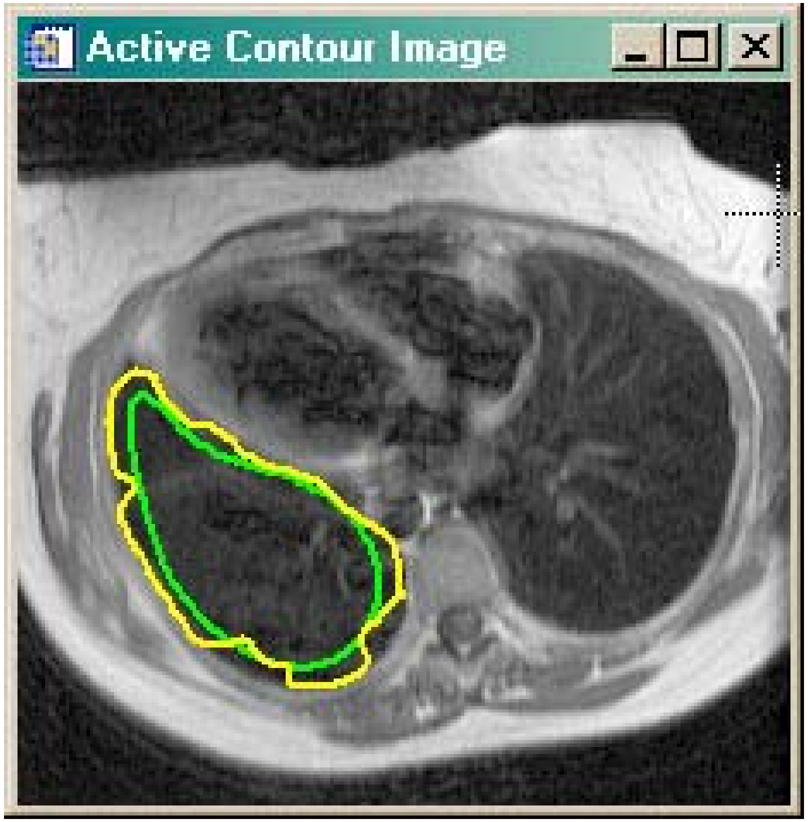
An Automated Segmentation

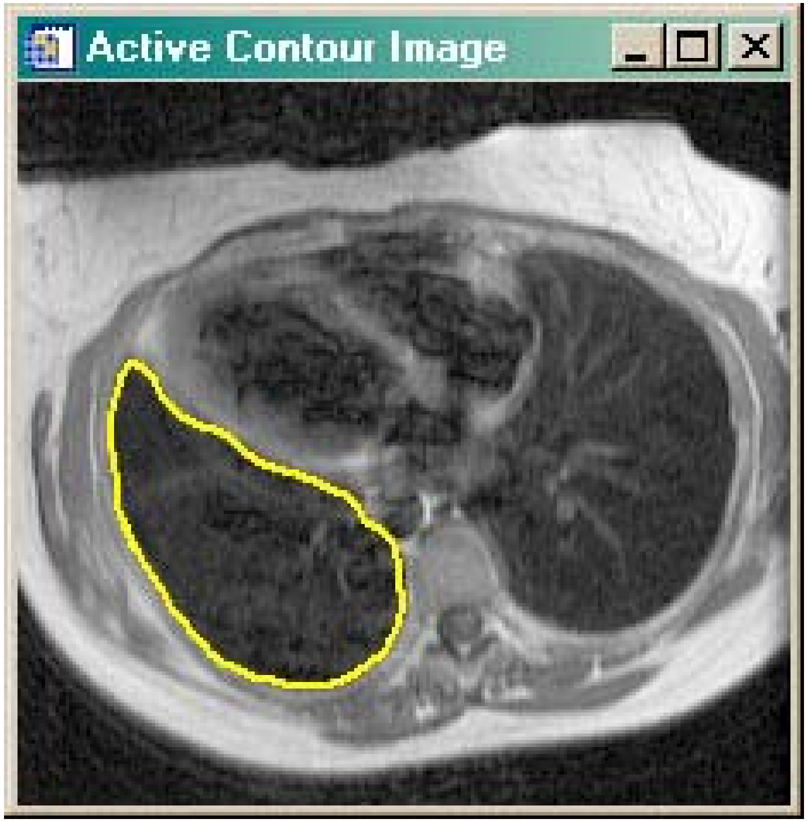


Active Contours (Snakes)

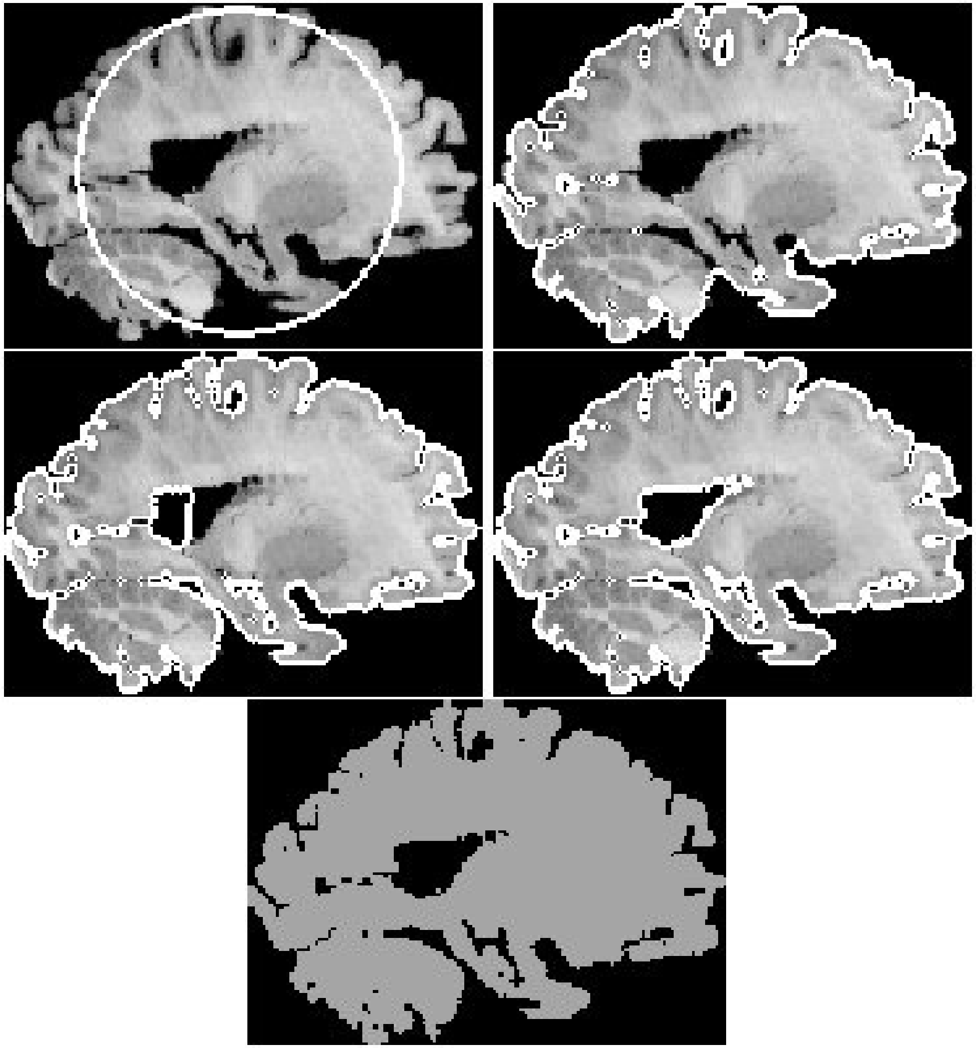
- Segmenting an object in an image with active contours involves minimizing a cost function based on certain properties of the desired object boundary and contrast in the image
- Smoothness of the boundary curve and local gradients in the image are usually considered
- Snake algorithms search the region about the current point and iteratively adjust the points of the boundary until an optimal, low cost boundary is found
- It may get caught in a local minimum (initial guess)

Example of A Snake Algorithm





Active Contour with Level-Set Method



End of Lecture

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