CoE4TN3
Image Processing

Image Enhancement in the Spatial Domain

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Image Enhancement

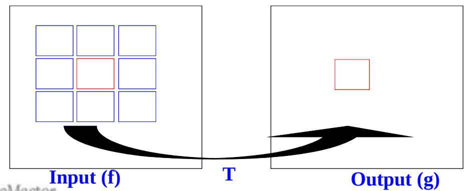
- Enhancement: to process an image so that the result is more suitable than the original image for a <u>specific</u> application.
- Enhancement approaches:
  1. Spatial domain
  2. Frequency domain

Image Enhancement

- Spatial domain techniques are techniques that operate directly on pixels.
- Frequency domain techniques are based on modifying the Fourier transform of an image.

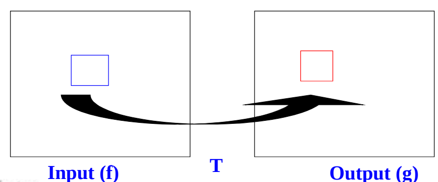
Spatial Domain: Background

- g(x,y)=T[f(x,y)]
  - f(x,y): input image, g(x,y): processed image
  - T: an operator



Spatial Domain: Point Processing

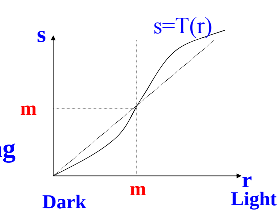
- s=T(r)
- r: gray-level at (x,y) in original image f(x,y)
- s: gray-level at (x,y) in processed image g(x,y)
- T is called gray-level transformation or mapping



Spatial Domain: Point Processing

- Contrast Stretching: to get an image with higher contrast than the original image
- The gray levels below m are darkened and the levels above m are brightened .

Contrast Stretching



Spatial Domain: Point Processing

Contrast Stretching

Original

Enhanced

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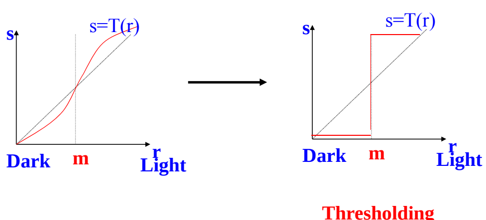
Spatial Domain: Point Processing

- Limiting case: produces a binary image (two level) from the input image



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Spatial Domain: Point Processing

Contrast Stretching: Thresholding

Original

Enhanced

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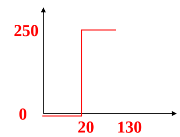
Exercise

- A 4x4 image is given as follow.

1) The image is transformed using the point transform shown. Find the pixel values of the output image.

17	64	128	128
15	63	132	133
11	60	142	140
11	60	142	138



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Gray-level Transforms

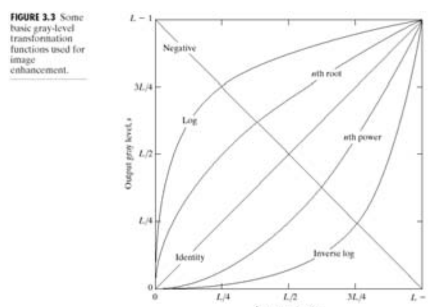

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Image Negative

- Suited for enhancing white detail embedded in dark regions.
- Has applications in medical imaging.

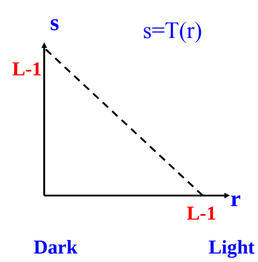

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Image Negative

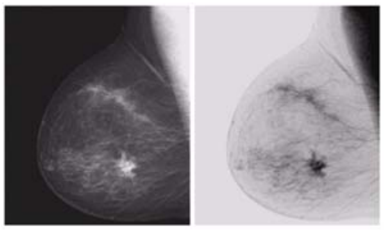

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

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Log Transformation

$$s = c\log(1 + r)$$

- Log transformation: maps a narrow range of low gray-level input image into a wider range of output levels.
- Expand the values of dark pixels in an image while compressing the higher-level values.

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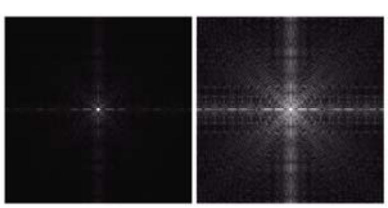
Log Transformation

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



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Power-law Transformation

$$s = cr^{\gamma}$$

- If γ <1: maps a narrow range of dark input values into a wider range of output values.
- If γ >1:opposite of the above effect.
- The process used to correct this power-law response phenomena is called *gamma correction*.

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Power-law Transformation

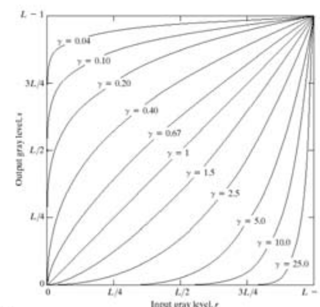

FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

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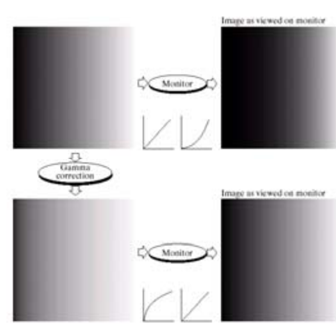
Power-law Transformation

FIGURE 3.7 (a) Linear-wedge gray-scale image. (b) Response of monitor to linear wedge. (c) Gamma-corrected wedge. (d) Output of monitor.



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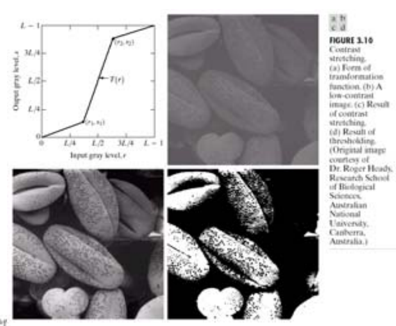
Power-law Transformation

Piecewise-Linear Transform

$$s = \begin{cases} \alpha r, & 0 \le r \le r_1 \\ \beta(r - r_1) + s_1, & r_1 \le r \le r_2 \\ \gamma(r - r_2) + s_2, & r_2 \le r \le L-1 \end{cases}$$

Contrast Stretching

Piecewise-Linear Transform

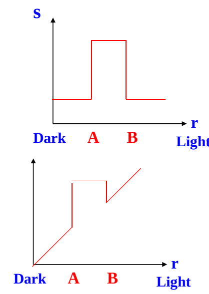


Gray-level Slicing

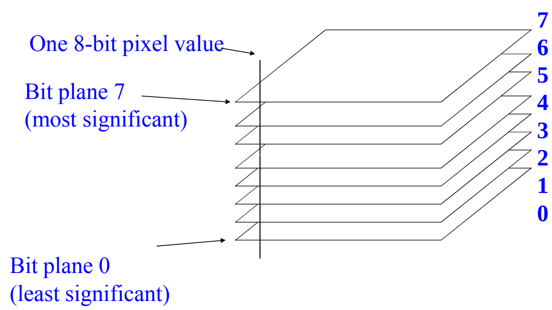
- Highlights a specific range of gray-levels in an image

2 basic methods:

1. Display a high value for all gray levels in the range of interest and a low value for all other
2. Brighten the desired range of gray levels but preserve the gray level tonalities



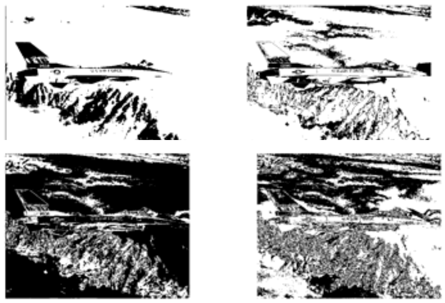
Bit-plane Slicing



Bit-plane Slicing

Original Image

Bit-plane Slicing



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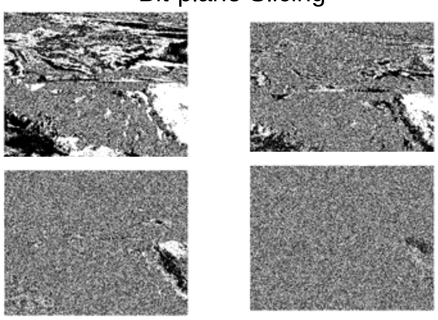
Bit-plane Slicing



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Bit-plane Slicing

- Higher order bit planes of an image carry a significant amount of visually relevant details.
- Lower order planes contribute more to fine (often imperceptible) details.

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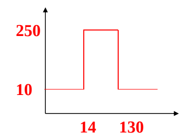
Exercise

- A 4x4 image is given as follow.

1) The image is transformed using the point transform shown. Find the pixel values of the output image.
2) What is the 7-th bit plane of this image

17	64	128	128
15	63	132	133
11	60	142	140
11	60	142	138



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Histogram Processing

- Histogram is a discrete function formed by counting the number of pixels that have a certain gray level in the image .
- In an image with gray levels in [0,L-1], the histogram is given by $p(r_k)= n_k/n$ where:
  - r_k is the k th gray level, k=0, 1, 2, …, L-1
  - n_k number of pixels in the image with gray level r_k
  - n total number of pixels in the image
- Loosely speaking, $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k.

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Histogram Processing

$$p_r(r_k) = \frac{n_k}{n}$$

- Problem: an image with gray levels between 0 and 7 is given below. Find the histogram of the image

1	6	2	2
1	3	3	3
4	6	4	0
1	6	4	7

0: 1/16	**4: 3/16**
1: 3/16	**5: 0/16**
2: 2/16	**6: 3/16**
3: 3/16	**7: 1/16**

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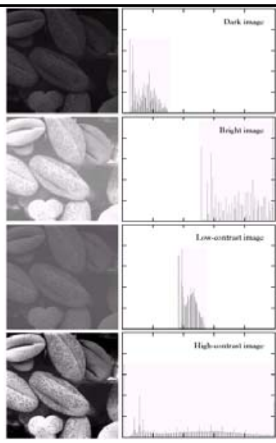

a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

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Histogram Equalization

- Goal: find a transform s=T(r) such that the transformed image has a flat (equalized) histogram.
- A) T(r) is signle-valued and monotonically increasing in interval [0,1];
- B) 0≤T(r) ≤1 for 0 ≤r ≤1.

Histogram Equalization

$$p_r(r_k) = n_k / n \quad k = 0,1,2,\ldots,L-1$$

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$

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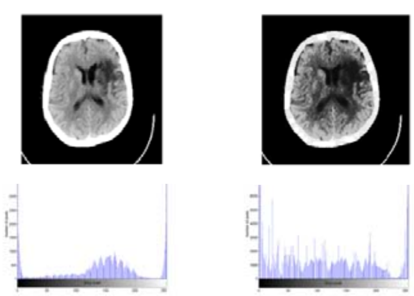
Histogram Equalization (HE): Example 1



Before HE **After HE**

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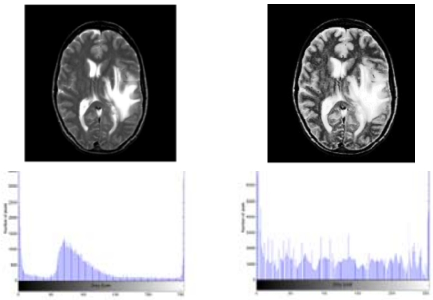
Histogram Equalization (HE): Example 2



Before HE **After HE**

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Local Histogram Processing

- Transformation should be based on gray-level distribution in the neighborhood of every pixel.
- Local histogram processing:
  - At each location the histogram of the points in the neighborhood is computed and a histogram equalization or histogram specification transformation function is obtained
  - The gray level of the pixel centered in the neighborhood is mapped
  - The center of the neighborhood is moved the next pixel and the procedure repeated

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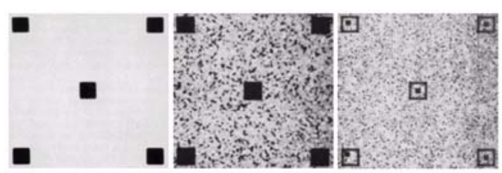
Local Histogram Processing



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

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Local Enhancement

- Mean of gray levels in an image: a measure of darkness, brightness of the image.
- Variance of gray levels in an image: a measure of average contrast.

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i)$$

$$\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 \, p(r_i)$$

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Local Enhancement

- Local mean and variance

$$m_{S_{xy}} = \sum_{(s,t)\in Sxy} r_{s,t} \, p(r_{s,t}) \qquad \sigma^2{}_{Sxy} = \sum_{(s,t)\in Sxy} (r_{s,t} - m_{Sxy})^2 \, p(r_{s,t})$$

- Local Enhancement

$$g(x,y) = \begin{cases} E \cdot f(x,y) & m_{S_{xy}} \le k_0 M_G, k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & otherwise \end{cases}$$

M_G : *Global mean* $\quad D_G$: *Global variance*

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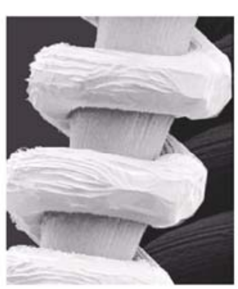
Local Enhancement

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).



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Local Enhancement

a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

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Local Enhancement

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

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Enhancement Using Arithmetic/Logic Operations

- Arithmetic/Logic operations are performed on the pixels of two or more images.
- Arithmetic: p and q are the pixel values at location (x,y) in first and second images respectively
  - Addition: p+q
  - Subtraction: p-q
  - Multiplication: p.q
  - Division: p/q

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Logic Operations

- When dealing with logic operations on gray-level images, pixel values are processed as strings of binary numbers.
- AND, OR, COMPLEMENT (NOT)

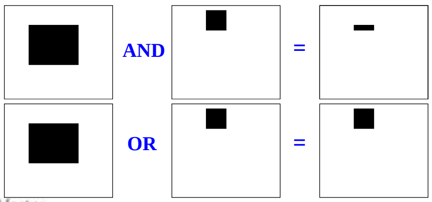
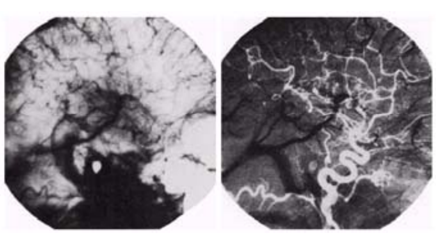


Image Subtraction

g(x,y)=f(x,y)-h(x,y)

- Example: imaging blood vessels and arteries in a body. Blood stream is injected with a dye and X-ray images are taken before and after the injection
  - f(x,y): image after injecting a dye
  - h(x,y): image before injecting the dye
- The difference of the 2 images yields a clear display of the blood flow paths.

Image Subtraction



a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

Image Averaging

K noisy observation images

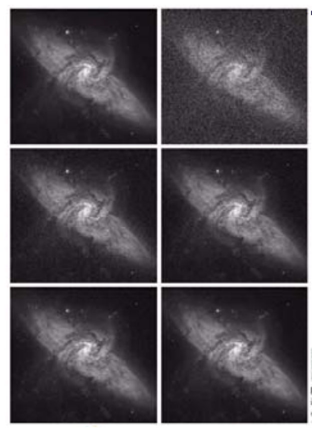
$$g_i(x,y) = f(x,y) + \eta_i(x,y)$$

Averaging

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

We have

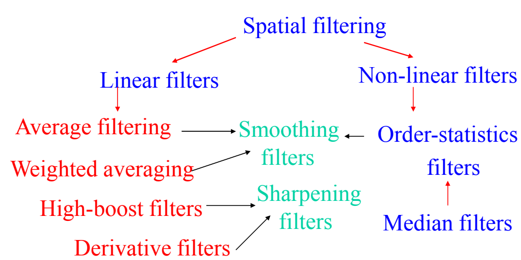
$$E\{\bar{g}(x,y)\} = f(x,y) \qquad \sigma^2_{\bar{g}(x,y)} = \frac{1}{M}\sigma^2_{\eta(x,y)}$$



a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64$, and 128 noisy images. (Original image courtesy of NASA.)

Spatial Filtering



Spatial Filtering

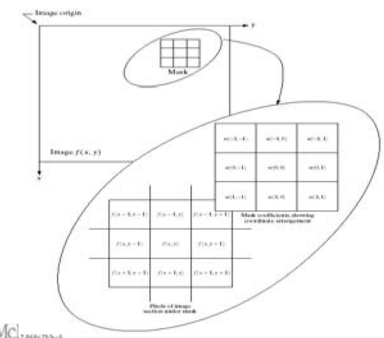


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Spatial Filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$w(s,t):(2a+1)\times(2b+1)\ mask$$

Filtering by using mask *w(s,t):* convolution

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$

Smoothing Filters

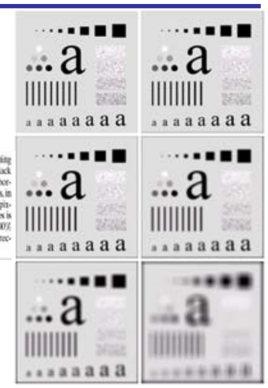
$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)}$$

$$\frac{1}{9}\times\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\qquad\frac{1}{16}\times\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

a b

FIGURE 3.34 Two 3 × 3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

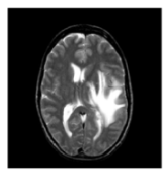
Example 1



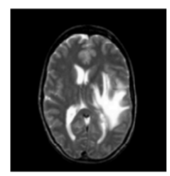
a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)-(f) Results of smoothing with square averaging filter masks of sizes n = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Example 2



1/16	2/16	1/16
2/16	4/16	2/16
1/16	2/16	1/16



Before smoothing **After smoothing**

Median Filtering

- Median filtering is particularly effective in the presence of impulse noise (salt and pepper noise).
- Unlike average filtering, median filtering does not blur too much image details.
- Example: Consider the example of filtering the sequence below using a 3-pt median filter:
  16 14 15 12 2 13 15 52 51 50 49
- The output of the median filter is:
  15 14 14 12 12 13 15 51 51 50 50

Median Filtering

- Advantages:
  - Removes impulsive noise
  - Preserves edges
- Disadvantages:
  - poor performance when # of noise pixels in the window is greater than 1/2 # in the window
  - poor performance with Gaussian noise

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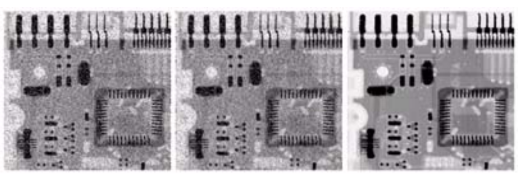
Median Filtering: Example 1



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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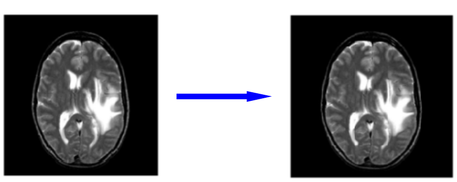
Median Filtering: Example 2



Before smoothing **After smoothing**

The smoothed MR brain image obtained by using median filtering over a fixed neighborhood of 3x3 pixels.

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Sharpening Filters

- Objective: highlight fine detail in an image or to enhance detail that has been blurred.
- First and second order derivatives are commonly used for sharpening:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

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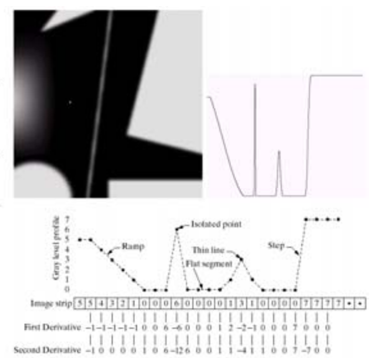
a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



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Sharpening Filters

1. First-order derivatives generally produce thicker edges in an image.
2. Second order derivatives have stronger responses to fine details such as thin lines and isolated points.
3. Second order derivates produce a double response at step changes in gray level.
4. For image sharpening, second order derivative has more applications because of the ability to enhance fine details.

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Laplacian

- The filter is expected to be isotropic: response of the filter is independent of the direction of discontinuities in an image.
- Simplest 2-D isotropic second order derivative is the Laplacian:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

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Laplacian

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

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Laplacian Enhancement

- Image background is removed by Laplacian filtering.
- Background can be recovered simply by adding original image to Laplacian output:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \textit{Laplacian mask center is negative} \\ f(x,y) + \nabla^2 f(x,y) & \textit{Laplacian mask center is positive} \end{cases}$$

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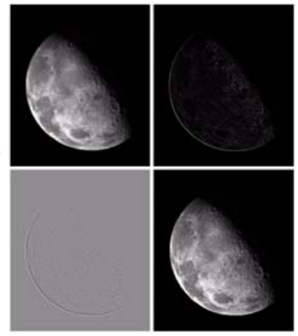
Laplacian



a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

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Laplacian

- Subtracting the Laplacian filtered image from the original image can be represented as:

G(x,y)= f(x,y)-[f(x+1,y)+ f(x-1,y)+ f(x,y+1)
+f(x,y-1)]+ 4f(x,y)
= 5f(x,y)-[f(x+1,y)+ f(x-1,y)+ f(x,y+1)
+ f(x,y-1)]

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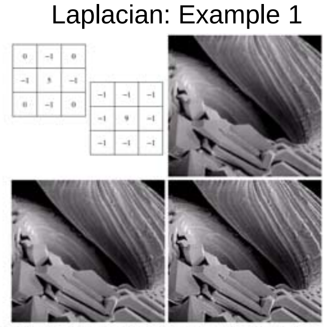
Laplacian: Example 1



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

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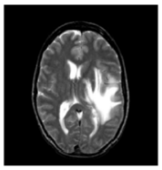
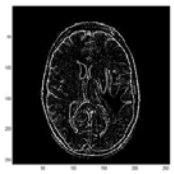
Laplacian: Example 2



0	-1	0
-1	**8**	-1
0	-1	0



Before filtering → **After filtering**

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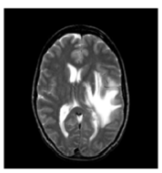
Laplacian: Example 2



-1	-1	-1
-1	**9**	-1
-1	-1	-1

Before filtering → **After filtering**

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Unsharp masking & High-boost filtering

- Unsharp masking: $f_s(x,y) = f(x,y) - \bar{f}(x,y)$

  $\bar{f}(x,y)$: blurred version of original image

- High-boost filtering:

$$f_{hb}(x,y) = Af(x,y) - \bar{f}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - \bar{f}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f_s(x,y)$$

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High-boost Filtering with Laplacian

0	−1	0
−1	$A+4$	−1
0	−1	0

−1	−1	−1
−1	$A+8$	−1
−1	−1	−1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

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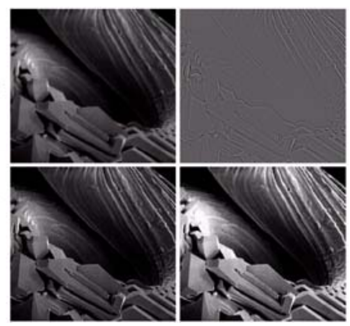
High-boost Filtering

a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (b) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



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Exercises

- A 4x4 image is given as follow.

1) Suppose that we want to process this image by replacing each pixel by the difference between the pixels to the top and bottom. Give a 3x1 mask that performs this.
2) Apply the mask to the second row of the image
3) Design a mask that can detect vertical edges, and process the following image with this mask.

17	64	128	128
15	63	132	133
11	60	142	140
11	60	142	138

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