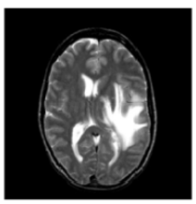
CoE4TN3
Image Processing

Image Enhancement in Frequency Domain

McMaster University

Image Enhancement



Original image

Enhanced image

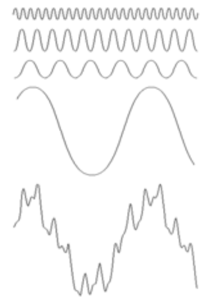
Enhancement: to process an image for more suitable output for a specific application.

Image Enhancement

- Image enhancement techniques:
  - Spatial domain methods
  - Frequency domain methods
- Spatial (time) domain techniques are techniques that operate directly on pixels.
- Frequency domain techniques are based on modifying the Fourier transform of an image.

Fourier Transform: a review

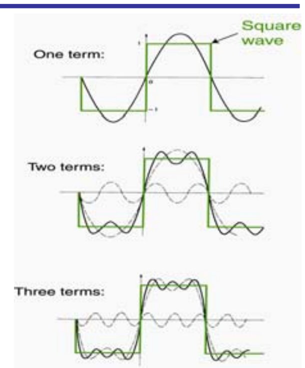
- Basic ideas:
  - A periodic function can be represented by the sum of sines/cosines functions of different frequencies, multiplied by a different coefficient.
  - Non-periodic functions can also be represented as the integral of sines/cosines multiplied by weighing function.



Joseph Fourier (1768-1830)

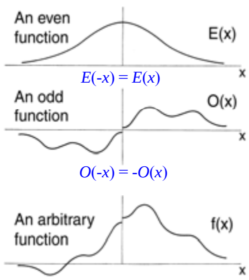
Fourier was obsessed with the physics of heat and developed the Fourier transform theory to model heat-flow problems.

Fourier transform basis functions



Approximating a square wave as the sum of sine waves.

Any function can be written as the sum of an even and an odd function



$$E(x) \equiv [f(x) + f(-x)]/2$$

$$O(x) \equiv [f(x) - f(-x)]/2$$

$$\Downarrow$$

$$f(x) = E(x) + O(x)$$

Fourier Cosine Series

Because $\cos(mt)$ is an even function, we can write an even function, *f(t)*, as:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F_m \cos(mt)$$

where series F_m is computed as

$$F_m = \int_{-\pi}^{\pi} f(t) \cos(mt)\, dt$$

Here we suppose *f(t)* is over the interval $(-\pi, \pi)$.

Fourier Sine Series

Because $\sin(mt)$ is an odd function, we can write any odd function, *f(t)*, as:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F'_m \sin(mt)$$

where the series F'_m is computed as

$$F'_m = \int_{-\pi}^{\pi} f(t) \sin(mt)\, dt$$

Fourier Series

So if $f(t)$ is a general function, neither even nor odd, it can be written:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F_m \cos(mt) + \frac{1}{\pi} \sum_{m=0}^{\infty} F'_m \sin(mt)$$

Even component — Odd component

where the Fourier series is

$$F_m = \int f(t) \cos(mt)\, dt \qquad F'_m = \int f(t) \sin(mt)\, dt$$

The Fourier Transform

Let $F(m)$ incorporates both cosine and sine series coefficients, with the sine series distinguished by making it the imaginary component:

$$F(m) = F_m - jF'_m = \int f(t) \cos(mt) dt - j \cdot \int f(t) \sin(mt) dt$$

Let's now allow *f(t)* range from $-\infty$ to ∞, we rewrite:

$$\Im\{f(t)\} = F(u) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi ut) dt$$

F(u) is called the **Fourier Transform** of *f(t)*. We say that *f(t)* lives in the "**time domain**," and ***F(u)*** lives in the "**frequency domain**." ***u*** is called the frequency variable.

The Inverse Fourier Transform

We go from *f(t)* to *F(u)* by

$$\Im\{f(t)\} = F(u) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi ut) dt$$

Fourier Transform

Given $F(u)$, *f(t)* can be obtained by the inverse Fourier transform

$$\Im^{-1}\{F(u)\} = f(t) = \int_{-\infty}^{\infty} F(u) \exp(j2\pi ut) du$$

Inverse Fourier Transform

2-D Fourier Transform

Fourier transform for *f(x,y)* with two variables

$$\Im\{f(x,y)\} = F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp(-j2\pi(ux+vy))dxdy$$

and the inverse Fourier transform

$$\Im^{-1}\{F(u,v)\} = f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\exp(j2\pi(ux+vy))dudv$$

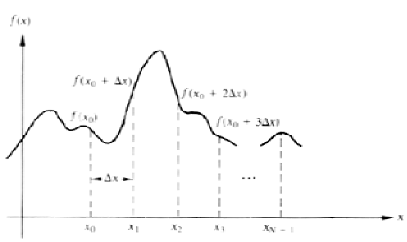
McMaster University

13

Discrete Fourier Transform (DFT)

- A continuous function f(x) is discretized as:

$$\{f(x_0), f(x_0+\Delta x), f(x_0+2\Delta x), ..., f(x_0+(M-1)\Delta x)\}$$



McMaster University

14

Discrete Fourier Transform (DFT)

Let x denote the discrete values (x=0,1,2,…,M-1), i.e.

$$f(x) = f(x_0 + x\Delta x)$$

then

$$\{f(x_0), f(x_0+\Delta x), f(x_0+2\Delta x), ..., f(x_0+(M-1)\Delta x)\}$$

⬇

$$\{f(0), f(1), f(2), ..., f(M-1)\}$$

McMaster University

15

Discrete Fourier Transform (DFT)

- The discrete Fourier transform pair that applies to sampled functions is given by:

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\exp(-j2\pi ux/M) \quad u=0,1,2,\ldots,M-1$$

and

$$f(x) = \sum_{u=0}^{M-1} F(u)\exp(j2\pi ux/M) \quad x=0,1,2,\ldots,M-1$$

McMaster University

16

2-D Discrete Fourier Transform

- In 2-D case, the DFT pair is:

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp(-j2\pi(ux/M+vy/N))$$

and:

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\exp(j2\pi(ux/M+vy/N))$$

McMaster University

17

Polar Coordinate Representation of FT

- The Fourier transform of a real function is generally complex and we use polar coordinates:

$$F(u,v) = R(u,v) + j\cdot I(u,v)$$

⬇ Polar coordinate

$$F(u,v) = |F(u,v)|\exp(j\phi(u,v))$$

Magnitude: $|F(u,v)| = [R^2(u,v) + I^2(u,v)]^{1/2}$

Phase: $\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$

McMaster University

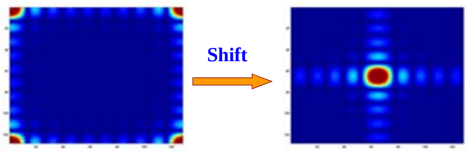
18

Fourier Transform: shift

- It is common to multiply input image by $(-1)^{x+y}$ prior to computing the FT. This shift the center of the FT to (M/2,N/2).

$$\Im\{f(x,y)\} = F(u,v)$$

$$\Im\{f(x,y)(-1)^{x+y}\} = F(u-M/2, v-N/2)$$



Symmetry of FT

- For real image f(x,y), FT is conjugate symmetric:

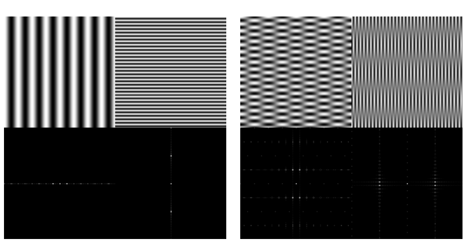
$$F(u,v) = F^*(-u,-v)$$

- The magnitude of FT is symmetric:

$$|F(u,v)| = |F(-u,-v)|$$

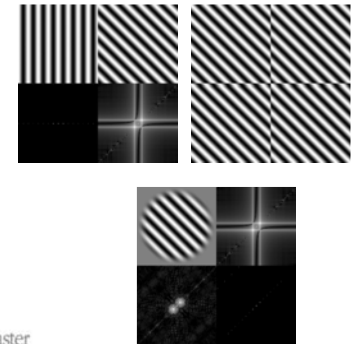
McMaster University

Simple Cases



McMaster University

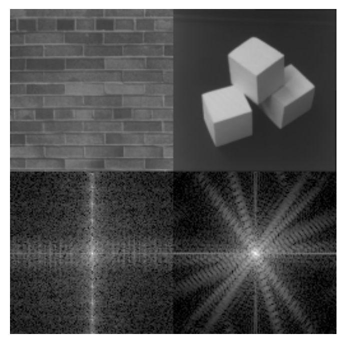
Edge Effect on FT



Periodic signal

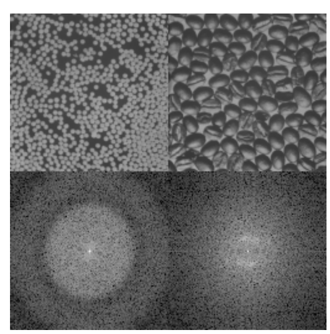
McMaster University

Edge Effect

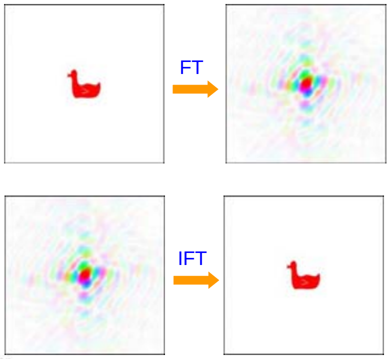


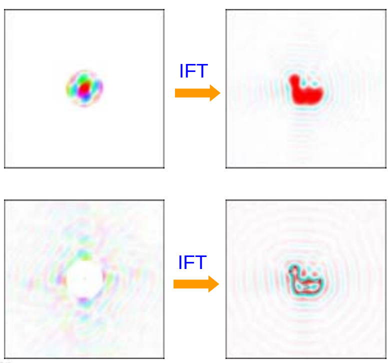
McMaster University

Magnitude (how much) & Phase (where)

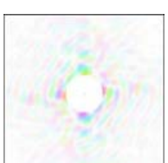


McMaster University



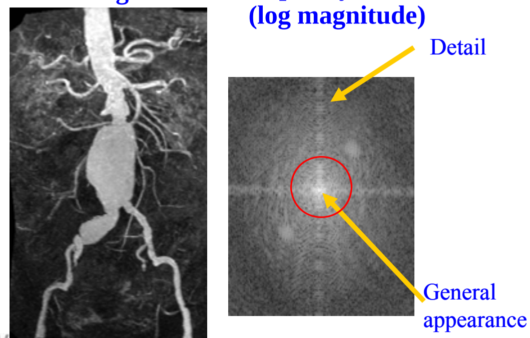


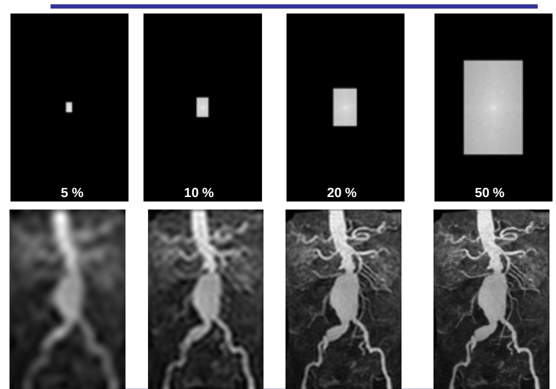
The central part of FT, i.e. the low frequency components are responsible for the general gray-level appearance of an image.



The high frequency components of FT are responsible for the detail information of an image.

Image **Frequency Domain (log magnitude)**





Frequency Domain Filtering

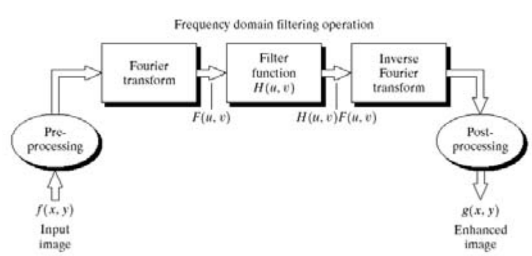


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Frequency Domain Filtering

- Edges and sharp transitions (e.g., noise) in an image contribute significantly to high-frequency content of FT.
- Low frequency contents in the FT are responsible to the general appearance of the image over smooth areas.
- Blurring (smoothing) is achieved by attenuating range of high frequency components of FT.

31

Convolution Theorem

G(u,v)=F(u,v)•H(u,v) — **Multiplication in Frequency Domain**

⇩ ⇳

g(x,y)=h(x,y)*f(x,y) — **Convolution in Time Domain**

- f(x,y) is the input image
- g(x,y) is the filtered
- h(x,y): impulse response

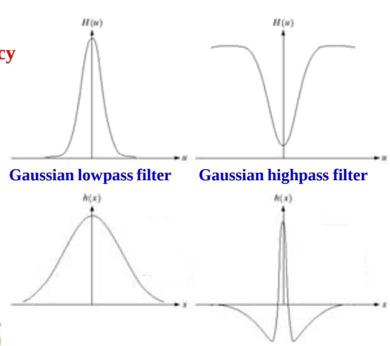
- Filtering in Frequency Domain with H(u,v) is equivalent to filtering in Spatial Domain with f(x,y).

32

Examples of Filters

Frequency domain

Spatial domain



33

Ideal low-pass filter (ILPF)

$$H(u,v)=\begin{cases}1 & D(u,v)\le D_0\\ 0 & D(u,v)>D_0\end{cases}$$

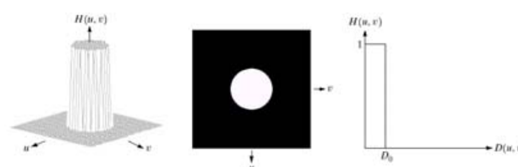
$$D(u,v)=[(u-M/2)^2+(v-N/2)^2]^{1/2}$$

(M/2,N/2): center in frequency domain

D_0 is called the ***cutoff*** frequency.

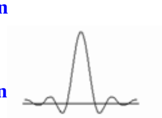
34

Shape of ILPF

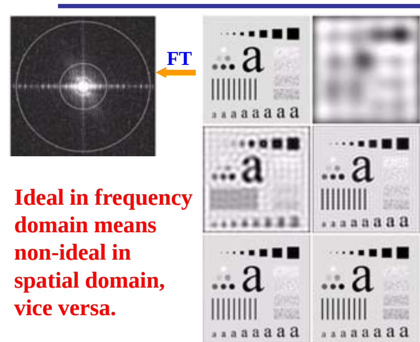


Frequency domain

Spatial domain



35



Ideal in frequency domain means non-ideal in spatial domain, vice versa.

ringing and blurring

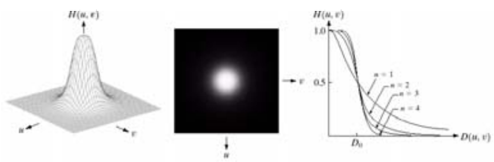
FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

36

Butterworth Lowpass Filters (BLPF)

- Smooth transfer function, no sharp discontinuity, no clear cutoff frequency.

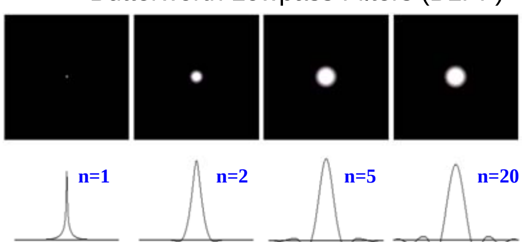
$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$



a b c

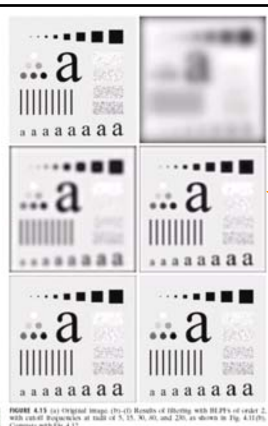
FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filters (BLPF)



$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

McMaster University



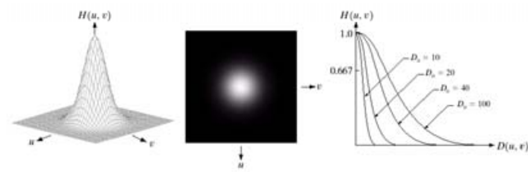
No serious ringing artifacts

McMaster University

Gaussian Lowpass Filters (GLPF)

- Smooth transfer function, smooth impulse response, no ringing

$$H(u,v)=e^{-\frac{D^2(u,v)}{2D_0^2}}$$

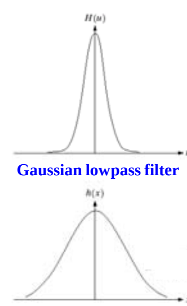


a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

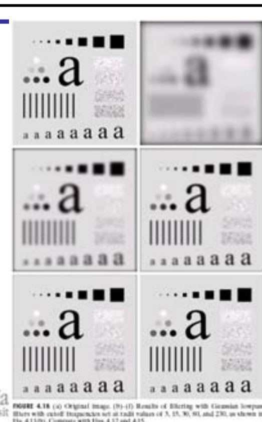
GLPF

Frequency domain



Spatial domain

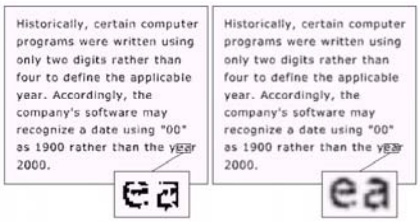
McMaster University



No ringing artifacts

McMaster University

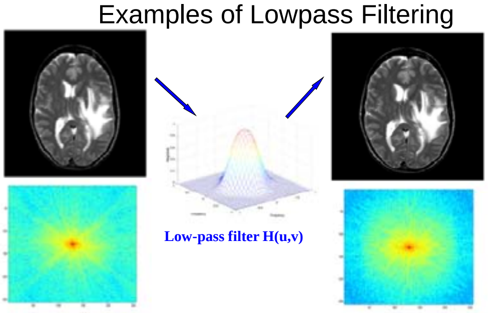
Examples of Lowpass Filtering



a b
FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

43

Examples of Lowpass Filtering

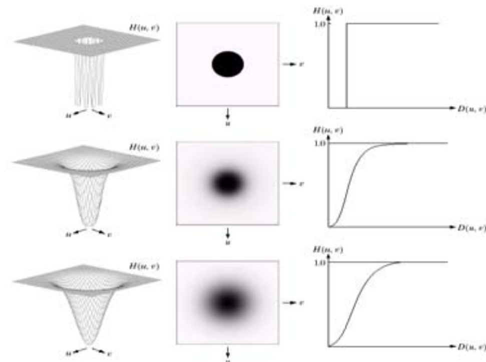


44

Sharpening High-pass Filters

- $H_{hp}(u,v)=1-H_{lp}(u,v)$
- Ideal: $H(u,v)=\begin{cases}1 & D(u,v)>D_0\\ 0 & D(u,v)\le D_0\end{cases}$
- Butterworth: $|H(u,v)|^2=\dfrac{1}{1+\left[\dfrac{D_0}{D(u,v)}\right]^{2n}}$
- Gaussian: $H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$

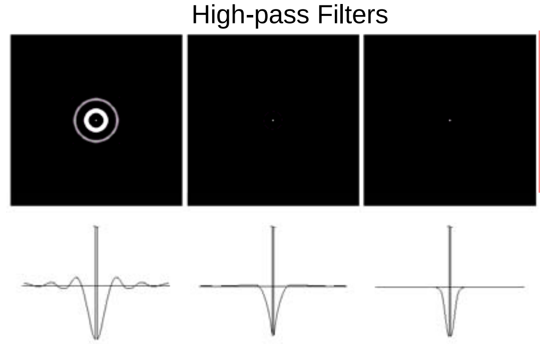
45



a b c
d e f
g h i
FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

46

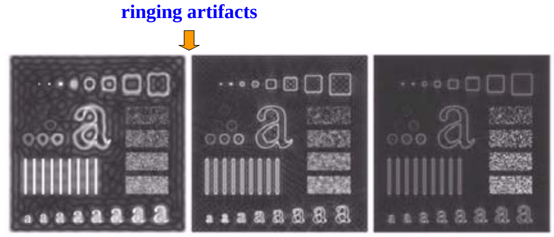
High-pass Filters



a b c
FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Ideal High-pass Filtering

ringing artifacts



a b c
FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with D_0 = 15, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

48

Butterworth High-pass Filtering

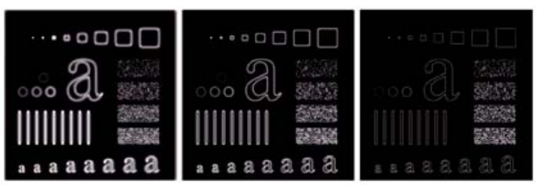
a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

McMaster University

49

Gaussian High-pass Filtering



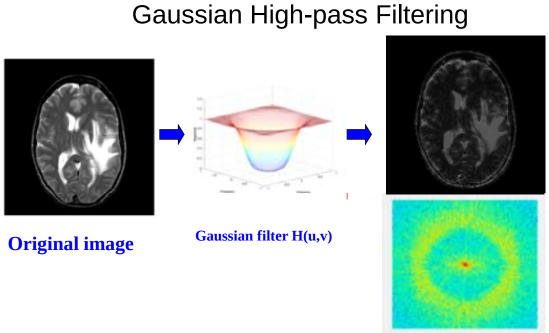
a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

McMaster University

50

Gaussian High-pass Filtering



McMaster University

51

Laplacian in Frequency Domain

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = -(u^2+v^2)F(u,v)$$

$$H_1(u,v) = -(u^2+v^2)$$

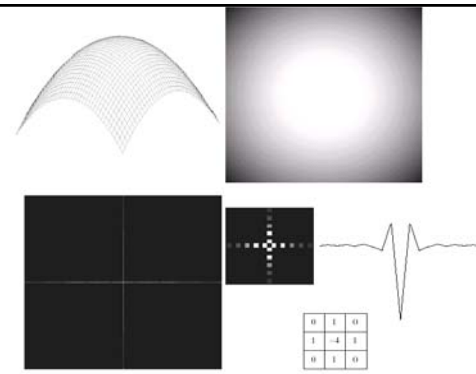
Frequency domain

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Spatial domain → **Laplacian operator**

McMaster University

52



a b
c d e
f

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

53

Subtract Laplacian from the Original Image to Enhance It

enhanced image **Original image** **Laplacian output**

Spatial domain $g(x,y) = f(x,y) - \nabla^2 f(x,y)$

Frequency domain $G(u,v) = F(u,v) + (u^2+v^2)F(u,v)$

new operator $H_2(u,v) = 1 + (u^2+v^2) = 1 - H_1(u,v)$

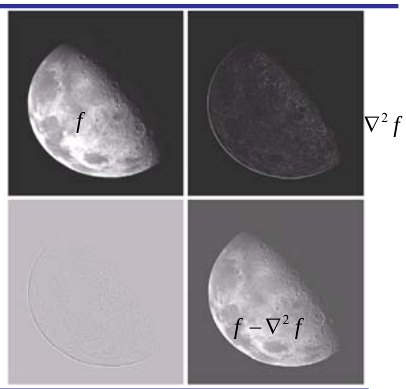
Laplacian

McMaster University

54

a b
c d

FIGURE 4.28
(a) Image of the North Pole of the moon.
(b) Laplacian filtered image.
(c) Laplacian image scaled.
(d) Image enhanced by using Eq. (4.4-12). (Original image courtesy of NASA.)



Unsharp Masking, High-boost Filtering

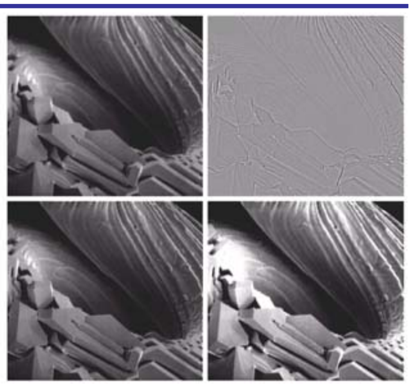
- Unsharp masking: $f_{hp}(x,y)=f(x,y)-f_{lp}(x,y)$
- $H_{hp}(u,v)=1-H_{lp}(u,v)$

- High-boost filtering: → **One more parameter to adjust the enhancement**

  $f_{hb}(x,y)=Af(x,y)-f_{lp}(x,y)$
- $f_{hb}(x,y)=(A-1)f(x,y)+f_{hp}(x,y)$
- $H_{hb}(u,v)=(A-1)+H_{hp}(u,v)$

a b
c d

FIGURE 4.29
Same as Fig. 3.43, but using frequency domain filtering. (a) Input image.
(b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



An image formation model

- We can view an image f(x,y) as a product of two components:

$$f(x,y) = i(x,y) \cdot r(x,y)$$
$$0 < i(x,y) < \infty$$
$$0 < r(x,y) < 1$$

- i(x,y): illumination. It is determined by the illumination source.
- r(x,y): reflectance (or transmissivity). It is determined by the characteristics of imaged objects.

Homomorphic Filtering

- In some images, the quality of the image has reduced because of non-uniform illumination.
- Homomorphic filtering can be used to perform illumination correction.

$$f(x,y) = i(x,y) \cdot r(x,y)$$

- The above equation cannot be used directly in order to operate separately on the frequency components of illumination and reflectance.

Homomorphic Filtering

ln : $z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

DFT : $Z(u,v) = F_i(u,v) + F_r(u,v)$

H(u,v) : $S(u,v) = H(u,v)Z(u,v)$

(DFT)$^{-1}$: $s(x,y) = i'(x,y) + r'(x,y)$

exp : $g(x,y) = e^{s(x,y)} = i_0(x,y)r_0(x,y)$

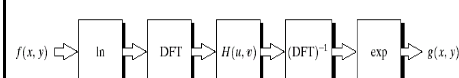


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

Homomorphic Filtering

- By separating the illumination and reflectance components, homomorphic filter can then operate on them separately.
- Illumination component of an image generally has slow variations, while the reflectance component vary abruptly.
- By removing the low frequencies (highpass filtering) the effects of illumination can be removed .

McMaster University

61

Homomorphic Filtering

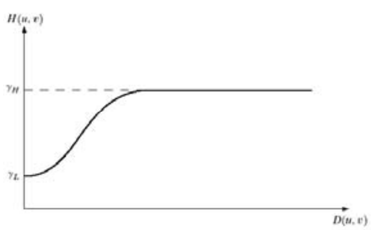


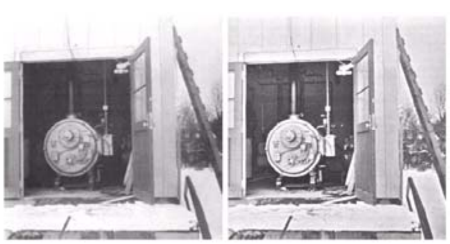
FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

McMaster University

62

Homomorphic Filtering: Example 1

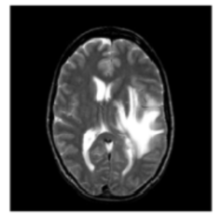
a b
FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)



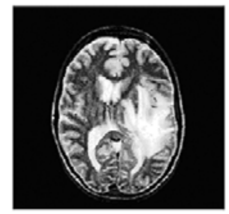
McMaster University

63

Homomorphic Filtering: Example 2



Original image



Filtered image

McMaster University

64

End of Lecture

McMaster University

65